

Burst-Mode Imaging Analysis of Flicker Oscillations in an Unsteady Hydrogen/Air Jet Flame Using Proper Orthogonal Decomposition

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ABSTRACT

Proper orthogonal decomposition (POD) was applied to 5-kHz images of an unsteady H₂/air jet flame to demonstrate its suitability for recovering dynamical information from high-repetition-rate burst-mode measurements. To make progress toward obtaining robust descriptions of turbulent flame dynamics, an unsteady flickering flame with a well-defined low-frequency oscillation was first considered. The POD approach was validated using synthetic flame images, where the oscillation frequency at 10 Hz was successfully estimated from long, continuous recordings and ensembles of short bursts, demonstrating the efficacy of the analysis even under the acquisition constraints characteristic of burst-mode laser systems. Further analyses of synthetic oscillations at 100 Hz and 500 Hz elucidated the primary error-reduction mechanisms and showed that the estimation error can be reduced below 1% with appropriate choices of the burst length and the number of bursts. POD was then applied to experimental ensembles of image bursts with an underlying flicker frequency of 11.4 Hz. For these cases, the oscillation frequency was successfully recovered, along with the detailed spatiotemporal representation of the flame. These findings establish POD as a useful tool for burst-mode imaging analysis and provide a foundation for its application to faster dynamics and with laser-based imaging diagnostics such as OH-PLIF, NO-PLIF, and PIV.

1. Introduction

Understanding the unsteady behavior of hydrogen (H₂) flames is essential for developing decarbonized combustion systems for the hard-to-abate aviation sector (Zaporozhets et al., 2021). While the unique thermophysical properties of H₂ establish the potential for improved performance and reduced operating costs for medium- and long-range aircraft (Hoelzen et al., 2022; Adler & Martins, 2023), they also present new constraints for next-generation combustion technologies. Imaging and laser-based diagnostics will continue to drive developments in this domain, for example with low-NO_x kerosene injectors (Legros et al., 2021) and H₂ strut injectors (Gope et al., 2025). However, the rapid flow and combustion timescales of H₂, mostly inaccessible to low-speed laser and imaging systems, demand the development of new high-speed diagnostics. A burst-mode laser-measurement platform combining high-speed imaging, planar laser-induced fluorescence (PLIF), and particle image velocimetry (PIV) has been incrementally developed for

this purpose (Schweizer et al., 2024, 2025). To fully exploit these advanced optical diagnostics, new data analysis methods for high-speed and burst-mode measurements must also be developed.

Decomposition-based approaches offer promising capabilities for analyzing unsteady flames, such as detecting and compactly representing the coherent flow structures. In a recent study, dynamic mode decomposition (DMD) was applied to images of coaxial H₂/air jet flames recorded at 5 kHz. The method was successful at representing and reconstructing the dominant low-frequency flickering dynamics. Despite this demonstrated effectiveness, the implementation relied on long, continuous acquisitions (~200-ms recordings). This constraint limits the applicability of DMD to measurements made with burst-mode laser systems, which have maximum burst durations on the order of tens of milliseconds.

To further explore the capabilities of decomposition approaches with respect to burst-mode measurement systems and data, the current study applies proper orthogonal decomposition (POD) to flame images from the same H₂/air jet configuration. Through the analysis of synthetic and experimental flame images, we demonstrate that POD is indeed suitable for capturing the low-frequency oscillation of flickering flames from both long and burst-mode recordings. Details of the experimental apparatus, test conditions, and data analysis are reported in Section 2. The results from synthetic and experimental flame images are presented and discussed in Section 3. Finally, Section 4 outlines the prospects for upcoming work, namely the application of POD to burst-mode laser imaging measurements.

2. Methods

2.1 Experimental apparatus

A coaxial jet burner, shown in Fig. 1 and described further in (Schweizer et al., 2024, 2025), was used to produce jet diffusion flames of H₂ in air. The burner comprised a central 1.84-mm fuel nozzle, a 20-mm air nozzle, and a 150-mm outer annulus. For the validation and application of POD in this study, the burner was operated with a fuel-jet Reynolds number $Re_f = 1000$ ($U_f = 57.14$ m/s) and no air coflow. Thus, the flame-flicker oscillation could be interrogated without interference from additional shear-layer interactions and high-frequency turbulent fluctuations.

To resolve the flame dynamics, 5-kHz images of the luminous flame emission were recorded with a high-speed camera (Phantom TMX 7510) equipped with an intensified relay optic (LaVision High Speed IRO), a UV-sensitive lens (CERCO UV, 100-mm focal length, f/2.8), and a long-pass filter. While not strictly necessary for this experiment, the intensifying and filtering elements enable flexibility to accommodate future studies of interest requiring faster repetition rates (5–30 kHz) and laser diagnostics (PIV/PLIF). For this study, an intensifier gain of 60% and a gate width of 150 μ s were used to obtain flame images with a signal-to-noise ratio (SNR) of approximately 25.

Experimental flame images were recorded in various configurations to obtain POD results under different acquisition constraints. While this work did not couple a burst-mode laser source to the imaging system, certain image datasets were captured in the burst-mode format in anticipation of forthcoming laser-diagnostic experiments. These acquisition configurations are detailed in Section 2.2.

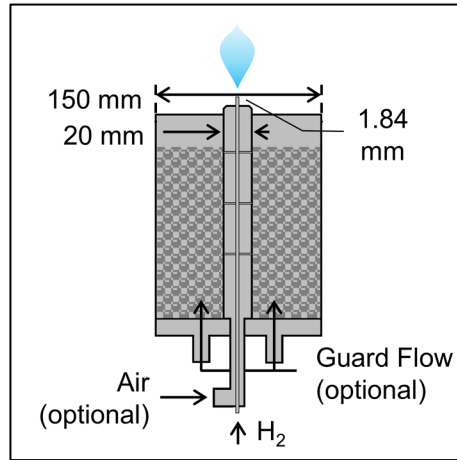


Figure 1. Experimental schematic for the coaxial H₂/air jet burner.

