

Quantifying spatial-averaging effects in background-oriented Schlieren measurements

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ABSTRACT

Background-oriented Schlieren (BOS) is a simple, non-contact optical measurement technique for visualizing density-gradient fields from background-image displacement. In the BOS technique, accurate estimation of the light-ray deviation angle is important. Although increasing the background-to-object distance improves sensitivity, it can degrade spatial resolution because the finite camera aperture forms a cone-shaped sampling volume on the target. For millimeter- and micrometer-scale phenomena, this spatial averaging can reduce the apparent deviation angle and distort the measured displacement field. In this study, the influence of spatial resolution on the measured deviation angle was experimentally investigated using a sinusoidal reference object. The object generated a controlled deviation-angle distribution with a wavelength of 3 mm and an amplitude of 0.2 mm. A projected BOS system was constructed, and the background-to-object distance was varied to change the spatial resolution while the displacement field was measured using fast checkerboard demodulation. The relationship between the normalized deviation angle amplitude and normalized spatial resolution was compared with theoretical models. The conventional simple moving average filter (SMAF) captures the essential trend, but quantitative discrepancies remained in the present optical configuration. To account for the circular camera aperture and the resulting two-dimensional spatial averaging, a Simple Radial Circular Moving Average Filter (SRCMAF) was introduced. The SRCMAF reduced the maximum discrepancy from approximately 31% for the SMAF to approximately 9%, demonstrating better agreement with the experiments. These results indicate that the spatial averaging effect in BOS should be evaluated relative to the wavelength of the target phenomenon. Based on the SRCMAF model, the spatial resolution should be less than approximately 20.3% of the shortest wavelength to be resolved in order to keep the deviation-angle error below 5%.

1. Introduction

Background-oriented Schlieren (BOS) is an optical visualization technique that utilizes refractive-index changes caused by variations in fluid density. The BOS technique uses a digital camera placed behind the object of observation to capture a background image and visualizes the density field by calculating the displacement of the background image caused by density changes using image processing. Furthermore, the obtained displacement allows for quantitative measurements of the density field. It has attracted considerable attention for quantitative measurements because of its

non-contact nature and the simplicity of the optical system. In recent years, BOS has also been actively applied to millimeter- and micrometer-scale measurements, including underwater ultrasound (Pulkkinen et al., 2017; Luo et al., 2020; Yamagishi et al., 2024), shock waves (Ichihara et al., 2022; Yamamoto et al., 2022; Jin et al., 2011), sparks (Kottakalam et al., 2024), and plasma (Kaneko et al., 2021; Komuro et al., 2019).

BOS is an image-based technique that uses only a background pattern and a camera, as shown in Fig. 1. The principle of BOS is based on the relationship between refractive index and density connected by the Gladstone–Dale equation (Venkatakrishnan and Meier, 2004; Raffel, 2015). The solid red and dashed red lines indicate the cases in which the light ray passes through the target and does not, respectively, as shown in Fig. 1 (a). The difference in refractive index between the target and the surrounding fluid produces a deviation angle ε (Venkatakrishnan and Meier, 2004). This deviation angle ε appears as a displacement Δh on the image sensor, as shown in Fig. 1 (a). Accurate measurement of the deviation angle ε is essential for quantitative BOS measurements. To achieve high sensitivity in the deviation angle, many studies have focused on the sensitivity factor S determined by the optical alignment (Gojani and Obayashi, 2012; Yevtikhieva et al., 2009).

$$\Delta h = S \cdot \varepsilon = M l_b \cdot \varepsilon, \quad (1)$$

where l_b is the distance between the target and the background, M is the magnification of the camera, and S is the sensitivity.

