

On the Detectability of the Turbulent/Non-Turbulent Interface in Free-Shear Flow Using Combined PIV and Mie Scattering

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ABSTRACT

The turbulent/non-turbulent interface (TNTI) plays a central role in turbulent entrainment and mixing processes by separating turbulent fluid from the surrounding ambient environment. Although the TNTI has been extensively investigated through direct numerical simulations, experimental studies remain comparatively scarce, particularly in high-Reynolds-number compressible flows. This limitation is largely due to the difficulty of simultaneously acquiring velocity and scalar information with sufficient spatial and temporal correspondence to accurately identify the interface. The present work introduces a unified experimental methodology for detecting and characterizing TNTI in a weakly underexpanded supersonic jet. Time-resolved particle image velocimetry (TR-PIV) measurements are combined with Mie-scattering imaging, exploiting the same image acquisition to obtain both velocity and scalar information. The recorded image intensity is used as a scalar proxy for particle concentration, enabling extraction of a TNTI candidate without additional optical instrumentation. The flow under investigation consists of a supersonic jet issuing from a circular nozzle of diameter $D = 5$ mm at a nozzle pressure ratio of $NPR = 5$, corresponding to a Reynolds number of approximately 4×10^5 . Time-resolved measurements are performed in the self-similar region of the jet $x/D \approx 100$ using a high-speed camera and continuous laser illumination. A dedicated image-processing framework based on intensity thresholding, connected-component analysis, and contour extraction is developed to identify instantaneous interface realizations. Subsequently, a signed-distance formulation is introduced to establish a local coordinate system centered on the TNTI, enabling conditional statistical analysis of scalar and velocity quantities. The proposed methodology can extract continuous TNTI realizations across the time-resolved sequence of frames. Conditional profiles of scalar intensity, velocity magnitude, and velocity components normal and tangential to the interface reveal clear transitions between the turbulent jet and the surrounding ambient fluid. The results demonstrate the feasibility of combining PIV and Mie-scattering measurements for TNTI detection in high-Reynolds-number supersonic flows and lay the foundation for future investigations of interface dynamics, intermittency, and entrainment mechanisms in compressible turbulent jets.

1. Introduction

Free shear flows constitute one of the fundamental classes of turbulent flows encountered in fluid dynamics. Canonical configurations such as jets, wakes, mixing layers, and plumes play a central role in numerous natural and engineering applications involving momentum, heat, and mass transport Dauxois et al. (2021). In particular, turbulent jets are characterized by their ability to entrain surrounding fluid, promoting mixing and dispersion over a broad range of spatial and temporal scales. Understanding the mechanisms governing this process remains essential for applications ranging from environmental pollutant transport and mixing, process safety, and accidental gas releases.

Supersonic underexpanded jets provide a particularly relevant example of such phenomena. These flows can arise during the accidental release of pressurized gases through leaks or ruptures. In such situations, the dispersion and dilution of the released substance are strongly influenced by turbulent mixing and entrainment processes, making the understanding of these mechanisms essential for accurate prediction of the resulting flow evolution. Moreover, when these releases occur in outdoor environments, the jet interacts with the atmospheric boundary layer (ABL), whose turbulence, stratification, and mean wind conditions can significantly affect entrainment, dispersion patterns, and the downstream transport Sanapo et al. (2026).

Overall, the interaction of the turbulent flows with the surrounding fluid at rest is commonly described through the concept of turbulent entrainment, whereby ambient fluid is incorporated into the turbulent region and subsequently mixed. The importance of entrainment was first recognized by Taylor (1946) during World War II while investigating the use of large fires to disperse fog from airfields. The resulting entrainment hypothesis was subsequently refined through the theoretical and experimental work of Batchelor (1954), culminating in the seminal study of Morton et al. (1956); Turner (1986). Since then, entrainment has become one of the central concepts in the description of free shear flows. A particularly successful aspect of this framework is the recognition that entrainment is primarily governed by the largest scales of motion. Large-scale eddies located near the edge of the flow engulf the surrounding fluid and transport it into the turbulent region, while smaller-scale motions are mainly responsible for its subsequent mixing Cenedese (2024).

Several definitions of entrainment have been proposed in the literature. One interpretation defines the entrainment velocity as the rate at which ambient fluid crosses the turbulent boundary, while another relates entrainment to the outward growth of the turbulent region by engulfing surrounding fluid. Although these viewpoints differ in formulation, they both emphasize the importance of accurately identifying the boundary separating turbulent and non-turbulent fluid.

Indeed, in parallel with the development of entrainment theory, Corrsin & Kistler (n.d.) proposed that this boundary is not a diffuse transition zone but rather a thin interface separating turbulent and irrotational regions of the flow. This interface, now commonly referred to as the

turbulent/non-turbulent interface (TNTI), is characterized by strong gradients of vorticity and scalar concentration and plays a central role in the exchange of momentum, heat, and mass between the turbulent flow and its surroundings. The TNTI delineates the largest scales of the turbulent velocity field, governs the spatial intermittency of turbulence, and contains regions of intense molecular diffusion. Consequently, it provides a natural framework for investigating entrainment and mixing processes.