2.2 Data analysis

To assess the suitability of POD for extracting dynamical information from high-speed and burst-mode measurements of flickering flames, a structured analysis was performed on both synthetic and experimental image data. In this section, we first present the POD implementation used to obtain a separable spatiotemporal representation of the flame dynamics. We then introduce a complex-signal framework for the decomposition's temporal component, which was developed to estimate the frequency of the flame oscillation even under burst-mode acquisition constraints. The procedure for generating synthetic flame images and emulating their acquisition is then detailed. Finally, with these elements in place, we describe the synthetic and experimental test cases used to valorize the burst-mode POD approach.

2.2.1 POD implementation

The suitability of POD for burst-mode measurements stems from its formulation as an ensemble-based decomposition that does not require temporal continuity. In other words, POD can be applied to both long-duration, time-resolved recordings and large collections of independently acquired snapshots. Burst-mode datasets occupy an intermediate regime, comprising ensembles of short, time-resolved sequences. For periodic dynamics, POD yields pairs of modes with phase-shifted temporal coefficients. This phase relationship can be exploited to

recover the underlying oscillation frequency. A similar approach has been used to recover some dynamical information from flows, even without time-resolved measurements (Oudheusden et al., 2005; Iudiciani et al., 2010; Stöhr et al., 2011). In the present work, a POD-based analysis to represent the dynamics of an unsteady H₂/air flame and estimate its flickering frequency is developed. The analysis is then extended to ensembles of independently captured high-speed bursts.

For the proposed POD implementation, 5-kHz images were organized into ensembles of N_B bursts of equal length L_B . Each 1200-pixel (height) $\times$ 800-pixel (width) image was downsampled by a factor of 3 to reduce the computational cost, reshaped to a column vector, and mean-subtracted to isolate the fluctuating component of the flame dynamics. The resulting data matrix was assembled as $\mathbf{X} = [\mathbf{q}(\mathbf{x}, t_1) \mathbf{q}(\mathbf{x}, t_2) \dots \mathbf{q}(\mathbf{x}, t_m)]$, where $\mathbf{q}(\mathbf{x}, t_k)$ is the pre-processed image (column vector of intensity values) acquired at time t_k . POD represents this data with a separable (space/time) series expansion: $\mathbf{q}(\mathbf{x}, t_k) = \sum_{r=1}^R a_r(t_k) \phi_r(\mathbf{x})$, where $a_r(t_k)$ is the r -th temporal coefficient, $\phi_r(\mathbf{x})$ is the corresponding spatial mode, and R is the maximum number of modes supported by the data matrix (truncating to a smaller value yields a reduced-order approximation of the data). The POD representation was obtained by performing the singular value decomposition $\mathbf{X} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^*$, where $\mathbf{U}$ contains the spatial modes, $\mathbf{\Sigma}$ is the diagonal matrix of singular values, and $\mathbf{V}^*$ contains the unscaled temporal coefficients. The POD eigenvalues are given by $\lambda_r = \sigma_r^2$, where $\sigma_r = \Sigma_{rr}$ is the r -th singular value of $\mathbf{X}$.

After decomposition of the data matrix, the leading pair of temporal coefficients (the first and second columns of $\mathbf{\Sigma} \mathbf{V}^*$) were assembled into the complex signal $z_k = a_1(t_k) + ia_2(t_k)$, with the phase given by $\theta_k = \arg(z_k)$. Previous studies have estimated the dominant oscillation frequency through differentiation of the phase evolution (Mugundhan et al., 2025) or a power spectral density analysis of the temporal coefficients (Sieber et al., 2016). While effective on continuously sampled data, these approaches do not readily extend to burst-mode datasets, which introduce temporal and phase discontinuities between each burst. Ensemble separation and averaging strategies are also ineffective when each burst is not sufficiently long to support an independent frequency estimate. To address these constraints, the complex signal was modeled (in a manner analogous to DMD) as $z_k \approx A_B e^{i\omega_0 t_k}$. Here, the complex amplitude A_B was allowed to vary across bursts to accommodate phase discontinuities within the ensemble. An estimate of the global frequency $\widehat{\omega}_0 = 2\pi \widehat{f}_0$ was then obtained by minimizing the residual between z_k and $A_B e^{i\omega_0 t_k}$ across all bursts.

2.2.2 Synthetic data generation and sampling

To assess the performance of the proposed approach under controlled conditions, the analysis was first applied to a set of synthetic flame images, which were generated using a simple parametric model. Rather than simulating the full physics of combustion, the flame model was constructed to qualitatively reproduce the spatiotemporal features of a flickering flame. The flame was approximated as a vertical column of bright pixels spanning the image height, with a traveling wave imposed along the boundary to mimic the bulging and pinching caused by periodically shed toroidal vortices (Xia & Zhang, 2018). The half-width of the flame was modulated row-wise according to $w(y, t) = w_0[1 + A_0 \sin(2\pi f_0 t - k_0 y)]$, where w_0 is the initial normalized half-width, A_0 is the bulge amplitude, f_0 is the linear oscillation frequency, k_0 is the vertical spatial wavenumber, and y is the position along the vertical axis.

The intensity values were then scaled to produce a flame with bright bulged regions, dim pinched regions, and a narrow luminous edge. Furthermore, a lateral phase offset $\psi(x, y) = \psi_e |x - x_c|/w(y, t)$ was applied, where ψ_e is the phase shift at the flame edge, x_c is the position of the flame centerline, and x is the position along the lateral axis. This offset profile caused the interior intensity modulation to lag the boundary oscillation. For each burst ensemble, a random (uniformly distributed) initial flame phase was selected with $\psi_{f,i} \sim \mathcal{U}[0, 2\pi)$. The flame phase was then advanced according to $\psi_f(k) = \psi_{f,i} + k\Delta\psi_f$, with $\Delta\psi_f = 2\pi f_0 \Delta t$. A representative synthetic flame image is presented in Fig. 2(a). To obtain good qualitative agreement with the experimental image in Fig. 2(b), the synthetic flame image was generated with $w_0 = 0.08$, $A_0 = 0.25$, $f_0 = 11.4$ Hz, $|k_0| = 3.5\pi$, and $|\psi_e| = \pi/2$. As shown in Figs. 2(c) and (d), the synthetic flame image also reproduces the characteristic modal features that were observed experimentally, namely the alternating band pattern with edge lobes leading the interior.

