

A meshless data-tailored approach to compute statistics from scattered data with adaptive radial basis functions.

Damien Rigutto^{1,2,*}, Manuel Ratz^{1,3} and Miguel Alfonso Mendez^{1,3,4}

1: Environmental and Applied Fluid Dynamics, von Karman Institute for Fluid Dynamics, Waterloosesteenweg 72, Sint-Genesius-Rode, 1640, Belgium

2: Transferts, Interfaces Et Procédés (TIPs), Université Libre de Bruxelles, Av. Franklin Roosevelt 50, Brussels, 1050, Belgium

3: Aero-Thermo-Mechanics Laboratory, Ecole Polytechnique de Bruxelles, Université Libre de Bruxelles, Brussels, Belgium

4: Experimental Aerodynamics and Propulsion Lab, Universidad Carlos III de Madrid, Leganés, Spain

*Corresponding author: damien.rigutto@vki.ac.be

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ABSTRACT

Radial basis function (RBF) regression has emerged as a powerful meshless tool for reconstructing continuous velocity fields from scattered velocimetry measurements. However, conventional isotropic formulations often exhibit oscillations in regions with strong gradients and anisotropic flow structures. This work presents an adaptive, anisotropic, and gradient-informed RBF framework for data assimilation of flow fields from scattered data. Local flow gradients, estimated from neighboring measurements, are used to guide adaptive sampling, define anisotropic kernel deformations, and introduce derivative-based regularization constraints. The methodology is tested on particle tracking velocimetry data of a turbulent round jet, characterized by highly non-uniform sampling and strong shear-layer gradients. Results demonstrate reduced oscillatory artifacts and better gradient capturing while requiring significantly fewer basis functions. The proposed framework provides an efficient and flexible meshless approach for the analysis of scattered experimental velocimetry data in complex turbulent flows.

1. Introduction

Constrained radial basis function (RBF) regression methods have recently emerged as attractive tools for the reconstruction of continuous velocity fields from scattered measurements obtained through particle-based velocimetry techniques such as Lagrangian particle tracking velocimetry (LPT, Schröder & Schanz (2023)). Unlike traditional interpolation approaches designed to project scattered measurements onto structured grids (Schneiders & Scarano, 2016; Scarano et al., 2022), constrained RBF methods directly reconstruct an analytical representation of the flow field from the scattered data. This meshless formulation enables analytical differentiation, adaptive local refinement, and the computation of derived quantities such as turbulence statistics or pressure fields without relying on a mesh (Sperotto et al., 2022; Li & Pan, 2024; Ratz & Mendez, 2024).

Recent neural-network-based approaches, including physics-informed neural networks (PINNs) and sinusoidal representation networks (SIRENs), also provide meshless continuous flow representations (Zhou & Grauer, 2023; Miotto et al., 2025). Similarly, established reconstruction approaches such as VIC-type methods and FlowFit also rely on optimization procedures to infer the underlying flow field from scattered measurements (Gesemann et al., 2016; Godbersen et al., 2024). However, network-based methods generally require solving large nonlinear optimization problems involving extensive hyperparameter tuning and regularization strategies, while FlowFit and VIC-type formulations typically involve iterative nonlinear optimization procedures associated with the reconstruction constraints.

In contrast, constrained RBF regression remains a linear framework requiring the solution of a single least-squares problem while still allowing physical constraints to be incorporated efficiently. This makes RBF-based approaches particularly attractive for experimental fluid mechanics applications where robustness and computational efficiency are essential.

In the context of image based velocimetry, the reconstruction often targets the computation of statistical quantities from large ensembles of scattered measurements. Several approaches based on local averaging or polynomial regression have been proposed to estimate turbulence statistics directly from scattered particle data (Agüera et al., 2016; Raiola et al., 2020). These methods demonstrated the importance of adapting the local support to the flow directionality and sampling distribution, particularly within turbulent shear layers.

Classical isotropic RBF formulations often struggle to provide such adaptation and suffer in regions of sharp gradients and strongly anisotropic flow structures. In these regions, the reconstruction may exhibit spurious oscillations. This work presents an adaptive, anisotropic, gradient-informed extension of the constrained RBF framework for experimental velocimetry. The method merges adaptive sampling, anisotropic basis deformation, and gradient-informed regularization in a fully meshless regression scheme. Local flow gradients from scattered measurements guide both adaptive collocation of basis functions and kernel deformation along local flow directions. Additional gradient-based regularization stabilizes the reconstruction in regions of strong shear. The methodology is demonstrated on particle tracking velocimetry of a turbulent round jet. This work assesses the applicability of the adaptive anisotropic framework to experimental velocimetry data with irregular sampling and strong shear-layer gradients. A more extensive validation and benchmarking, also including a direct numerical simulation (DNS) dataset is given in Rigutto et al. (2025).