Direct numerical simulation (DNS) has enabled detailed investigations of the TNTI by providing simultaneous access to velocity^{*}, vorticity, and scalar fields with complete spatial resolution. Foundational studies have demonstrated that the TNTI can be identified through vorticity-based criteria and have revealed many of its geometrical and dynamical properties Bisset et al. (n.d.). Nevertheless, experimental investigations remain essential (Westerweel et al. (2005, 2002)) because they provide access to realistic flow configurations, high Reynolds numbers, and complex boundary conditions that are currently inaccessible to DNS.

Experimental identification of the TNTI, however, presents significant challenges. Traditional approaches often rely on separate velocity and scalar measurements obtained from different optical systems and, in many cases, from different seeding strategies. The resulting spatial and temporal registration issues complicate the direct comparison between velocity- and scalar-based interface definitions and introduce additional uncertainties in the determination of the interface location.

The present study explores an alternative experimental approach based on the simultaneous use of time-resolved particle image velocimetry (PIV) and Mie scattering. By exploiting the same image acquisition used for the velocity measurements, the scattered light intensity is used as a scalar proxy for particle concentration, enabling the extraction of a TNTI candidate without additional optical instrumentation. This approach offers the advantage of perfect spatial and temporal correspondence between the velocity and scalar information, while maintaining a relatively simple experimental implementation. At the same time, the method inherits the limitations associated with using image intensity as a surrogate scalar quantity and therefore requires careful processing of the resulting scalar fields.

The objective of the present work is to introduce and assess a unified PIV–Mie methodology for the detection and characterization of the TNTI in a supersonic underexpanded jet. Despite the extensive body of research devoted to the TNTI, experimental investigations remain comparatively scarce at high Reynolds numbers, particularly in the range of approximately 10^5 to 10^6 , where the complexity of the flow and measurement challenges significantly increase. The present study, therefore, aims to contribute to the experimental characterization of the TNTI under such conditions while providing a practical methodology based on simultaneous velocity and scalar information. A robust image-processing framework is developed to extract instantaneous interface realizations from time-resolved measurements. Subsequently, a signed-distance formulation

^{*}It provides the 3-components of the velocity vector.

is employed to establish a local coordinate system centered on the interface, enabling conditional statistics of scalar and velocity quantities. The results demonstrate the feasibility of the proposed methodology and provide a foundation for future investigations of entrainment and interface dynamics in compressible turbulent flows.

2. Experimental Setup

The present investigation focuses on a weakly underexpanded supersonic jet issuing into a quiescent atmosphere through a circular orifice nozzle of diameter $D = 5$ mm and aspect ratio $AR_N = L/D = 6.2$, where L denotes the nozzle length. The flow is supplied from a (100)-liter pressurized reservoir maintained at an upstream stagnation pressure of $p_0 = 5 \times 10^5$ Pa and a stagnation temperature of $T_0 \approx 291.15$ K, corresponding to a nozzle pressure ratio $NPR = p_0/p_a = 5$. Under these operating conditions, the flow reaches sonic conditions at the nozzle exit, with a throat pressure of 2.64×10^5 Pa, a throat temperature of 242.6 K, and a throat density of 3.79 kg m^{-3} . The corresponding speed of sound is 312.2 m s^{-1} , yielding a mass flow rate of approximately $2.33 \times 10^{-2} \text{ kg s}^{-1}$ and a Reynolds number of approximately 4.0×10^5 , based on the nozzle diameter and throat conditions (all the previous throat conditions have been calculated using the isentropic relations). The resulting flow therefore lies within the high-Reynolds-number regime characteristic of practical underexpanded gas releases. The jet is seeded with IMERYS Jetfine 2 talc particles, characterized by a mean particle diameter between 1 and $2 \mu\text{m}$. The tracer particles are introduced directly into the pressurized reservoir using a Dantec Dynamics powder seeding generator designed for high-pressure environments. Time-resolved measurements are performed using a Phantom V2012 high-speed camera synchronized with a continuous-wave Spectra-Physics Stabilite 2017 laser. The measurement region covers a field of view of approximately $75 \times 75 \text{ mm}^2$, corresponding to $15 \times 15 D^2$, and is centered at a streamwise location of $x/D = 100$ and a radial position of $r/D \approx 40$. This region lies within the fully developed self-similar turbulent portion of the jet.

Additional large-scale, non-time-resolved PIV measurements are performed using a Litron Bernoulli 200 pulsed laser in combination with a LaVision Imager CX 12 camera. These measurements are employed to characterize the global jet development, including centerline velocity decay, radial velocity distributions, and self-similar behavior in the far field. A schematic view of the experimental setup is reported in Fig. 1.

[†]A rigorous mass flow characterization must include a discharge coefficient characterization, here not reported.