This synthetic data generation and sampling scheme enabled a robust validation of the POD-based approach under idealized conditions (with the oscillation and acquisition parameters known exactly and without experimental noise or real flame variability). The controlled environment also allowed assessment of the performance in terms of the accuracy and precision of the frequency estimate. For future work, the synthetic framework can be expanded to include a model for image noise, different flame shapes, and non-uniform probability distributions to govern the burst acquisition.

2.2.3 Synthetic test cases

The POD analysis was first validated on three illustrative synthetic configurations. **Synthetic Case 1** (SC1: $N_B = 1$ burst, $L_B = 500$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz) represents a long, continuous recording that provides full temporal coverage of the oscillation cycle. This case

establishes the baseline behavior of the frequency-estimation approach under favorable acquisition conditions. **Synthetic Case 2** (SC2: $N_B = 10$ bursts, $L_B = 50$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz) uses the same total number of frames ($F_{tot} = L_B \times N_B$) but segments the dataset into short bursts, mimicking the acquisition pattern of burst-mode laser and imaging systems. This case demonstrates the feasibility of the burst-mode POD concept but highlights the degradation of performance when coverage of the oscillation cycle is sparse. Finally, **Synthetic Case 3** (SC3: $N_B = 50$ bursts, $L_B = 50$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz) preserves the short-burst structure of SC2 but increases the total number of frames. This case demonstrates how repeated sampling can recover performance, albeit at the cost of collecting and processing a larger dataset. The performance of the frequency estimator was assessed using a Monte-Carlo (MC) framework. For each MC evaluation, N_B synthetic bursts of length L_B were simulated at the prescribed frequency f_0 , with the full processing routine (POD + frequency estimation) repeated n_{MC} times (each burst with a random initial flame phase) to obtain a sampling distribution for the estimated frequency $\hat{f}_0$. This distribution was used to compute the mean absolute error of the estimator for SC1–SC3.

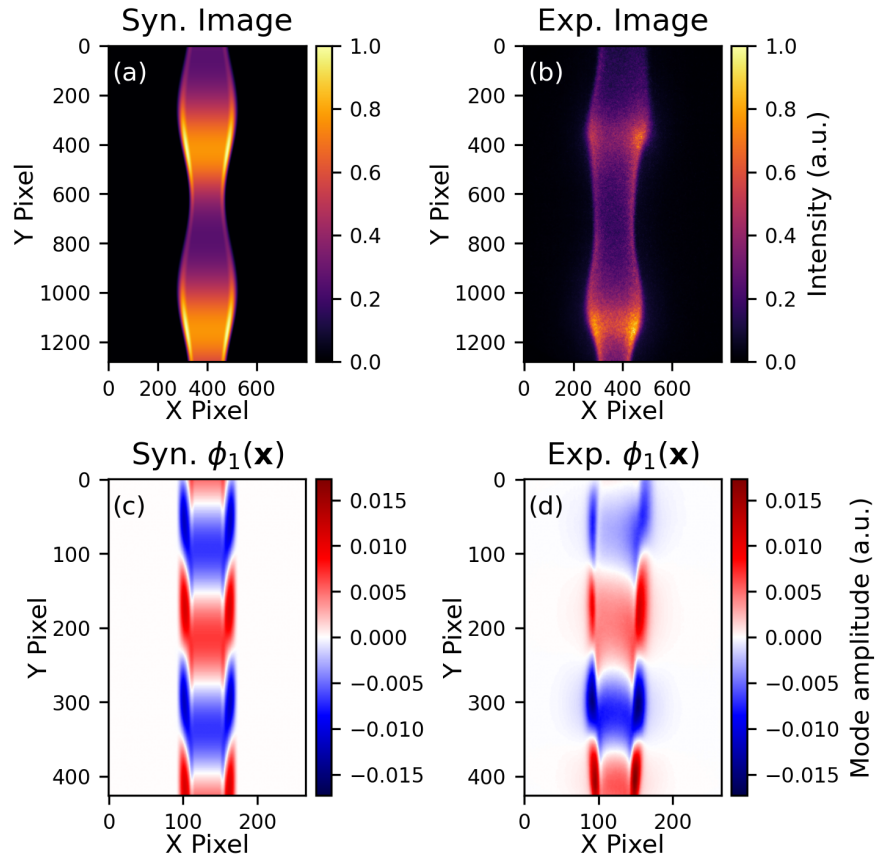


Figure 2. (a) Synthetic and (b) experimental flame images. (c) Synthetic and (b) experimental POD spatial modes: $\phi_1(\mathbf{x})$. The reduced pixel resolution in (c) and (d) reflects the spatial downsampling applied during processing.

2.2.4 Monte-Carlo scaling analysis

A broader MC scaling analysis was then performed with $f_0 = 100$ Hz (MC1) to determine the impact of the two controllable acquisition parameters L_B and N_B on the estimation error. This analysis is framed by an important practical question: for a given oscillation frequency, which burst mode acquisition strategies are appropriate for accurately characterizing the dynamics? The 100-Hz case represents an order-of-magnitude increase relative to the 10-Hz baseline and moves the analysis into the frequency range of turbulent flow and combustion instabilities (Iudiciani et al., 2010; Stöhr et al., 2011). A complementary set of evaluations was also performed at $f_0 = 500$ Hz (MC2) to assess whether the scaling trends persist when the oscillation frequency is further increased.

2.2.5 Experimental test cases

After validating the POD-based analysis on synthetic data, the methodology was applied to the experimental measurements. **Experimental Case 1** (EC1: $N_B = 1$ burst, $L_B = 440$ frames/burst, $f_0 = 11.4$ Hz, $f_s = 5$ kHz) provides a long, continuous acquisition analogous to SC1. Next, **Experimental Case 2** (EC2: $N_B = 10$ bursts, $L_B = 44$ frames/burst, $f_0 = 11.4$ Hz, $f_s = 5$ kHz) reproduces the burst structure of SC2 and demonstrates how sparse sampling and burst-to-burst variability impact the representation of the dynamics in practice. Finally, **Experimental Case 3** (EC3: $N_B = 50$ bursts, $L_B = 44$ frames/burst, $f_0 = 11.4$ Hz, $f_s = 5$ kHz) mirrors SC3 and assesses the degree to which improved sampling can recover performance despite these experimental uncertainties. For this work, successive recordings were separated by a sufficiently long time (> 10 seconds) to obtain an independent sample of the flame phase and avoid temporal correlation between the bursts. This delay also mimics the inter-burst delay imposed on burst-mode laser systems to allow for thermal relaxation of the lasing media.