The remainder of the paper is organized as follows. Section 2 presents the adaptive anisotropic RBF methodology. Section 3 describes the experimental turbulent jet dataset. Section 4 presents the reconstruction results and discusses the impact of the adaptive formulation on the computation of flow statistics. Finally, Section 5 summarizes the main findings and perspectives.

2. Methodology

The methodology extends constrained radial basis function regression with adaptive sampling, anisotropic basis deformation, and gradient-informed regularization to more accurately reconstruct scattered experimental velocimetry data in regions with strong gradients and anisotropic flow. A brief overview is provided here; full details are available in Rigutto et al. (2025).

2.1. Constrained meshless RBF reconstruction

The reconstruction is based on a linear combination of n_B radial basis functions collocated throughout the measurement domain. For a scalar quantity $f(\mathbf{x})$, the approximation reads

$$\tilde{f}(\mathbf{x}) = \sum_{m=1}^{n_B} w_m \gamma_m(\mathbf{x}), \quad (1)$$

where w_m are the unknown regression coefficients and γ_m denotes the m -th basis function centered at the collocation point $\mathbf{x}_{c,m}$. In the original formulation, γ_m is isotropic, and for the most common Gaussian basis, it reads:

$$\gamma_m(\mathbf{x}; \mathbf{x}_{c,m}, c_m) = \exp \left(-c_m^2 \|\mathbf{x} - \mathbf{x}_{c,m}\|_2^2 \right), \quad (2)$$

where c_m is the only hyper-parameter besides the collocation point $\mathbf{x}_{c,m}$.

Following previous constrained RBF formulations for experimental fluid mechanics (Sperotto et al., 2022; Ratz & Mendez, 2024), the reconstruction coefficients are obtained from a penalized least-squares problem accounting both for data fidelity and regularization terms, but also physical constraints such as boundary conditions or free divergence of the flow, to mention only a few.

2.2. Adaptive sampling and basis collocation

The proposed adaptive formulation exploits local gradient information $\mathbf{G}_g(\mathbf{x})$, estimated directly from the scattered measurements using weighted polynomial regression. A smoothed version of the gradient field, denoted $\tilde{\mathbf{G}}_g(\mathbf{x})$, is subsequently used to guide both the adaptive downsampling of the experimental dataset and the spatial distribution of the basis functions.

The smoothing step ensures progressive transitions between regions populated by small, densely distributed bases and regions characterized by larger and sparser basis supports. This gradual variation in the basis distribution improves both the stability and the robustness of the regression while preventing abrupt changes in the local reconstruction resolution.

Collocation points are distributed with a variable-density Poisson disk sampling strategy (Slak & Kosec, 2019; Dwork et al., 2021; Lawrence et al., 2024), yielding a well-conditioned, spatially adaptive basis. The local basis spacing $r_c(\mathbf{x})$ sets the target sampling radius and is controlled by the smoothed gradient magnitude according to

$$r_c(\mathbf{x}) = (r_{\max} - r_{\min}) \left(1 - \sqrt{\|\tilde{\mathbf{G}}_g(\mathbf{x})\|_2} \right) + r_{\min}, \quad (3)$$

where $\tilde{\mathbf{G}}$ denotes the normalized smoothed gradient field and $r_{\min}, r_{\max}$ define the minimum and maximum local basis spacing.

2.3. Anisotropic basis adaptation

To better capture directional flow structures such as turbulent shear layers, the isotropic RBF kernels are locally deformed according to the estimated flow gradients. The anisotropic basis functions are defined as

$$\gamma_m(\mathbf{x}; \mathbf{x}_{c,m}, \mathbf{M}_m) = \exp \left(-\|\mathbf{M}_m(\mathbf{x} - \mathbf{x}_{c,m})\|_2^2 \right), \quad (4)$$

where the metric tensor $\mathbf{M}_m$ controls the local orientation and stretching of each basis function.

The deformation is constructed such that the bases become sharper in the direction of the strongest gradients while being elongated along the other direction. The stretching is therefore applied tangentially to the shear layers, with an adaptive aspect ratio (AR) ranging from 1 to $\text{AR}_{\max}$ according to the local gradient magnitude.

Similar concepts have previously been explored in anisotropic interpolation and velocimetry applications (Beatson et al., 2010; Raiola et al., 2020). In the present work, the anisotropic deformation is directly embedded within the constrained RBF regression framework.