For example, Schwarz & Braukmann (Schwarz and Braukmann, 2023) proposed the optimal distance l_b for achieving a large displacement in BOS measurements of vortices. Yamagishi et al. (2024) mentioned that the distance l_b could change the range of measurable amplitudes in underwater ultrasound measurements. Gojani et al. (2013) defined the measurement uncertainties arising from the optical alignment. Sensitivity S and spatial resolution δ_t are dominant factors in measurement accuracy. The effect of “spatial resolution δ_t ”, which is the cone of light on the target, is often negligible. On the other hand, several studies have suggested measurement limitations caused by spatial resolution, when measuring periodic waves on the millimeter or micrometer scale (Pulkkinen et al., 2017; Yamagishi et al., 2024). In practice, the light ray has a finite volume δ_t , referred to as the spatial resolution, as illustrated by the pink region in Fig. 1(b) (Gojani et al., 2013). When the observed object changes periodically on the order of millimeter to micrometer, the magnitude of the spatial resolution δ_t causes a change in the apparent deviation angle ε due to the averaging effect within the ray cone extending in the line of sight. In this case, the light ray is the solid red line. The angle between the solid red and dashed red lines is smaller than $\varepsilon_{\max}$. Here, $\varepsilon_{\max}$ ideally occurs when $l_b = 0$, i.e., $\delta_t = 0$, as the light ray incident along the optical axis reaches the inflection point of the sinusoidal target, producing the maximum deviation angle. Alternatively, $\varepsilon_{\max}$ can be interpreted as the refraction angle measured by an ideal pinhole camera with an infinitesimally small aperture, as illustrated in Fig. 1. For millimeter- or micrometer-scale targets, the spatial resolution δ_t should be considered for accurate measurement of the deviation angle.

To quantify the averaging effect of spatial resolution, Yamagishi (2025) measured the response of the refraction angle to l_b using a 1 mm thick acrylic plate with sinusoidal grooves of 3 mm wavelength and 0.4 mm amplitude as the observation target. They proposed that the wavelength-normalized spatial resolution and normalized deviation angle amplitude follow a simple moving average filter (SMAF)

for sinusoidal waveforms. The sinusoidal SMAF is a sinc function normalized such that its first zero occurs at unity. This model is useful because it captures the essential dependence of the measured deviation angle on the spatial resolution relative to the object wavelength, thereby providing practical guidelines for optical system design.

In addition, in an actual BOS optical system, the finite aperture of the camera lens forms a cone of rays on the target plane, and the effective spatial averaging may depend on the aperture geometry. Since typical camera lenses have a circular aperture, the measured refraction angle may include the effect of two-dimensional circular averaging. Therefore, in addition to evaluating the applicability of the SMAF, it is useful to examine whether a circular-averaging response model can describe the experimental results more appropriately.

In this study, we examine the influence of spatial resolution on the measured angular deviation of millimeter-order targets using a sinusoidal reference object. By varying the background-to-object distance l_b and the background wavelength, we quantitatively evaluate the effect of spatial resolution on the measured displacement field. Furthermore, we compare the SMAF with a circular-averaging response model, referred to here as the Simple Radial Circular Moving Average Filter (SRCMAF), to investigate the influence of aperture geometry on the spatial-averaging response.

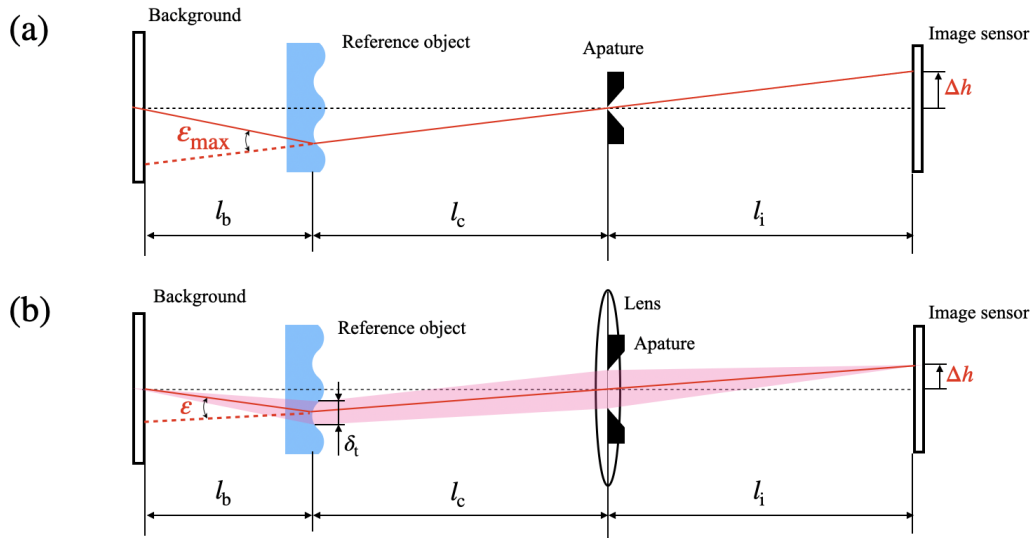


Figure 1. A schematic of BOS and the spatial averaging effect on the target.