The experimental results were compared to those from synthetic datasets with identical acquisition parameters, enabling a direct comparison between idealized and real conditions. For the experimental recordings, the flame sometimes exhibited spatial asymmetry and intermittent fluctuations that disturbed the dominant flickering motion. Furthermore, the bulged and pinched regions did not follow a perfect sinusoidal wave shape but showed locally stretched and compressed segments. With the full set of synthetic, Monte-Carlo, and experimental test cases now defined, Table 1 summarizes the configurations used to valorize the POD-based analysis of flame flicker.

Table 1. Summary of Synthetic and Experimental Datasets

Case	N_B	L_B	F_{tot}	f_0	f_s	C_B
ID	[bursts]	[frames/burst]	[frames]	[Hz]	[kHz]	[cycles/burst]
SC1	1	500	500	10.0	5	1.0
SC2	10	50	500	10.0	5	0.1
SC3	50	50	2500	10.0	5	0.1
MC1	1–500	3–100	3–5000	100	5	0.06–2.0
MC2	1–200	2–20	2–4000	500	5	0.2–2.0
EC1	1	440	440	11.4	5	1.0
EC2	10	44	440	11.4	5	0.1
EC3	50	44	2200	11.4	5	0.1

3. Results and discussion

3.1 POD analysis of synthetic flame images

The suitability of POD for the analysis of burst-mode datasets is first validated with the long-acquisition case (SC1). A good estimate of the oscillation frequency is expected, since the burst length is sufficient to capture a full oscillation cycle ($C_B = L_B f_0 / f_s = 1$ cycle/burst). Since the periodic component of the dynamics is well-captured in the dataset (i.e., no portion of the cycle is missing), the POD eigenvalue spectrum adopts the paired structure shown in Fig. 3(a). Here the leading pair of modes account for the majority of variance in the dataset (~85%), with the subsequent pairs accounting for increasingly smaller fractions of the total. The corresponding spatial modes $\phi_1(\mathbf{x})$ and $\phi_2(\mathbf{x})$ are shown in Figs. 3(b) and (c). These modes, nearly identical apart from a spatial phase shift, form an orthogonal basis for the evolution at the fundamental angular frequency ω_0 .

The corresponding temporal coefficients $a_1(t_k)$ and $a_2(t_k)$ are presented in Fig. 4(a), revealing a shifted pair of sinusoid signals. This pairing of modes is analogous to the sine-cosine basis established with a Fourier decomposition or the complex-conjugate pairing found in DMD (Sieber et al., 2016; Mugundhan et al., 2025). The trajectory traced by the temporal coefficients is then plotted on Fig. 4(b) after centering the data and normalizing by $\sqrt{2\lambda_k}$ (Oudheusden et al., 2005; Stöhr et al., 2011), revealing a loop that closely follows the unit circle. Based on this correlation between the temporal coefficients, a frequency $\hat{f}_0 = 10.06$ Hz was estimated using the methodology described in Section 2.2, yielding an absolute error of 0.6% for this dataset. The subsequent MC evaluation ($n_{MC} = 100$) of additional SC1 datasets yielded a mean absolute error of 0.7%. This estimator performance demonstrates that under favorable acquisition conditions, the

POD-based analysis provides an accurate representation of the dynamics and a highly reliable estimate of the flickering frequency.

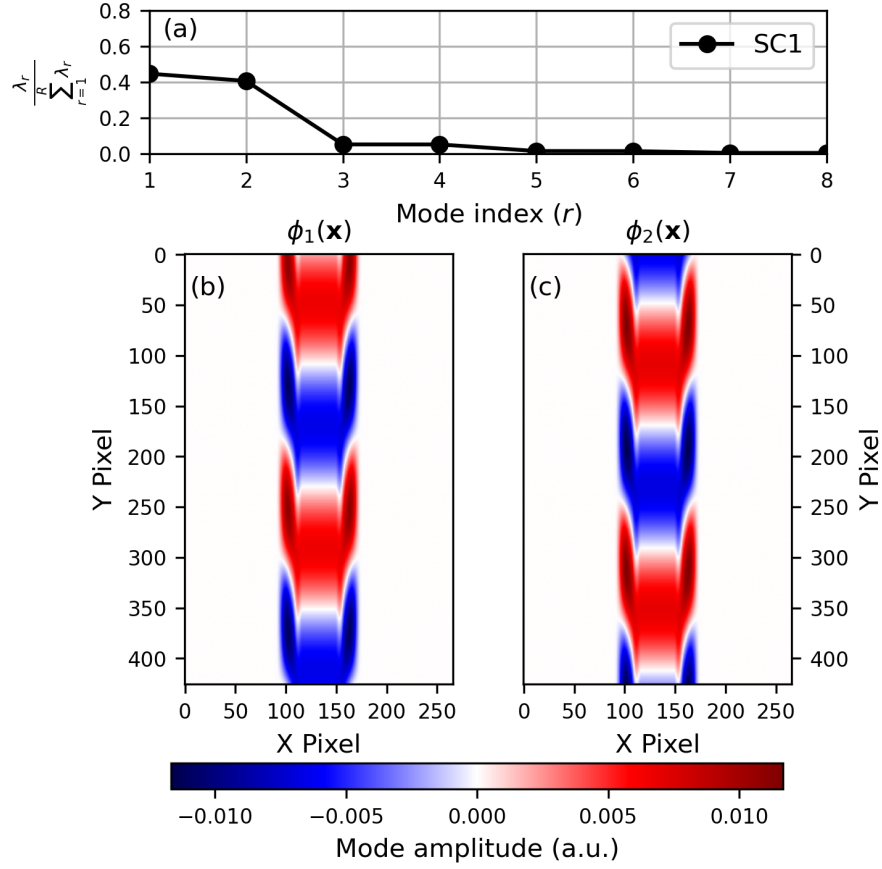


Figure 3. (a) Eigenvalue spectrum (first eight modes), along with the (b) first and (c) second spatial modes produced by POD of the SC1 dataset ($N_B = 1$ burst, $L_B = 500$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz).