The resulting anisotropic basis distribution substantially improves the reconstruction of sharp gradients while reducing the oscillatory artifacts commonly observed with isotropic formulations.

2.4. Gradient-informed regularization

Additional regularization terms are introduced by penalizing discrepancies between reconstructed gradients and locally estimated gradients obtained from polynomial regression. The resulting formulation stabilizes the reconstruction in regions characterized by strong shear while preserving the sharpness of relevant flow structures.

This approach is inspired by gradient-enhanced interpolation methods (Laurent et al., 2019) and gradient-informed Gaussian process formulations (Bhaduri et al., 2020; Määtä et al., 2021), where derivative information is used to improve local adaptivity and reconstruction smoothness.

3. Experimental test case

The proposed adaptive anisotropic reconstruction framework is demonstrated using experimental data from a turbulent round jet measured with time-resolved three-dimensional particle tracking velocimetry (3D-PTV). The goal is to assess its applicability to scattered experimental velocimetry data with irregular sampling, strong shear-layer gradients, and turbulence-induced spatial variability.

The dataset corresponds to a turbulent water jet in a hexagonal acrylic tank. The coordinate system is defined with x along the jet centerline (axial), y radial, and z out of plane. The jet issues upward from a nozzle of diameter $D = 15$ mm, giving a Reynolds number $Re \approx 6750$. The measurement volume spans $x/D \times y/D \times z/D \in [0.25, 4.25] \times [-1.3, 1.3] \times [-0.5, 0.5]$. Time-resolved volumetric data were acquired with a four-camera 3D-PTV system at 1000 Hz, yielding 2000 snapshots. Further details on the setup and acquisition are given in Ratz & Mendez (2024). The measurements comprise scattered velocity samples with strongly non-uniform spatial distributions, especially in the turbulent shear layer, making this dataset well suited for testing adaptive meshless reconstruction methods.

For the present analysis, the three-dimensional particle distribution is projected onto the x - y plane by grouping measurements according to radial distance from the jet centerline. This leverages the statistical axisymmetry of turbulent round jets, where mean flow and second-order statistics depend mainly on axial and radial coordinates. The projection markedly changes the sampling, producing a strong accumulation of points away from the centerline. The adaptive downsampling strategy from Section 2 is therefore applied to rebalance the spatial distribution before regression.

Figure 1 illustrates the scattered experimental measurements used for the reconstruction. Figure 1(a) shows the complete ensemble colored by the streamwise velocity component, while Figure 1(b) presents a representative slice around $x/D \approx 1$, highlighting both the strong velocity gradients and the high turbulence intensity within the shear layer.

4. Results

The proposed adaptive anisotropic reconstruction framework is assessed on the experimental turbulent jet dataset introduced in Section 3. Among the full dataset, only a subset of 5×10^4 and 5×10^5 are used for the regression of the mean and the fluctuation components, respectively. Three

regression strategies are compared throughout the analysis:

- isotropic RBFs with uniform collocation (IsoUni),
- isotropic RBFs with adaptive collocation (IsoAdapt),
- the proposed adaptive anisotropic formulation (AnisoAdapt).

The objective of this comparison is to isolate the effects of adaptive basis placement and anisotropic deformation on the reconstruction of turbulent flow statistics from scattered experimental measurements.

The reconstruction procedure begins with the estimation of local gradients from the scattered velocity measurements. These gradients are subsequently used to guide both the adaptive sampling strategy and the anisotropic deformation of the basis functions. Figure 2 presents the estimated gradient field (a) together with its smoothed representation (b) used for the adaptive collocation.

The resulting adaptive basis distributions are shown in Figure 3, along with the number of points supporting the bases (N_{support}) and the bases' aspect ratio. Compared to the uniform isotropic