2. Theory

2.1. Background-oriented schlieren technique

BOS uses only the background and a camera to measure the density field. The optical alignment is shown in Fig. 1. The camera is focused on the background (Venkatakrishnan, 2005; Venkatakrishnan and Meier, 2004). The density gradient in the target fluid causes the light rays to be slightly refracted.

As a result, the refraction is observed as a displacement Δh of the background pattern on the image. This relationship is expressed as follows:

$$\Delta h = \frac{l_i l_b}{l_b + l_c} \varepsilon. \quad (2)$$

Here, l_c is the distance between the target and the lens, and l_i is the distance between the lens and the image sensor. Δh is scaled by the geometrical conditions of the optical path length. The ray deviation is obtained by integrating the refractive-index gradient along the line of sight, which is expressed as

$$\varepsilon = \frac{1}{n_0} \int \frac{\partial n}{\partial r} dz, \quad (3)$$

where n is the refractive index of the observation target, n_0 is the refractive index of the surrounding fluid, r denotes the spatial coordinate in the direction of the measured displacement, and z is the coordinate along the line of sight. The Gladstone–Dale relation relates the refractive index to the density (Kottakalam et al., 2024). Using the density ρ and the Gladstone–Dale constant K , this relation is written as

$$n = \rho K + 1. \quad (4)$$

2.2. Sensitivity

In BOS, the angle of refraction appears as a displacement due to its product with l_b . If the cone of light rays is negligible, the deviation angle ε would be constant. The parameter $l_i l_b / (l_b + l_c)$ can be modified by changing the experimental conditions. Here, the magnification of the camera M is defined as $l_i / (l_b + l_c)$ in the BOS configuration. The sensitivity S is given by

$$S = M l_b. \quad (5)$$

Sensitivity can be increased by increasing the background distance l_b . However, this also reduces the spatial resolution δ_t (described below), making it difficult to achieve both high sensitivity and high spatial resolution.

2.3. Spatial resolution

In BOS, the background is captured by the camera. The background is discretized by the pixel size of the image sensor. When the pixel size on the image sensor is Δ_i , the light ray imaged onto the sensor has a finite volume, as shown by the pink area in Fig. 2. The spatial resolution at the deviation point T corresponds to the diameter of the projection of the cone of light rays on the target, as defined by

$$\delta_t = \frac{M}{M+1} \frac{l_b}{F_{\#}} + \frac{l_c}{l_i} \Delta_i. \quad (6)$$

Here, $F_{\#} = f/d$ is the focal ratio of the lens (F-number), d is the aperture of the camera, f is the focal length of the lens. The first and second terms correspond to the red and yellow lines in Fig. 2, respectively, and are derived from the lens formula and geometry. When the magnification satisfies $M > 0.1$, the contribution of the second term is small; therefore, δ_t is mainly determined by l_b and $F_{\#}$ as follows:

$$\delta_t = \frac{M}{M+1} \frac{l_b}{F_{\#}} \quad (M > 0.1). \quad (7)$$

Increasing the F-number, i.e., narrowing the aperture, reduces the spatial resolution δ_t and thus improves the spatial resolution. However, this also decreases the amount of light reaching the image sensor and reduces the signal-to-noise ratio. In measurements of small-scale flow fields, the camera F-number is also affected by the diffraction limit. The spatial resolution limited by diffraction is given by the following equation using the wavelength λ of light and the distance l_i between the lens and the image sensor.

$$\delta = 1.22 \frac{l_i}{d} \lambda. \quad (8)$$

When the diffraction-limited spot diameter δ exceeds the image sensor pixel pitch Δ_i , diffraction becomes dominant in determining the effective image-plane resolution. Therefore, an optimal F-number exists that provides the highest spatial resolution (Cozzi et al., 2017), and an appropriate balance must be determined based on the experimental conditions.