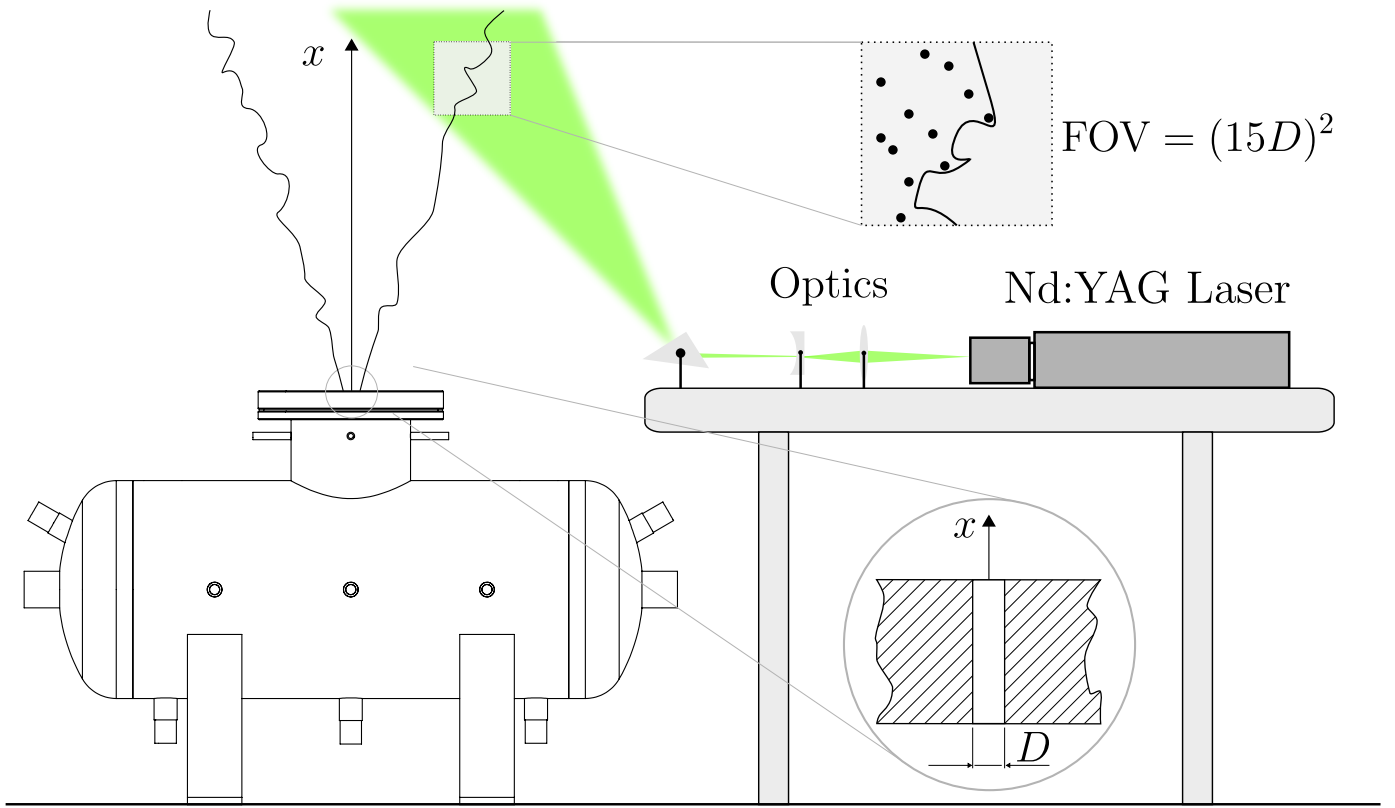


Figure 1. Experimental setup and measurement configuration. The jet issues from a circular nozzle of diameter $D = 5 \text{ mm}$ into a quiescent atmosphere. Time-resolved PIV and Mie-scattering measurements are performed within a $75 \times 75 \text{ mm}^2$ field of view.

All PIV image pairs are processed using an in-house software implementing an iterative multi-grid image deformation algorithm (Scarano & Riethmuller (1999)). The processing framework incorporates FFT-based cross-correlation, iterative window deformation, sub-pixel peak estimation, and outlier detection. The code is accelerated on graphics processing units (GPUs) through CUDA-enabled CuPy implementations.

3. TNTI Detection Methodology

The TNTI was identified from the time-resolved frame acquisition conducted for the PIV measurements. In the present work, the recorded image intensity is employed as a scalar proxy for the local concentration of seeding particles. Under the Beer–Lambert law, the attenuation and scattering of light are related to the concentration of scattering particles; consequently, for the dilute seeding conditions considered here, the scattered light intensity can be assumed to be proportional to the local particle concentration. Prior to jet release, the ambient region was lightly seeded to enable flow visualization and PIV measurements. Nevertheless, the jet flow contained a significantly higher particle concentration than the surrounding ambient fluid, providing sufficient scalar con-

trast to distinguish the turbulent jet from the non-turbulent environment.

3.1. Image Pre-processing and Thresholding

The instantaneous raw frames were acquired directly in grayscale format and subsequently subjected to a Gaussian smoothing operation in order to reduce high-frequency noise while preserving the large-scale structure of the interface. The resulting scalar field, $I(x, y)$, was subsequently thresholded according to

$$\chi(x, y) = \begin{cases} 1, & I(x, y) > I_{th}, \\ 0, & I(x, y) \leq I_{th}, \end{cases} \quad (1)$$

where I_{th} denotes the selected intensity threshold and χ is the binary intermittency function. Pixels associated with the turbulent jet are therefore assigned a value of unity, whereas pixels belonging to the surrounding non-turbulent region are assigned a value of zero.