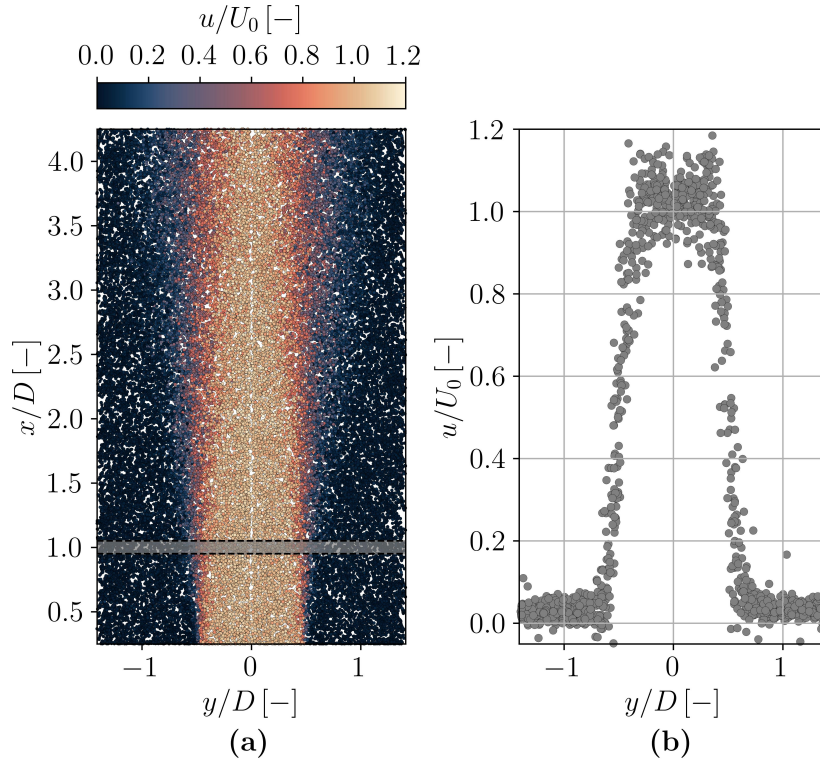


Figure 1. Experimental turbulent jet dataset used for the adaptive anisotropic reconstruction. (a) Scatter plot of the scattered velocity ensemble colored by the streamwise velocity component. (b) Representative slice around $x/D \approx 1$, illustrating the strong gradients and turbulent fluctuations within the shear layer.

formulation, whose basis distribution can be seen in Figure 2 (c), the adaptive strategies concentrate basis functions within the turbulent shear layers while maintaining a support $N_{support} > 10^3$ in smoother regions of the flow, as shown in Figure 3(a). This adaptive collocation significantly reduces the total number of basis elements while preserving sufficient local resolution in regions characterized by strong velocity gradients.

The anisotropic formulation, shown in Figure 3(b), further stretches the basis functions according to the local flow directionality, allowing sharper gradients to be reconstructed without increasing the total number of basis elements. The deformation is primarily aligned with the shear layers, producing elongated bases in the streamwise direction while preserving narrow supports across the gradients.

Overall, the stretching preserves a nearly constant ratio of data points per basis throughout most of the domain, while improving the local support near the edges of the measurement region where the particle density becomes lower. This contributes to a more stable regression and smoother reconstruction close to the domain boundaries.

The spatial distribution of the basis aspect ratios is presented in Figure 3(c). The strongest stretching occurs near the jet inlet and within the turbulent shear layers, where the velocity gradients are