As such, the sensitivity S and the spatial resolution δ_t are not independent of each other; they are key parameters that determine the measurable range and accuracy of BOS.

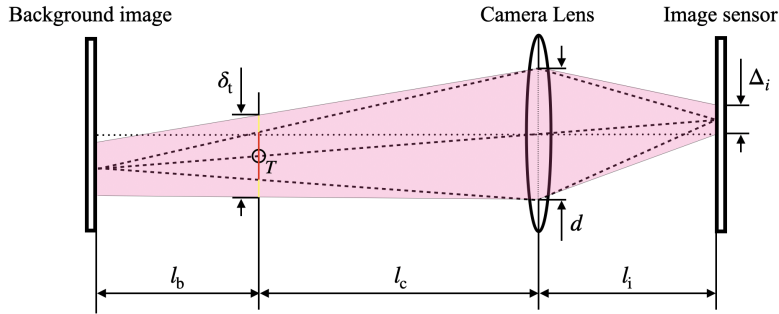


Figure 2. Spatial resolution δ_t on the target in BOS.

3. Experiments

3.1. Experimental setup

Fig. 3 shows a schematic diagram of the experimental optical system. To discuss the effects of sensitivity and spatial resolution on the deviation angle, a sinusoidal displacement field was obtained

by varying l_b in the experiment. In this experiment, the deviation angle was controlled by using a reference object. Fig. 4 shows the main components of the projected BOS system. The background, projection lens (Ai-Nikkor 105 mm F1.8S), reference object, camera lens (105 mm F2.8 EX DG MACRO, Sigma), and camera (EOS 80D, Canon) were positioned along the camera's line of sight, and the optical system was focused on the background plane. The background was displayed on a PC screen. Fast checkerboard demodulation (FCD) was applied to detect the displacement of the captured background image (Wildeman, 2018; Shimazaki et al., 2022).

The background projection was based on non-parallel projection (Yamagishi, 2025). The blue solid line represents the aperture angle of light rays passing through a point on the background and forming an image through the camera lens. Under non-parallel projection, the aperture angle is determined mainly by the camera lens aperture. The spatial resolution δ_t was controlled by the distance l_b between the reference object and the projected background and by the F-number.

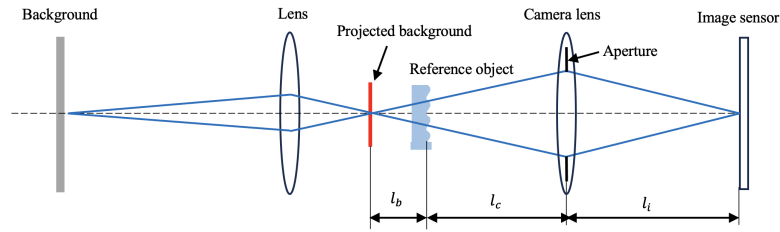


Figure 3. The optical alignment of projected-BOS system.

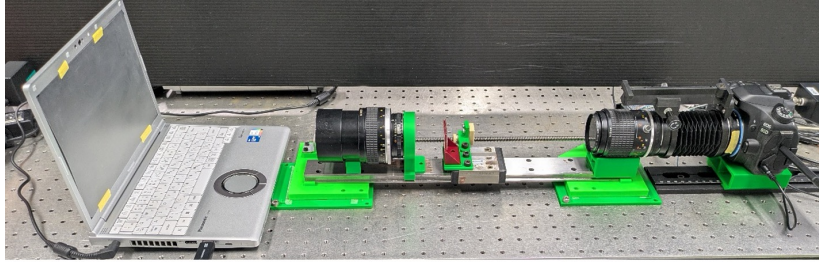


Figure 4. The optical alignment of the BOS experimental setup.