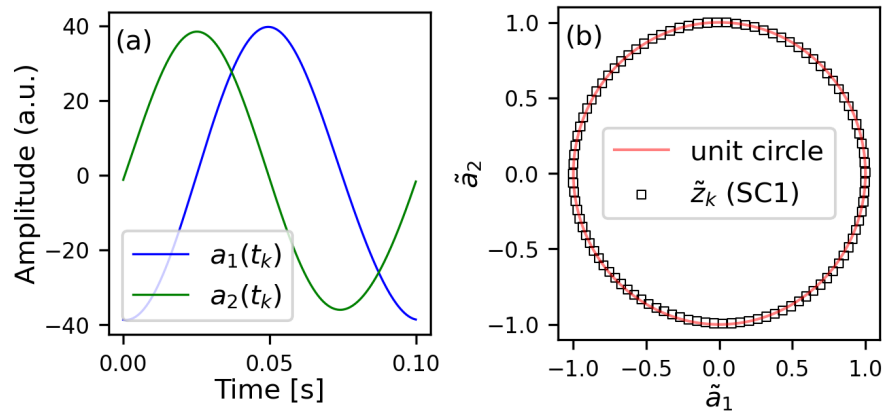


Figure 4. (a) Leading temporal coefficients and (b) the corresponding phase portrait produced by POD of the SC1 dataset ($N_B = 1$ burst, $L_B = 500$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz). Every fifth data point is shown in (b).

Next, the POD approach is applied to the first synthetic burst-mode imaging case (SC2). In this case, the individual bursts cannot provide complete temporal coverage of the oscillation ($C_B = L_B f_0 / f_s = 0.1$ cycles/burst), so the POD-based analysis must rely on the ensemble of bursts to provide coverage of the global cycle in phase. Complete phase coverage is not guaranteed, as the bursts will likely sample similar portions of the cycle. Therefore, the representations and frequency estimates obtained from POD in SC2 are expected to be less reliable compared to the long-acquisition results in SC1.

Indeed, the eigenvalue spectrum in Fig. 5(a) lacks the paired structure seen in the previous case, instead showing a smoother decay. Furthermore, $\phi_2(\mathbf{x})$ in Fig. 5(c) shows the characteristic phase shift relative to $\phi_1(\mathbf{x})$ in Fig. 5(b), but the edge lobes are a different shape. This distortion further indicates that the decomposition has only partially resolved the underlying periodic structure.

Considering now the time-dependent component, the temporal coefficients in Fig. 6(a) appear irregular when plotted sequentially. This is a predictable consequence of the acquisition configuration, which introduces discontinuities between successive bursts that obscure the oscillation at 10 Hz. Nevertheless, the phase portrait in Fig. 6(b) confirms that the temporal coefficients retain their correlated structure. For SC2, a pure sine-cosine pairing is not observed as with the previous case, with the data instead tracing out elliptical arcs. In this case, complete coverage of the oscillation cycle is not obtained. Several bursts have overlapping trajectories, while other areas of the parameter space are left vacant.

Surprisingly, even with incomplete phase coverage, a reasonable frequency estimate $\hat{f}_0 = 10.17$ Hz was still obtained for this dataset, corresponding to an absolute error of 1.7%. Nevertheless, the subsequent MC evaluation ($n_{MC} = 100$) revealed the overall degraded performance, with a mean absolute error of 6.8%. This result demonstrates that the accuracy of the estimated frequency depends not only on the total amount of data acquired, but ultimately the phase coverage achieved with the given values of N_B and L_B . Furthermore, the SC2 analysis suggests that the decomposition components can be informative in the burst-mode acquisition regime. When the underlying dynamics are known to be periodic, the structure of the modes and temporal coefficients can indicate whether the acquisition parameters are sufficient. However, for unknown or poorly understood dynamics, it may be challenging to determine whether the decomposition has faithfully represented the flow behavior.

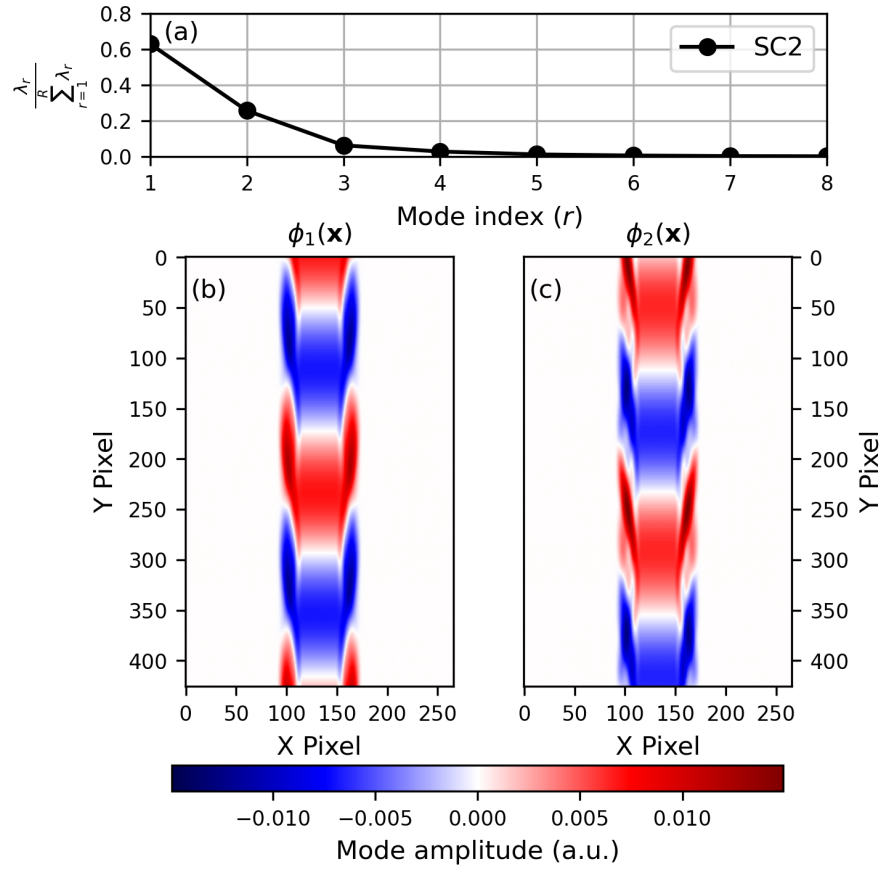


Figure 5. (a) Eigenvalue spectrum (first eight modes), along with the (b) first and (c) second spatial modes produced by POD of the SC2 dataset ($N_B = 10$ bursts, $L_B = 50$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz).