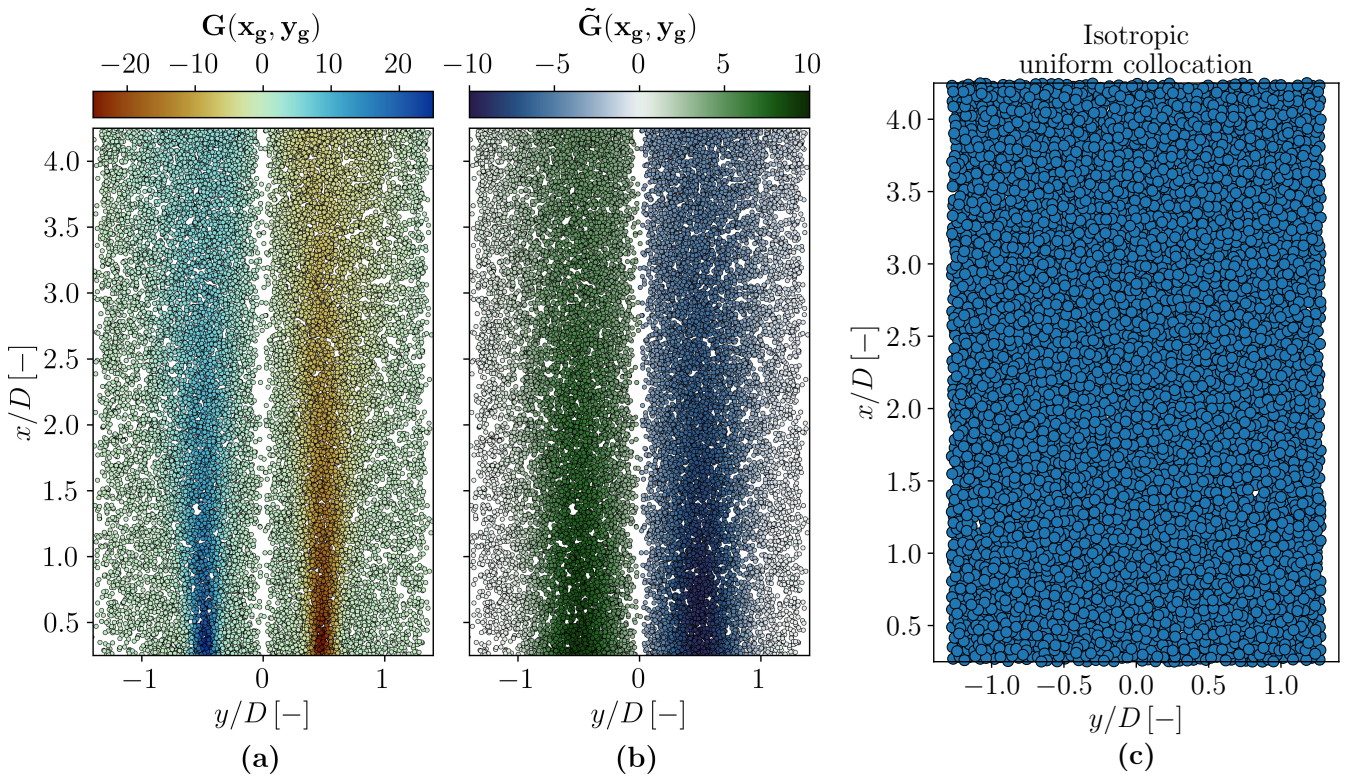


Figure 2. Estimated gradient field obtained from the experimental turbulent jet dataset: (a) raw estimated gradients, (b) smoothed gradients used for adaptive collocation and anisotropic basis deformation, and (c) bases placement for the Isotropic Uniform approach