3.2. Reference object

Fig. 5 shows the image and diagram of reference object. The reference object was used as the observation target to control the deviation angle. The reference object was made from transparent resin, glass plates, an aqueous sodium iodide solution (NaI), and a plastic frame. First, transparent resin was poured into a mold with a smooth surface patterned with sinusoidal grooves with a wavelength of 3 mm and an amplitude of 0.2 mm, and the resin was then cured. The glass plates were approximately 1 mm thick. The cured resin plate was placed in the frame and sandwiched between the glass plates. Next, an NaI solution was injected through a hole drilled in the side of the frame. Due to the manufacturing process, the reference object has a sinusoidal waveform with an amplitude of 0.2 mm. Without a contact liquid, the refraction angle would be too large due to the difference in refractive

index between air and the transparent resin. Therefore, NaI solution was used as the contact liquid, and the refraction angle was suppressed by adjusting its concentration. Note that the deviation angle of the reference object changes depending on the concentration of the NaI solution. After injecting the NaI solution, the injection port was sealed with adhesive or other means to prevent degradation of the solution.

The significance of using a sinusoidal reference field to investigate the spatial averaging effect is discussed below. Since any continuous function can be represented as a superposition of trigonometric functions through Fourier analysis, the displacement field induced by the density gradient of a real fluid can also be interpreted as a superposition of spatial frequency components in two-dimensional Fourier space. A sinusoidal reference field with a single wavenumber component is therefore useful for isolating the influence of spatial resolution on the measured refraction angle for a specific spatial frequency.

In the present study, the use of a sinusoidal reference field allowed the relationship between spatial resolution and the measured refraction angle to be formulated in a normalized form with respect to the wavenumber of the phenomenon. This relationship provides a criterion for estimating, in advance, the spatial-resolution condition required to resolve a structure of a given scale in a real fluid field, based on the highest wavenumber component of interest.

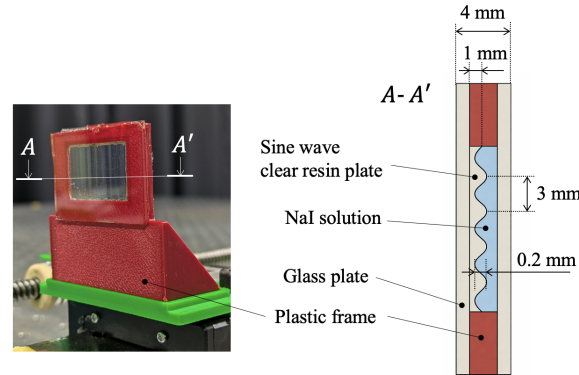


Figure 5. The image and diagram of the reference object.

3.3. Experimental condition

F-number was fixed at $f/8$. The distance l_b between the reference object and the projected background was varied in 2 mm increments by moving the reference object. The origin of l_b was determined by scanning the relative position between the projected background and the reference object over ± 0.5 mm in 0.1 mm increments around the visually estimated zero-displacement position. The position at which the FCD-measured displacement amplitude was minimized was defined as $l_b = 0$. The detailed experimental conditions are listed in Table 1. Here, λ_{bg} is defined as twice the checker size. Therefore, the relationship between λ_{bg} and the carrier wavenumber k_c is given by $k_c = 2\sqrt{2}\pi/\lambda_{bg}$. The carrier was oriented at 45° with respect to the displacement direction induced by the reference object.

Table 1. Measurement conditions.

Refractive index of NaI solution	1.4922
Background wavelength λ_{bg} [mm]	0.6 $\sim$ 2.2(0.1 mm step)
l_b [mm]	2 $\sim$ 72 (2 mm step)
$l_b + l_c$ [mm]	216
l_i [mm]	216
F-number	$f/8$

4. Results and Discussion

Representative displacement fields obtained at a background wavelength of $\lambda_{bg} = 1.6$ mm are shown in Fig. 6(a). Since the reference object used in this experiment has frequency components only in the x direction, the measured displacement appears only in the x -direction component.

At $l_b = 0$ mm, the displacement is 0, and as l_b increases, the displacement increases according to the sensitivity theory. The maximum displacement is recorded at $l_b = 32$ mm, but beyond that, the averaging effect due to spatial resolution becomes dominant, and the displacement decreases.