The threshold value was selected empirically after inspection of the intensity histograms and the resulting interface topology (see Westerweel et al. (2002)). This approach was found to provide a robust separation between the highly seeded jet region and the weakly seeded ambient flow throughout the time-resolved sequence.

3.2. Binary Mask Cleaning

Direct thresholding produces a binary field containing isolated regions arising from local intensity fluctuations, particle clustering, and intermittent pockets of entrained or detrained fluid. To obtain a physically meaningful interface, a sequence of cleaning operations was applied to the binary mask.

First, connected-component analysis was employed to remove isolated regions with areas smaller than a prescribed threshold. Later, only the largest connected turbulent structure spanning the measurement domain was retained. This procedure suppresses spurious external islands and prevents the interface from following the image boundaries.

The resulting binary field defines two distinct regions: the turbulent jet and the surrounding non-turbulent ambient fluid. The TNTI is then obtained as the contour separating these two regions.

3.3. TNTI Extraction

The TNTI is defined as the boundary of the cleaned binary mask corresponding to the iso-contour

$$\chi(x, y) = 0.5, \tag{2}$$

which represents the transition between the two discrete states of the binary intermittency function ($\chi = 0$ in the non-turbulent region and $\chi = 1$ in the turbulent region). The value 0.5 is therefore used as the conventional contour level separating the two classes, providing a sub-pixel representation of the interface location.

The extracted contour represents the instantaneous location of the turbulent/non-turbulent interface candidate and provides a geometrically consistent description of the jet boundary. Examples of the resulting TNTI superimposed on instantaneous scalar images are shown in Fig. 2.

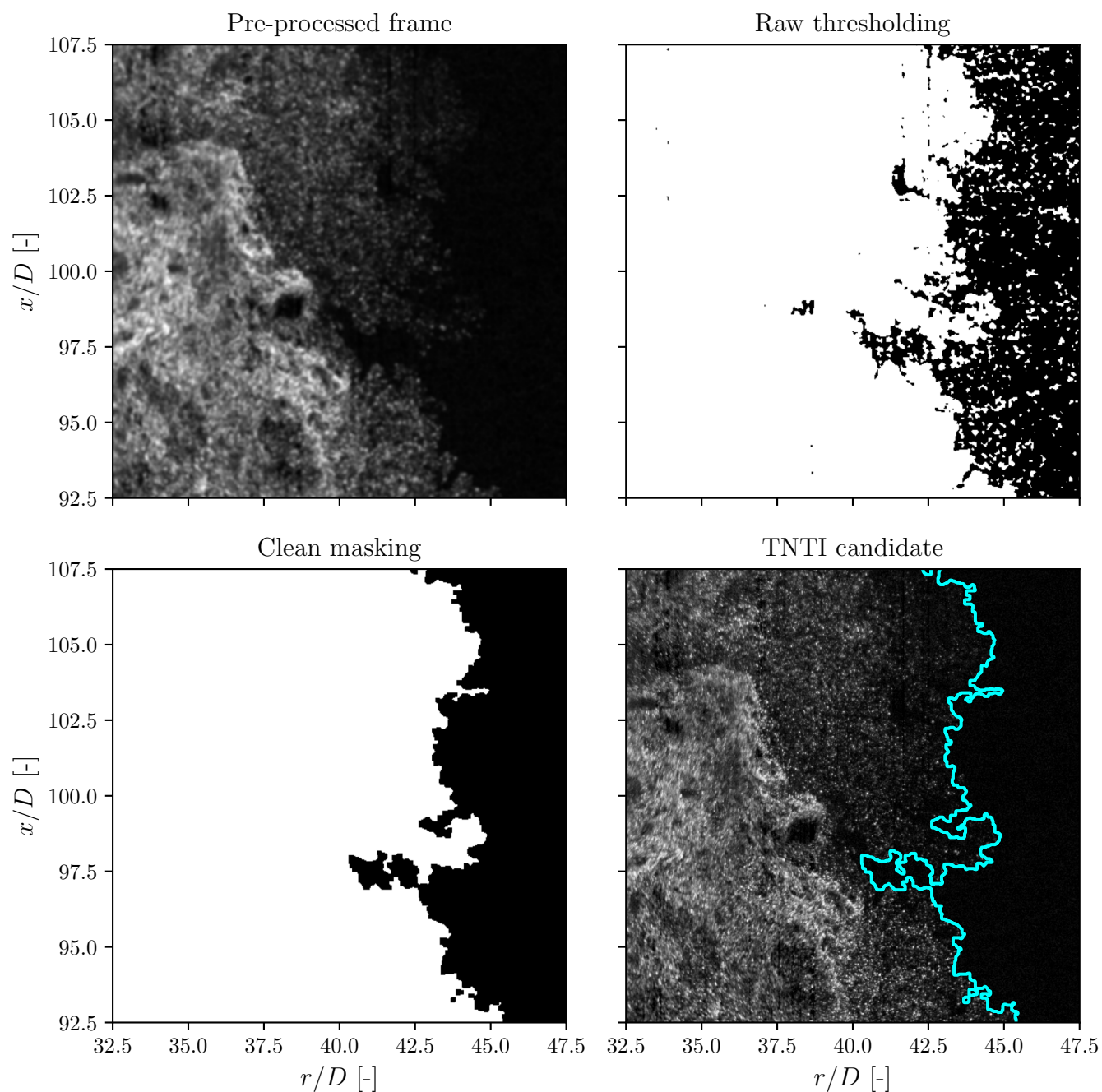


Figure 2. Illustration of the TNTI extraction procedure applied to an instantaneous raw Mie-scattering frame. The processing sequence consists of image thresholding, binary-mask cleaning through connected-component analysis, and contour extraction. The resulting TNTI candidate provides a continuous interface separating the highly seeded turbulent jet from the weakly seeded ambient fluid.