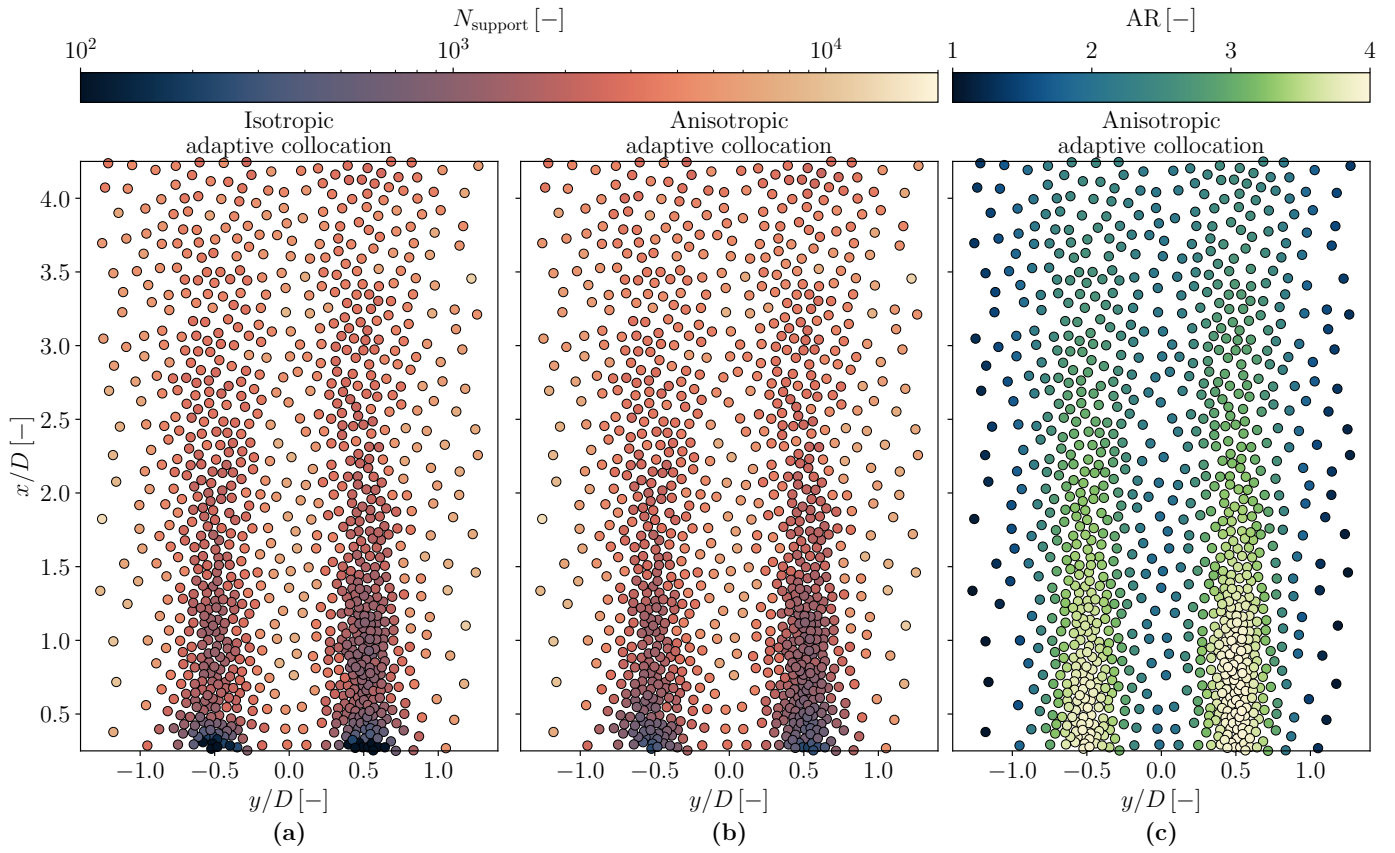


Figure 3. Adaptive basis distribution for the turbulent jet reconstruction: (a) isotropic adaptive collocation, (b) adaptive anisotropic collocation, (c) aspect ratio distribution of the anisotropic basis functions.

largest, while the bases progressively recover isotropic shapes farther downstream and outside the jet core.

Figure 4 presents the reconstructed mean streamwise velocity field obtained using the three regression strategies (Fig. 4a–c). Velocity profiles extracted at several downstream locations are additionally shown in Fig. 4(d)–(f) in order to better assess the reconstruction performance across the shear layer.

The IsoUni formulation (Fig. 4a,d) exhibits noticeable oscillatory artefacts throughout the flow field, particularly within the shear layer close to the nozzle exit. These oscillations originate from the difficulty of simultaneously resolving sharp gradients and maintaining smoothness using uniformly distributed isotropic bases.

The IsoAdapt formulation (Fig. 4b,e) significantly improves the reconstruction by removing most of the oscillations through adaptive basis placement. However, localized overshoots remain visible near the strongest shear-layer gradients. These overshoots are primarily caused by the isotropic shape of the basis functions, whose supports remain too large to accurately resolve the sharp velocity gradients close to the jet inlet. Further reducing the basis size would improve the gradient

resolution, but would simultaneously decrease the number of supporting data points per basis, potentially leading to regression instabilities and reduced statistical convergence.

The proposed AnisoAdapt formulation (Fig. 4c,f) further improves the reconstruction quality by combining adaptive collocation with anisotropic basis deformation. The stretching enables the basis functions to remain narrow across the gradients while being elongated along the local flow directionality, thereby improving gradient resolution without reducing the effective number of supporting data points. As a result, the reconstructed velocity profiles exhibit both sharper shear-layer gradients and substantially smoother spatial distributions throughout the domain. Regarding computational cost, the decrease in the number of bases allows the regression time to be re-