The displacement averaged along the y -axis is shown in Fig. 6(b). The increase in the wavelength of the sinusoidal displacement field with increasing l_b is attributed to the perspective effect. The ratios of spatial resolution to displacement wavelength at $l_b = 60$ and 72 mm are $\delta_t/\lambda_p = 1.25$ and 1.5, respectively. These values indicate that the observed displacement field corresponds to the refractive-angle distribution averaged within the cone-shaped spatial resolution volume. As a result, the displacement distribution becomes progressively flattened, and the sinusoidal structure becomes indistinct. In addition, grid-like artifacts are observed due to the degradation of the signal-to-noise ratio. The size of these artifacts corresponds to the background wavelength, suggesting that they originate from the FCD image-processing procedure.

The relationship between normalized deviation angle amplitude and spatial resolution is shown in Fig. 7. The spatial resolution δ_t is calculated from Eq. (7). The wavelength λ_p is a parameter of the reference object. The deviation angle is estimated from the measured displacement: $\varepsilon = A/S$, where A is the displacement amplitude and S is the sensitivity.

For normalization of the deviation angle, it is important to accurately evaluate the maximum deviation angle used as the reference value. Ideally, $\varepsilon_{\max}$ is the refraction angle generated when a light ray incident along the optical axis passes through the inflection point of the sinusoidal reference object. In other words, it occurs at $l_b = 0$. Under this condition, however, no displacement appears in the background image, making it fundamentally difficult to measure such a refraction angle using BOS. Therefore, $\varepsilon_{\max}$ must be obtained separately under conditions that ensure sufficient sensitivity for BOS measurements while minimizing the spatial resolution δ_t among all experimental conditions. In this experiment, $\varepsilon_{\max}$ was evaluated from measurements obtained at the maximum F-number of the camera lens, $F_{\#} = 22$, and a background distance of $l_b = 2$ mm. To reduce systematic errors associated with the background wavelength, $\varepsilon_{\max}$ was evaluated separately for each background wavelength.

The black dashed line represents the sinusoidal simple moving average filter (SMAF). Here, SMAF

denotes the averaged value of a cosine wave, $\cos(2\pi x/\lambda_p)$, where x is a real number. Yamagishi (2025) found that the relationship between the normalized deviation angle amplitude and spatial resolution of a sinusoidal reference object is well described by the SMAF, and proposed that the effect of spatial resolution on deviation angle can be estimated using the SMAF when the object has a periodic waveform. However, in the present optical configuration, the finite circular aperture of the camera lens suggests that the aperture geometry should also be considered when quantitatively estimating the relationship between the normalized deviation angle amplitude and spatial resolution. Since the spatial resolution is determined within a cone incident on the reference object, the average value of the sinusoidal wavefront must take into account its two-dimensional nature. In this case, the curve corresponding to SMAF is the response obtained by averaging the cosine wavefront over a circular region with diameter δ_t . This curve can be expressed using the first-order Bessel function of the first kind, defined in Eq. (9), as shown in Eq. (10). Here, η denotes the normalized amplitude response of the circular averaging model, corresponding to $\varepsilon/\varepsilon_{\max}$.

$$J_1(x) = \frac{1}{\pi} \int_0^\pi \cos(\theta - x \sin \theta) d\theta, \quad (9)$$

$$\eta(\delta_t) = \frac{2|J_1(\pi\delta_t/\lambda_p)|}{\pi\delta_t/\lambda_p}. \quad (10)$$

In this paper, we refer to this as the Simple Radial Circular Moving Average Filter (SRCMAF) for a sinusoidal wavefront and show it in Fig. 7 as a solid red line. As shown in Fig. 7, the SRCMAF provides better agreement with the experimental results than the SMAF. The maximum discrepancy was reduced from approximately 31% for the SMAF to approximately 9% for the SRCMAF.

Using the SRCMAF response, the condition for keeping the deviation-angle measurement error below 5% for the highest wavenumber component to be resolved is obtained by solving $\eta(\delta_t) = 0.95$. This gives $\delta_t/\lambda_p = 0.203$, indicating that the spatial-resolution diameter should be less than approximately 20.3% of the shortest wavelength of interest.

The conical spread of the spatial resolution originates from the circular aperture of typical camera lenses. Thus, the spatial averaging effect should be modeled as circular averaging rather than one-dimensional averaging. From this viewpoint, SRCMAF, which accounts for circular averaging, is more generally applicable than SMAF. Therefore, the circular simple moving average theory for sinusoidal wavefronts provides an appropriate basis for quantitatively predicting the spatial averaging effect inherent in BOS measurements.