3.4. Signed-Distance Formulation

For the purpose of conditional statistical analysis, a signed-distance field, $n(x, y)$, was constructed from the instantaneous TNTI. For every point in the measurement domain, the shortest Euclidean distance to the interface was evaluated. By convention,

$$n(\mathbf{x}) = \begin{cases} -d(\mathbf{x}, \Gamma), & \mathbf{x} \in T, \\ +d(\mathbf{x}, \Gamma), & \mathbf{x} \in NT, \end{cases} \quad (3)$$

where $d(\mathbf{x}, \Gamma)$ denotes the minimum distance to the TNTI Γ , while T and NT indicate the turbulent and non-turbulent regions, respectively. The interface itself corresponds to

$$n = 0, \quad (4)$$

Here, negative values indicate points within the turbulent jet, and positive values indicate points in the surrounding ambient fluid. The signed-distance field, therefore, provides a local coordinate system centered on the TNTI, allowing scalar and velocity quantities to be conditionally averaged as a function of the distance from the interface.

An example of the resulting signed-distance field and the associated TNTI location is presented in Fig. 3, illustrating the sign convention adopted throughout the present analysis.

4. Conditional Statistical Framework

The signed-distance field introduced in the previous section provides a local coordinate system centered on the TNTI. Using this coordinate system, scalar and velocity quantities can be conditionally averaged as a function of the normal distance from the interface. For a generic quantity ϕ , the conditional average is defined as

$$\langle \phi | n \rangle = \frac{1}{N_n} \sum_{i=1}^{N_n} \phi_i, \quad (5)$$

where the summation is performed over all samples whose signed distance lies within a prescribed interval centered at n , and N_n denotes the number of samples belonging to that interval.

The conditional averaging procedure was applied to both the scalar field and the velocity measurements. In addition to the velocity magnitude,

$$|\mathbf{U}| = \sqrt{u^2 + v^2}, \quad (6)$$

the velocity field was decomposed into components normal and tangential to the TNTI. The unit normal vector $\hat{\mathbf{n}}$ was obtained from the gradient of the signed-distance field,

$$\hat{\mathbf{n}} = \frac{\nabla n}{|\nabla n|}, \quad (7)$$

while the unit tangent vector was defined as

$$\hat{\mathbf{t}} = (-\hat{n}_y, \hat{n}_x). \quad (8)$$

The corresponding velocity components are therefore given by

$$u_n = \mathbf{U} \cdot \hat{\mathbf{n}}, \quad (9)$$

and

$$u_t = \mathbf{U} \cdot \hat{\mathbf{t}}. \quad (10)$$

This framework enables the statistical characterization of the scalar and velocity fields relative to the TNTI and forms the basis of the conditional profiles presented in the following section.

5. Results and Discussion

5.1. TNTI Detection

The combination of intensity thresholding, binary-mask cleaning, and contour extraction successfully identifies a continuous interface separating the highly seeded turbulent jet from the weakly seeded surrounding fluid (see Fig. 2). The resulting TNTI captures the large-scale geometrical features of the jet boundary while suppressing isolated regions associated with local intensity fluctuations, particle clustering, and intermittent pockets of entrained or detrained fluid. The extracted interface exhibits the highly convoluted geometry typically associated with turbulent free shear flows. Large-scale structures are observed along the interface, reflecting the interaction between the turbulent jet and the *irrotational* ambient fluid. Such structures are generally associated with the engulfment mechanism responsible for entrainment (Westerweel et al. (2005); Turner (1986); Bisset et al. (n.d.)), whereby large turbulent eddies incorporate external fluid into the turbulent region.

5.2. Conditional Statistics Across the TNTI

The conditional statistics obtained using the signed-distance framework are presented in Fig. 4. The profiles represent averages computed over all instantaneous realizations as a function of the normal distance from the TNTI.

Figure 4(a) shows the conditional scalar intensity profile. A pronounced transition is observed across the interface, with the highest intensity values occurring inside the turbulent region and a rapid decrease towards the ambient fluid. The sharp variation around $n = 0$ confirms that the extracted interface corresponds to the boundary separating regions characterized by notably different particle concentrations. A similar behavior is observed for the conditional velocity magnitude shown in Fig. 4(b). Moving from the turbulent region towards the ambient fluid, the velocity magnitude decreases monotonically from approximately 26 ms^{-1} to values approaching 2 ms^{-1} . The smooth transition across the interface suggests the progressive exchange of momentum between the turbulent jet and the surrounding environment. The decomposition of the velocity field into normal and tangential components relative to the interface provides additional insight into the local flow structure.