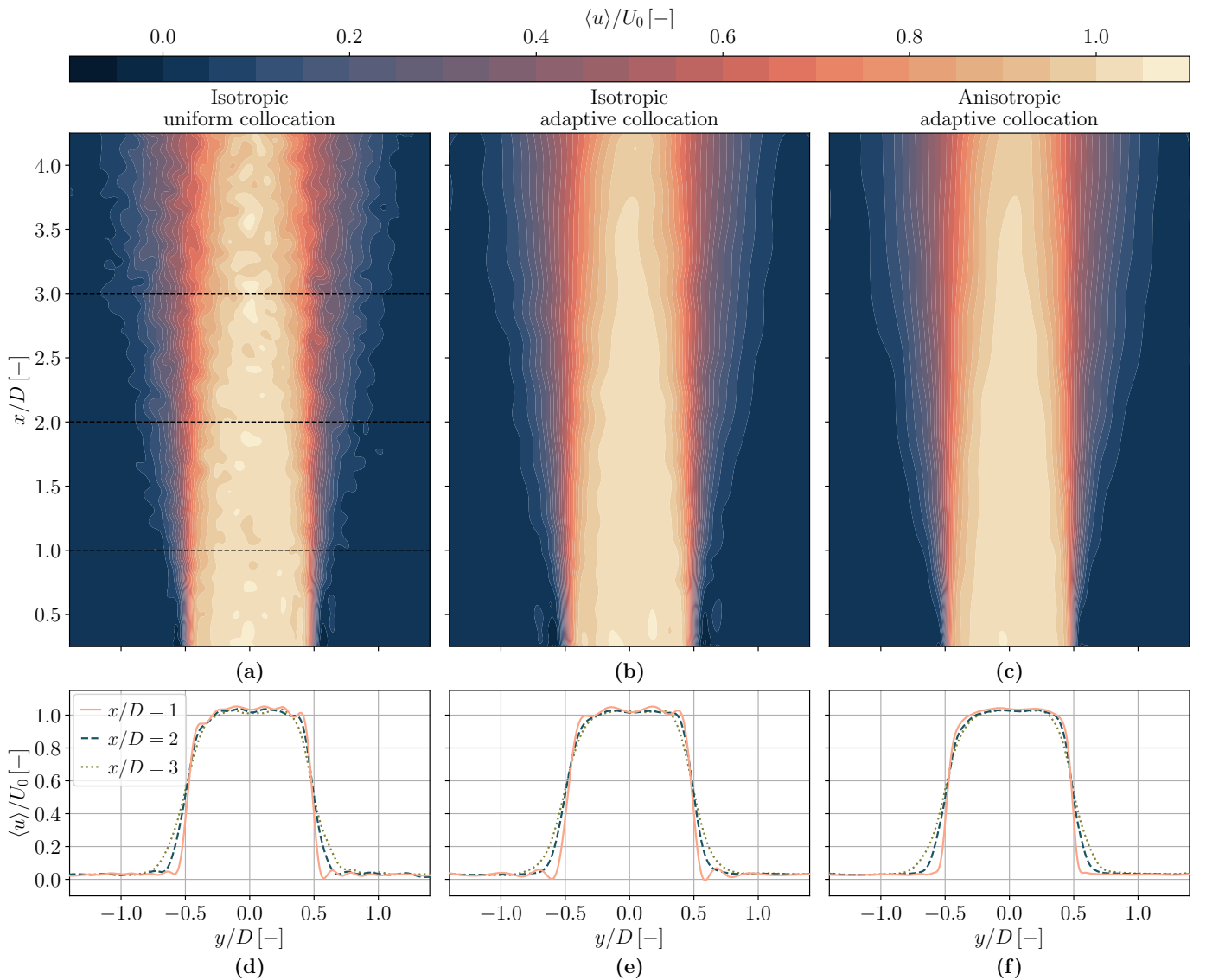


Figure 4. Mean streamwise velocity reconstruction: isotropic uniform collocation (a), isotropic adaptive collocation (b), adaptive anisotropic reconstruction (c). Extracted velocity profiles at several downstream locations are shown in (d-f).

duced to 166 seconds, whereas the original isotropic formulation took 214 seconds.

The same reconstruction strategies are subsequently applied to the streamwise Reynolds stress component $\langle u'u' \rangle$, shown in Figure 5. The IsoUni approach again exhibits strong oscillatory behavior together with artificially amplified fluctuation levels resulting from inaccuracies in the mean velocity reconstruction. Although the IsoAdapt approach substantially reduces these oscillations, localised overshoots remain visible within the turbulent shear layer at the jet exit.

The proposed AnisoAdapt formulation provides the smoothest and most physically consistent reconstruction of the turbulence statistics. The anisotropic basis deformation enables the sharp turbulent shear layers to be resolved while avoiding the spurious oscillations commonly associated with isotropic formulations. In addition, the resulting fluctuation levels remain lower and spatially smoother, indicating an improved estimation of the underlying mean velocity field.

The gain in computational time is remarkable, going from 3 026 seconds for the isotropic uniform approach to only 738 seconds for the anisotropic adaptive one. Overall, the proposed methodology demonstrates that combining adaptive sampling, anisotropic basis deformation, and gradient-informed regularization significantly improves the reconstruction of experimental turbulent flow statistics from scattered velocimetry measurements.

5. Conclusions and Perspectives

This work introduced an adaptive, anisotropic, gradient-informed extension of constrained radial basis function regression for reconstructing flow statistics from scattered velocimetry data. The method combines adaptive sampling, anisotropic basis deformation, and gradient-informed regularization in a fully meshless framework tailored to particle-based measurements.

We demonstrated the approach on 3D particle tracking velocimetry of a turbulent round jet with strong shear gradients and highly non-uniform sampling. The adaptive anisotropic formulation markedly improves the robustness and smoothness of reconstructed velocity fields and turbulence statistics compared with conventional isotropic RBFs.

The adaptive collocation strategy reduces the number of basis functions while preserving high spatial resolution in dynamically important regions. Anisotropic basis deformation improves reconstruction of sharp shear-layer gradients and suppresses oscillations typical of isotropic formulations. The resulting fields show more physical consistency and smoother spatial distributions of mean flow and Reynolds stresses.

The proposed formulation retains the core advantages of constrained RBF regression for experimental fluid mechanics: it is fully meshless, supports analytical differentiation, enables adaptive local refinement, and handles irregular measurement distributions without a mesh. These modifications extend classical RBF capabilities while preserving the flexibility and computational effi-

ciency that make meshless regression well suited to scattered experimental velocimetry data.