For $\delta_t/\lambda_p > 0.75$, a pronounced discrepancy between the experimental results and the theoretical predictions is observed. One possible cause of this discrepancy is that the spatial averaging in the actual optical system may not be represented by an ideal uniform circular average. Instead, the measured deviation angle may correspond to a weighted average over the ray cone, depending on the optical transfer characteristics of the imaging system. However, quantitative validation of this effect through ray-tracing analysis or theoretical modeling has not yet been conducted; therefore, the primary cause of the observed discrepancy remains unclear.

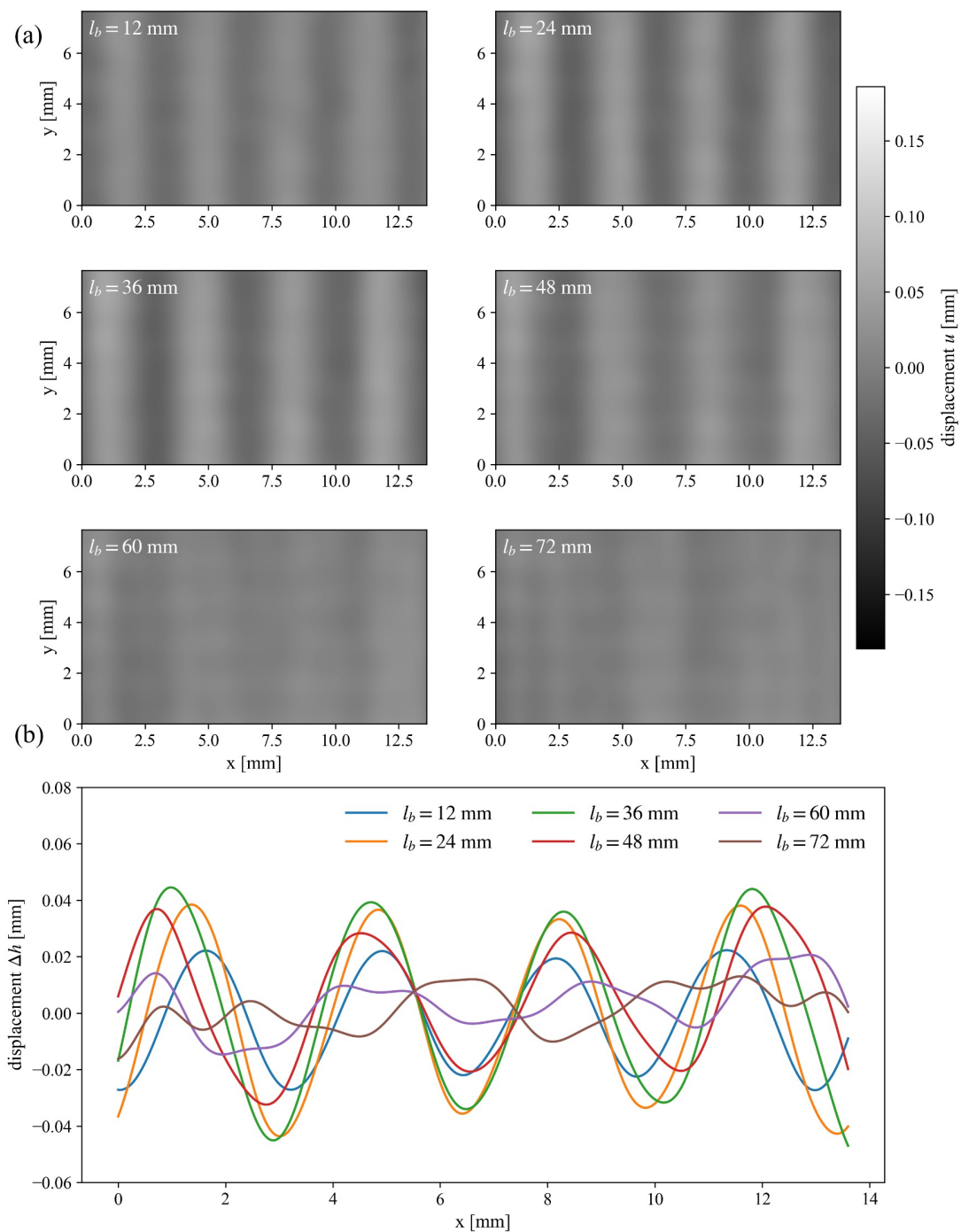


Figure 6. Displacement results for $\lambda_{bg} = 1.6$ mm measured using FCD at different l_b . (a) Displacement field, and (b) displacement averaged along the y-axis.