The conditional normal velocity profile, shown in Fig. 4(c), exhibits a behavior closely resembling that of the velocity magnitude, decreasing continuously from the turbulent core towards the ambient region and approaching zero at the interface. This observation indicates that the local velocity field is predominantly aligned with the interface normal direction in the present configuration.

In contrast, the conditional tangential velocity shown in Fig. 4(d) remains comparatively small

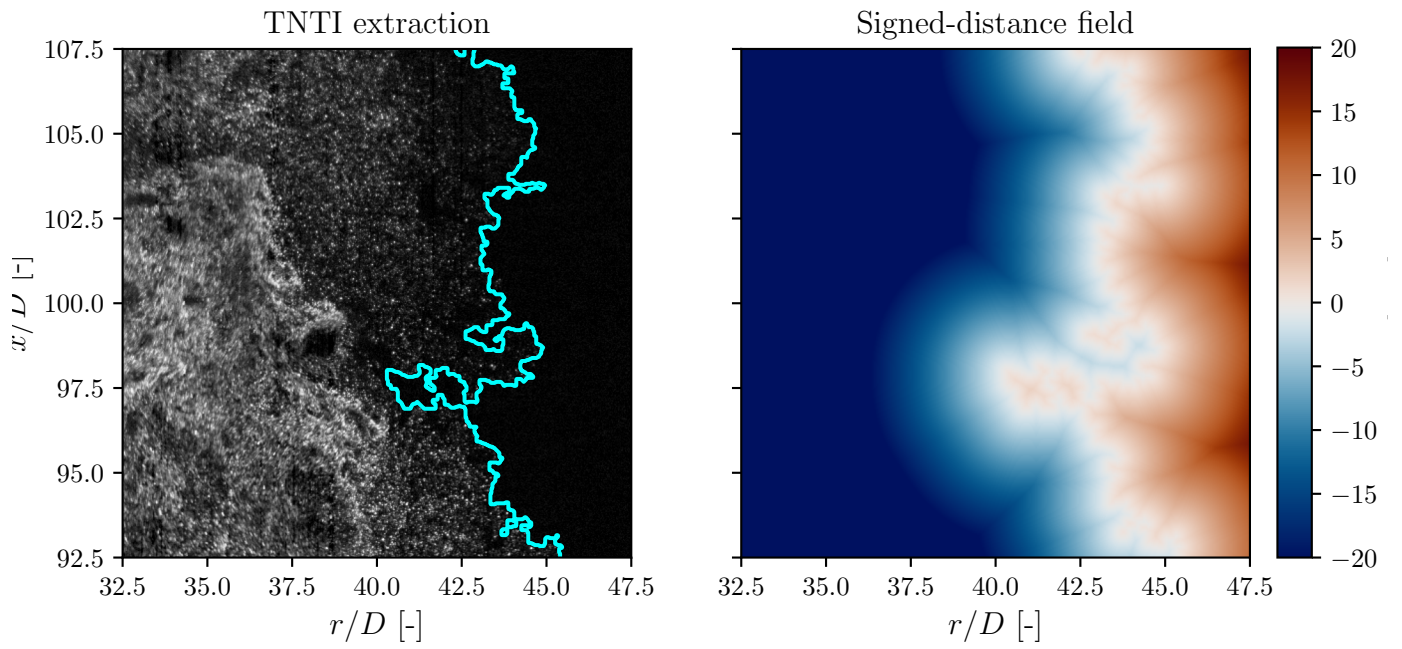


Figure 3. Example of the signed-distance field used for conditional statistical analysis. (a) Instantaneous TNTI candidate superimposed on the scalar field. (b) Corresponding signed-distance field n , obtained from the minimum Euclidean distance to the interface. The zero-level set ($n = 0$) coincides with the TNTI, while negative and positive values identify the turbulent and non-turbulent regions, respectively. This local coordinate system forms the basis of the conditional averaging framework adopted in the present study.