Future work will target pressure reconstruction, fully three-dimensional statistical analysis, and adaptive sampling based on both local turbulence intensity and gradient information.

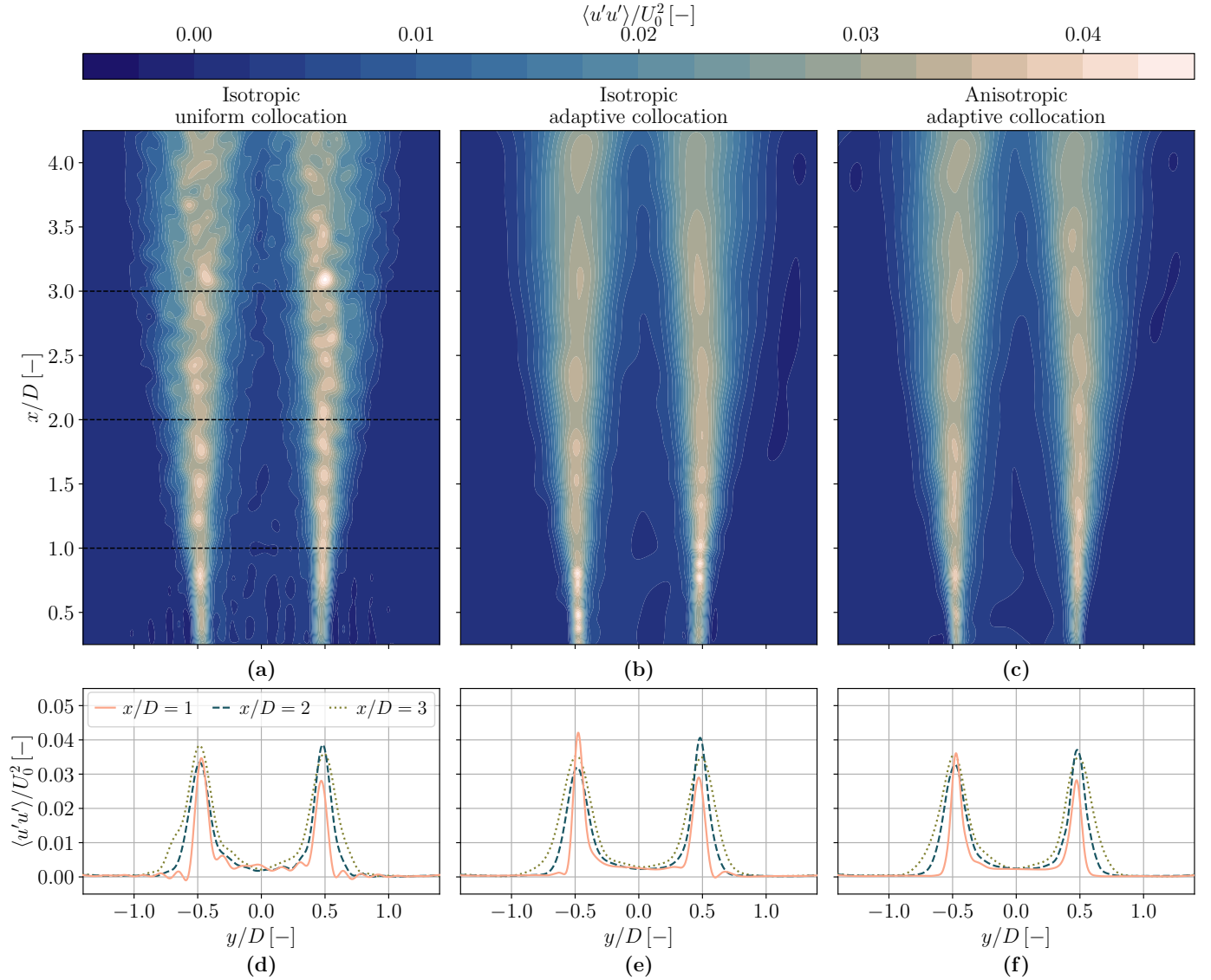


Figure 5. Reconstruction of the streamwise Reynolds stress field $\langle u'u' \rangle$: isotropic uniform collocation (a), isotropic adaptive collocation (b), adaptive anisotropic reconstruction (c). Extracted profiles are shown in (d–f).

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The data are visualized using perceptually uniform colormaps to prevent distortion of the data and exclusion of readers with color-vision deficiencies (Crameri et al., 2020; Crameri, 2023).

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