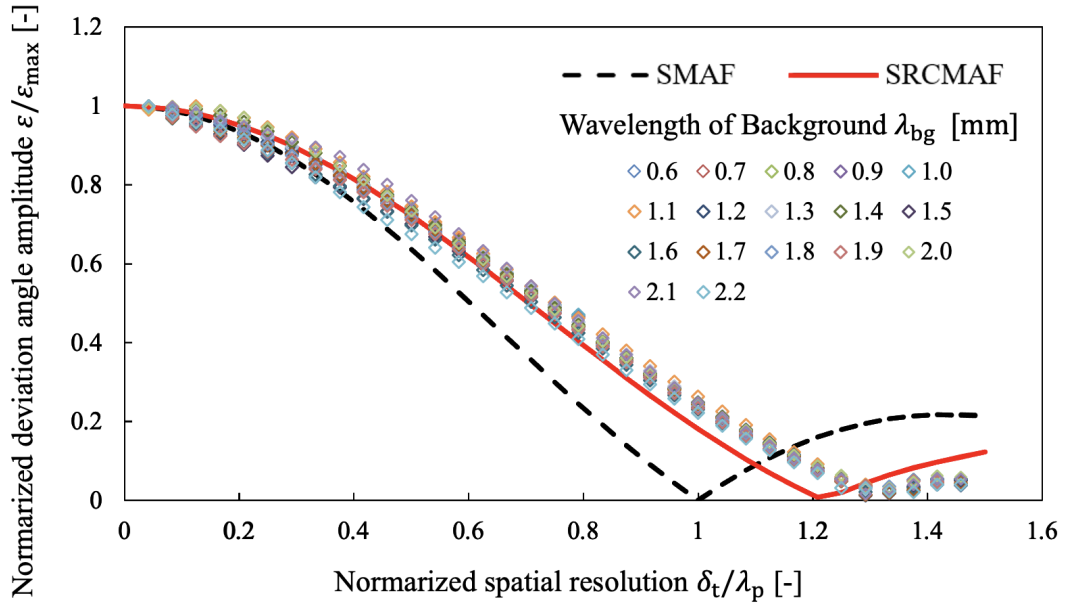


Figure 7. The relationship between $\varepsilon/\varepsilon_{\max}$ and δ_t/λ_p

5. Conclusion

In this study, we examined the relationship between the deviation angle ratio, $\varepsilon/\varepsilon_{\max}$, obtained by observing a sinusoidal reference target, and the normalized spatial resolution, δ_t/λ_p . The Simple Moving Average Filter (SMAF) proposed by Yamagishi (2025) provides a useful reference model for describing the influence of spatial averaging on the measured deviation angle. In the present experimental configuration, the finite aperture of the camera lens forms a circular averaging region on the target plane. Therefore, the effect of aperture geometry was also considered.

To account for this circular averaging effect, we introduced the Simple Radial Circular Moving Average Filter (SRCMAF) and compared it with the experimental results. The comparison showed that considering circular averaging improves the quantitative description of the experimental data, reducing the maximum discrepancy from approximately 31% to approximately 9%. This result indicates that aperture geometry can be an important factor when quantitatively evaluating spatial-averaging effects in BOS measurements.

The SRCMAF formulation enables the relationship between the spatial resolution and the measured refraction angle to be expressed in a normalized form when using a sinusoidal reference field. This relationship provides a criterion for estimating, in advance, the spatial resolution required to resolve a structure of a given scale in a real fluid field, based on the highest wavenumber component of interest.

Based on the SRCMAF model, the spatial resolution in BOS measurements should be less than 20.3% of the shortest wavelength that needs to be resolved in the target phenomenon in order to keep the deviation-angle measurement error below 5%. This study concludes that this is a requirement for the spatial resolution necessary in the BOS technique.

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