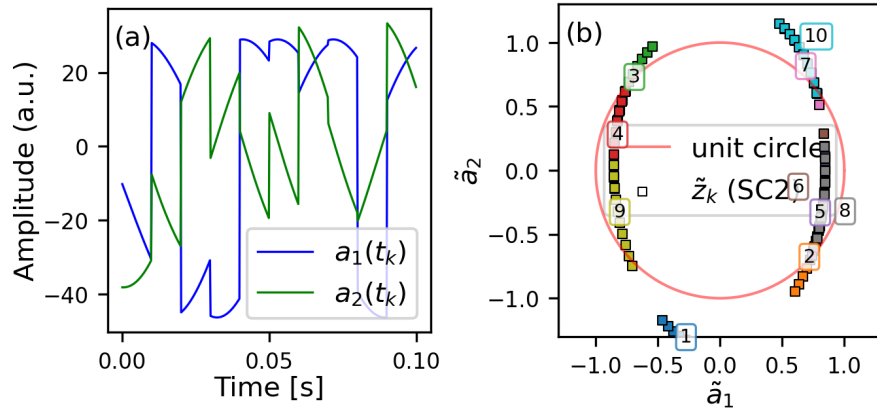


Figure 6. (a) Leading temporal coefficients and (b) the corresponding phase portrait produced by POD of the SC2 dataset ($N_B = 10$ bursts, $L_B = 50$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz). In (b), every fifth data point is shown, and the bursts are distinguished by different colors.

Finally, the POD approach is applied to the second synthetic burst-mode acquisition case (SC3). Once again, each burst is too short to capture a full oscillation cycle, although now the coverage has been improved by including $5\times$ the number of bursts compared to SC2. With improved sampling of the oscillation, the eigenvalue spectrum in Fig. 7(a) recovers the paired structure seen in the SC1 dataset. Furthermore, the dominant spatial modes in Figs. 7(b) and (c) lack the pronounced distortions seen in the previous case.

A similar situation is obtained with the decomposition's temporal component. As with the previous case, the temporal coefficients in Fig. 8(a) appear irregular when plotted sequentially, with the underlying periodicity revealed in Fig. 8(b). With the improved phase coverage afforded by the increased number of bursts, the elliptical distortion prominent in SC2 is much less pronounced. However, the trajectory still deviates from the one obtained in SC1, likely due to the phase discontinuities introduced with burst-mode sampling. Nevertheless, an excellent frequency estimate is obtained for this dataset, with an absolute error of 0.01%. The MC evaluation ($n_{MC} = 100$) of additional SC3 datasets yielded a mean absolute error of 0.9%, which is comparable to the accuracy achieved in SC1.

Here the tradeoff for multi-burst datasets is clarified: to preserve the fidelity of the POD result, a reduction in the burst length must be compensated by a disproportionate increase in the number of acquired bursts (From SC1 to SC3: a ten-fold decrease in L_B , a fifty-fold increase in N_B , and a five-fold increase in F_{tot}). This tradeoff has important implications for burst-mode and high-speed measurement systems, as L_B is likely to be the constrained parameter, while N_B can be increased at relatively little cost. Some factors influencing the maximum value of L_B could be heat transfer to the lasing media and the pulse-energy profile for burst-mode laser systems, and the amount of memory available at the desired resolution for a high-speed imaging system. Thus, the POD approach offers a flexible framework to accommodate a variety of realistic experimental designs.

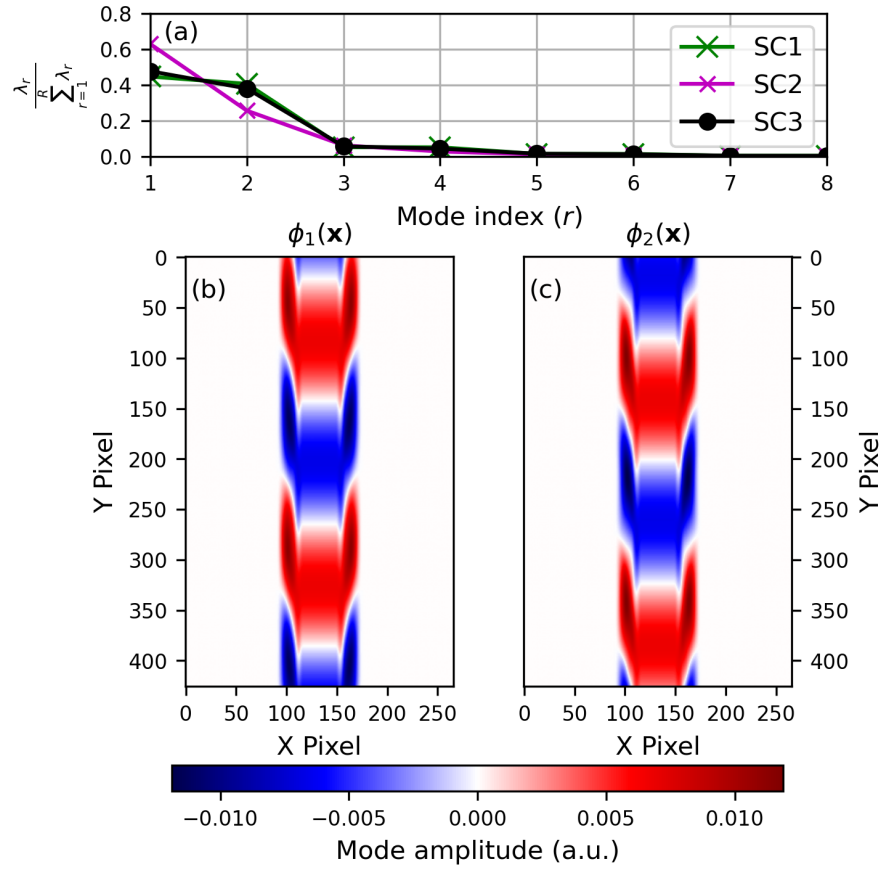


Figure 7. (a) Eigenvalue spectrum (first eight modes), along with the (b) first and (c) second spatial modes produced by POD of the SC3 dataset ($N_B = 50$ bursts, $L_B = 50$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz). The eigenvalue spectra for SC2 and SC3 are also included.