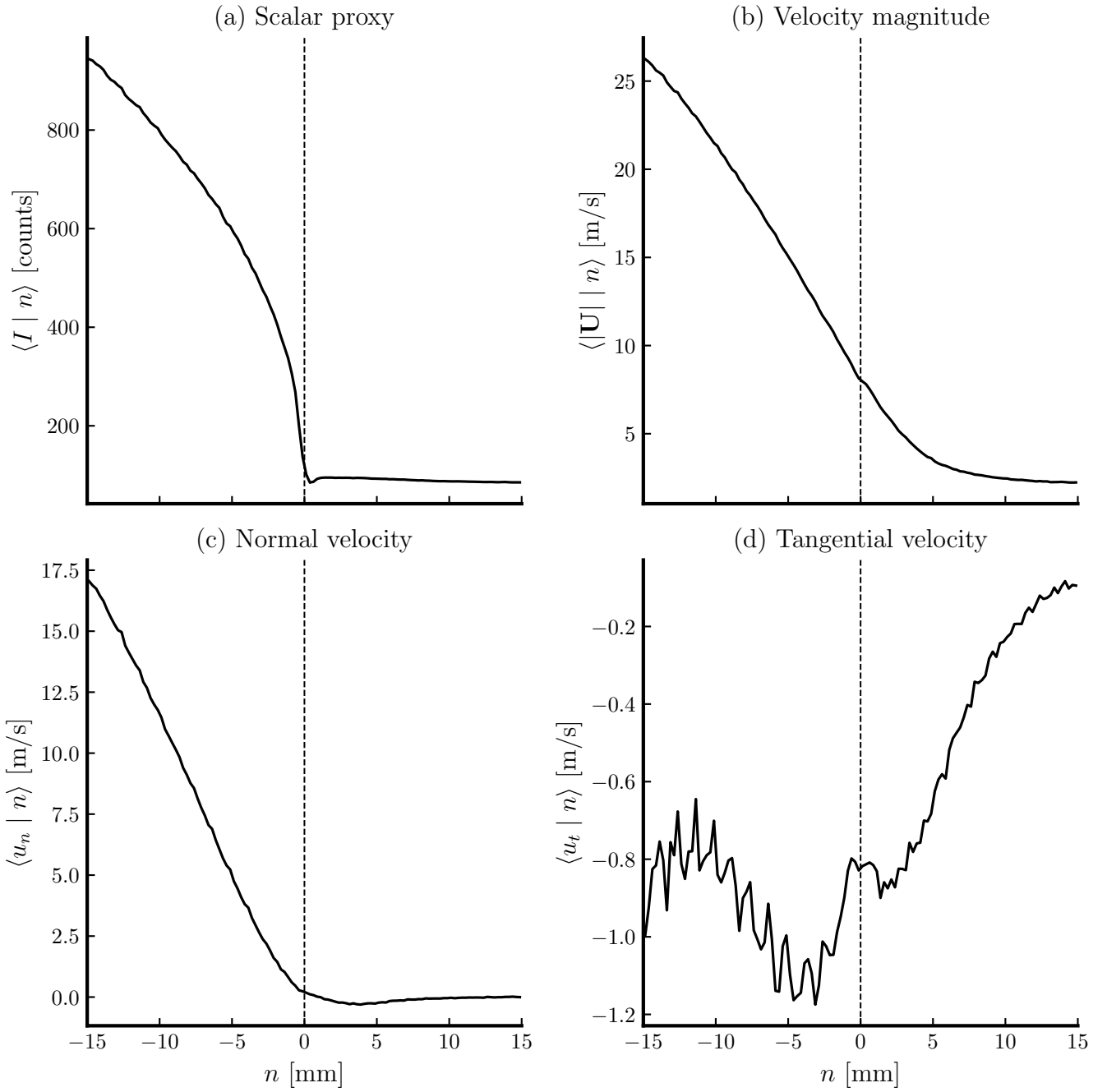


Figure 4. Conditional statistics computed as a function of the signed distance from the TNTI. (a) Conditional scalar intensity profile. (b) Conditional velocity magnitude. (c) Conditional velocity component normal to the interface. (d) Conditional velocity component tangential to the interface. The vertical dashed line denotes the interface location ($n = 0$). The profiles reveal the transition between the turbulent (jet) and non-turbulent (the surrounding ambient fluid) regions.

throughout the entire measurement domain. Its magnitude remains below approximately 1 ms^{-1} , which is significantly smaller than the corresponding normal component. The near-zero values observed on both sides of the interface suggest that the dominant velocity variations across the TNTI are associated with changes in the normal component rather than tangential motion.

The conditional profiles demonstrate that the proposed TNTI extraction methodology produces an interface that consistently separates regions characterized by distinct scalar concentration and velocity levels.

5.3. Temporal Robustness of the TNTI Detection

To assess the robustness of the proposed methodology, representative instantaneous TNTI realizations extracted from the time-resolved image sequence are presented in Fig. 5. Despite the significant temporal evolution of the turbulent structures, the extracted interface remains continuous and physically consistent throughout the sequence.

The methodology captures the temporal deformation of the interface while preserving its large-scale topology. The observed variations in the TNTI position reflect the natural unsteadiness of the high-speed turbulent jet and confirm the proposed approach's ability to track interface evolution over time. This capability constitutes a necessary prerequisite for future investigations of TNTI dynamics, intermittency, and entrainment processes using time-resolved measurements.

Overall, the results demonstrate the feasibility of combining time-resolved PIV and Mie-scattering measurements for TNTI identification in compressible high-speed free-shear flows. The proposed framework provides simultaneous access to velocity and scalar information within a common measurement system and establishes the basis for more detailed future analyses of interface geometry and turbulent entrainment.