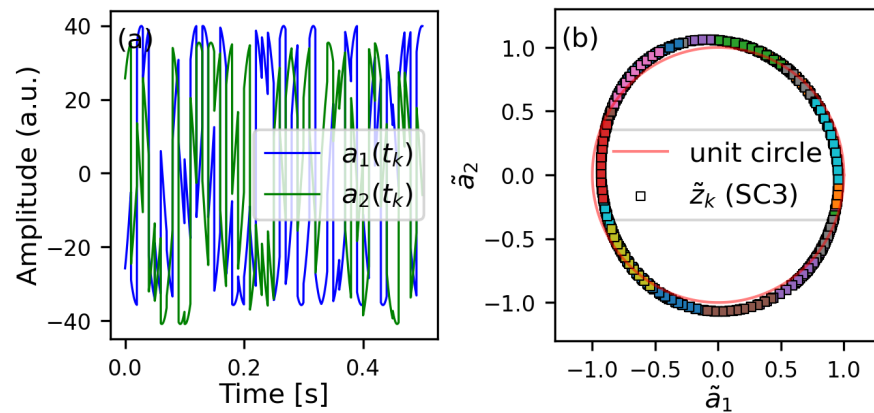


Figure 8. (a) Leading temporal coefficients and (b) the corresponding phase portrait produced by POD of the SC3 dataset ($N_B = 50$ bursts, $L_B = 50$ frames/burst, $f_0 = 10$ Hz, $f_s = 5$ kHz). In (b), every fifth data point is shown, and the bursts are distinguished by different colors.

3.2 POD analysis of synthetic flame images – Error scaling

The influence of L_B and N_B on the frequency-estimation error is now examined via a multi-parametric MC evaluation at $f_0 = 100$ Hz (MC1). This builds on the initial validations at $f_0 = 10$ Hz (a frequency representative of flicker dynamics) and extends the methodology to an oscillation frequency that better represents the flow and combustion instabilities found in turbulent flames (Iudiciani et al., 2010; Stöhr et al., 2011). Since individual frequency estimates could vary across different realizations for a given set of parameters (L_B , N_B), we report the mean absolute error, computed as the average over all realizations in that configuration. The error is first considered with respect to L_B , expressed through the number of oscillation cycles captured per burst, $C_B = L_B f_0 / f_s$. As seen in Fig. 9(a), the estimation error drops rapidly to sub-percent levels as C_B approaches unity. This behavior reflects an intuitive error-reduction mechanism driven by increased temporal coverage. For $C_B > 1$, the error plateaus or decreases more gradually, again marking $C_B = 1$ as the relevant transition value.

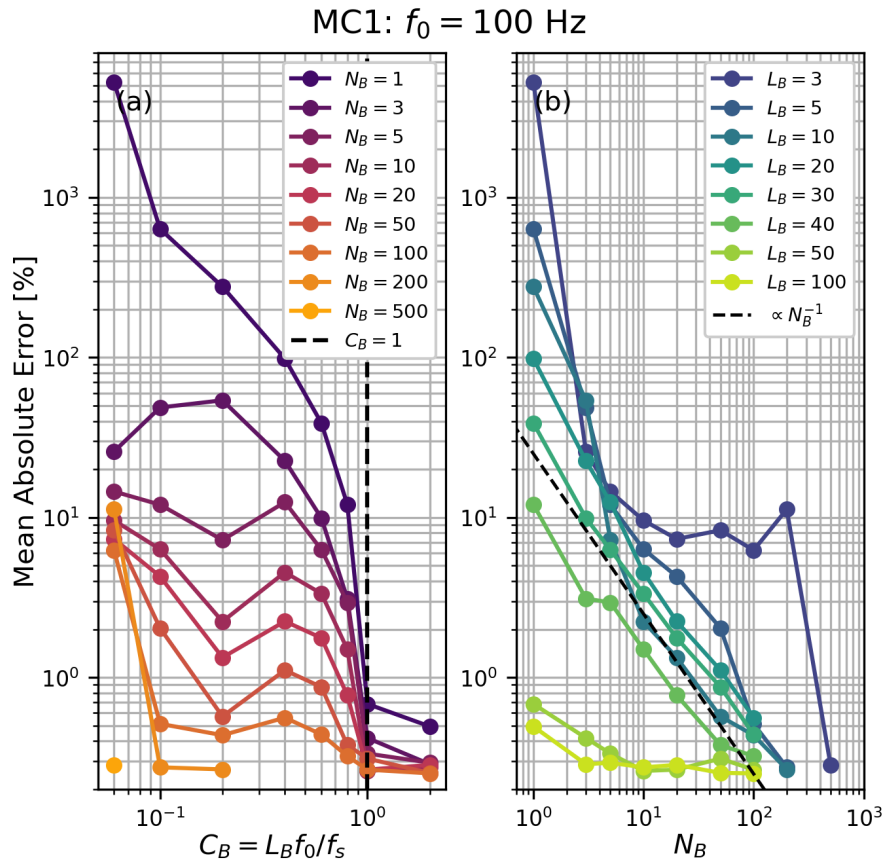


Figure 9. Mean absolute frequency-estimation error obtained from the MC1 evaluation ($f_0 = 100$ Hz, $n_{MC} = 30$) as a function of (a) $C_B = L_B f_0 / f_s$ and (b) the number of acquired bursts, N_B . In (a), each curve corresponds to a different value of N_B , with the dashed line indicating $C_B = 1$. In (b), each curve corresponds to a different value of L_B , with the dashed line showing a reference N_B^{-1} scaling.

This behavior has important implications for burst-mode measurement systems: if the acquisition does not occur near $C_B = 1$, the accuracy of the POD results may be fundamentally limited, especially for small values of N_B . Furthermore, in experimental settings where the dominant oscillation frequency is unknown, identifying this transition region may be nontrivial.

In addition to the dependence on L_B , Fig. 9(a) also introduces the role of N_B , with larger ensembles generally yielding lower errors across all values of C_B . This trend reveals a second, distinct error-reduction mechanism. As N_B increases, the probability that the ensemble collectively captures a sufficient portion of the global oscillation cycle improves, even when individual bursts provide incomplete temporal coverage (in this case as little as 6% per burst). This mechanism is inherently stochastic and depends on the phase distribution that was sampled during acquisition. For the synthetic data, the acquisition process is assumed to uniformly sample the full phase span. By contrast, experimental measurements offer no guarantee of uniform phase coverage and may follow a non-uniform sampling pattern.