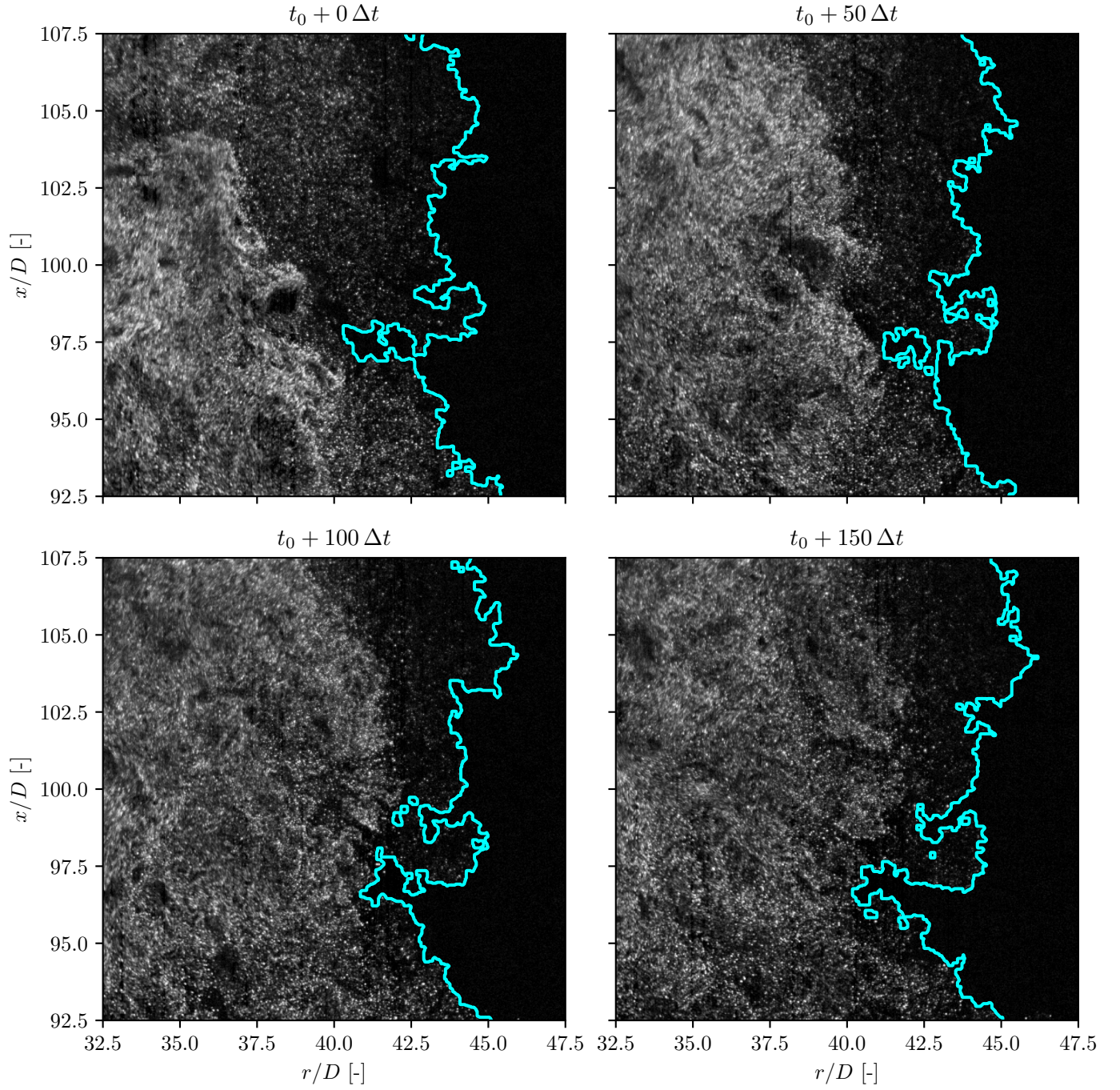


Figure 5. Instantaneous TNTI realizations extracted from the time-resolved image sequence. The contour line denotes the detected TNTI candidate. The interface remains continuous and physically consistent throughout the sequence while capturing the temporal deformation of the jet boundary, demonstrating the robustness of the proposed PIV–Mie detection methodology.

Nomenclature

AR_N	Nozzle aspect ratio L/D [-]
$d(\mathbf{x}, \Gamma)$	Minimum Euclidean distance from point $\mathbf{x}$ to the TNTI [mm]
D	Nozzle diameter [m]
I	Image intensity (scalar proxy) [-]
I_{th}	Intensity threshold [-]
L	Nozzle length [m]
$\dot{m}$	Mass flow rate [kg s^{-1}]
n	Signed distance from the TNTI [mm]
N_n	Number of samples within a distance bin [-]
NPR	Nozzle pressure ratio, p_0/p_a [-]
p_a	Ambient pressure [Pa]
p_0	Upstream stagnation pressure [Pa]
r	Radial coordinate [m]
T_0	Upstream stagnation temperature [K]
u	Radial velocity component [m s^{-1}]
u_n	Velocity component normal to the TNTI [m s^{-1}]
u_t	Velocity component tangential to the TNTI [m s^{-1}]
v	Axial velocity component [m s^{-1}]
$\mathbf{U}$	Velocity vector [m s^{-1}]
$ \mathbf{U} $	Velocity magnitude [m s^{-1}]
x	Axial coordinate [m]
Γ	Turbulent/non-turbulent interface (TNTI) [-]
χ	Binary intermittency function [-]
ϕ	Generic flow quantity [-]
$\langle \phi n \rangle$	Conditional average of ϕ at distance n from the TNTI [-]
$\hat{\mathbf{n}}$	Unit normal vector to the TNTI [-]
$\hat{\mathbf{t}}$	Unit tangent vector to the TNTI [-]
DNS	Direct Numerical Simulation
GPU	Graphics Processing Unit
PIV	Particle Image Velocimetry
TNTI	Turbulent/Non-Turbulent Interface
TR	Time-Resolved
PIV	Particle Image Velocimetry

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