The stochastic mechanism is further clarified in Fig. 9(b), where the error is plotted directly as a function of N_B . For very short bursts, the initial reduction in error with increasing N_B is quite dramatic, with sub-percent accuracy eventually obtained for sufficiently large ensembles. A unique plateau region is observed for $L_B = 3$, where the error stays at $\sim 10\%$ before steeply dropping after ~ 200 bursts have been captured. For large values of L_B , the error is already low due to adequate per-burst temporal coverage. As such, increasing N_B in this regime yields only modest additional improvements. Between these extremes in burst length, an intermediate regime is identified where the error follows an approximate N_B^{-1} scaling. In other words, the accuracy in this regime ($C_B \approx 0.6\text{--}0.8$) is roughly proportional to the number of independent samples. This scaling provides a useful practical heuristic for experiments: when operating below the critical $C_B = 1$ threshold, increasing N_B offers a systematic pathway for improving the accuracy of the POD results.

As seen in Fig. 10, the scaling trends for both L_B and N_B appear to hold at the higher oscillation frequency of $f_0 = 500$ Hz (MC2). The sharp error reduction at $C_B = 1$ is still present, along with the N_B^{-1} scaling for intermediate L_B values. The main qualitative difference appears in the small- L_B regime: the plateau region in MC1 is far less pronounced in MC2. This is expected, since increasing the oscillation frequency shortens the corresponding period and therefore increases the temporal coverage associated with a given value of L_B . Additionally, the range of L_B values spanning the $C_B = 1$ transition becomes compressed, meaning that $C_B \gtrsim 1$ can be obtained with fewer frames. The error reduction for large ensembles is very similar between MC1 and MC2, with most configurations achieving sub-percent error once N_B reaches 50–100 bursts. Thus, for increased

oscillation frequencies, the total amount of data needed to produce accurate POD results is decreased.

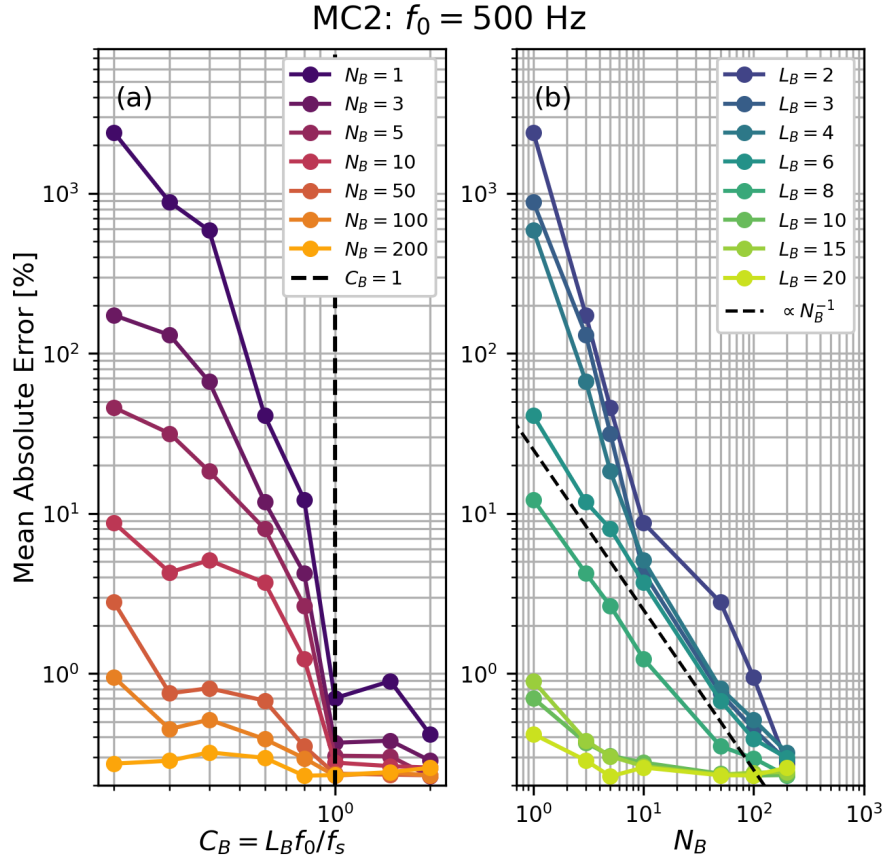


Figure 10. Mean absolute frequency-estimation error obtained from the MC2 evaluation ($f_0 = 500$ Hz, $n_{MC} = 100$) as a function of (a) $C_B = L_B f_0 / f_s$ and (b) the number of acquired bursts, N_B . In (a), each curve corresponds to a different value of N_B , with the dashed line indicating $C_B = 1$. In (b), each curve corresponds to a different value of L_B , with the dashed line showing a reference N_B^{-1} scaling.

3.3 POD analysis of experimental flame images

With the POD approach validated using the synthetic flame images, it is now applied to the experimental flame images in EC1–EC3. For these cases, a ground-truth oscillation frequency of 11.4 Hz is assumed based on previous long-acquisition DMD measurements. The experimental results are also compared directly to synthetic predictions generated with identical oscillation and acquisition parameters.

For the long-acquisition case (EC1), an excellent frequency estimate $\hat{f}_0 = 11.35$ (0.47% absolute error) is obtained, demonstrating the efficacy of the POD approach on real data under favorable acquisition conditions. The experimental eigenvalue spectrum shown in Fig. 11(a) exhibits the same paired structure previously identified in Section 3.2 and agrees closely with its synthetic

counterpart. The primary difference is a heavier tail, indicating that a larger proportion of the variance is contained in the higher-order modes ($r \geq 3$). This behavior is expected, since the experimental flame images contain additional sources of variability beyond the dominant oscillation feature, including camera noise and intermittent fluctuations of the flame shape. Nevertheless, the periodic component of the dynamics is well captured by the leading temporal coefficients, with the experimental trajectory shown in Fig. 11(d) closely matching both the synthetic and ideal-oscillator trajectories.

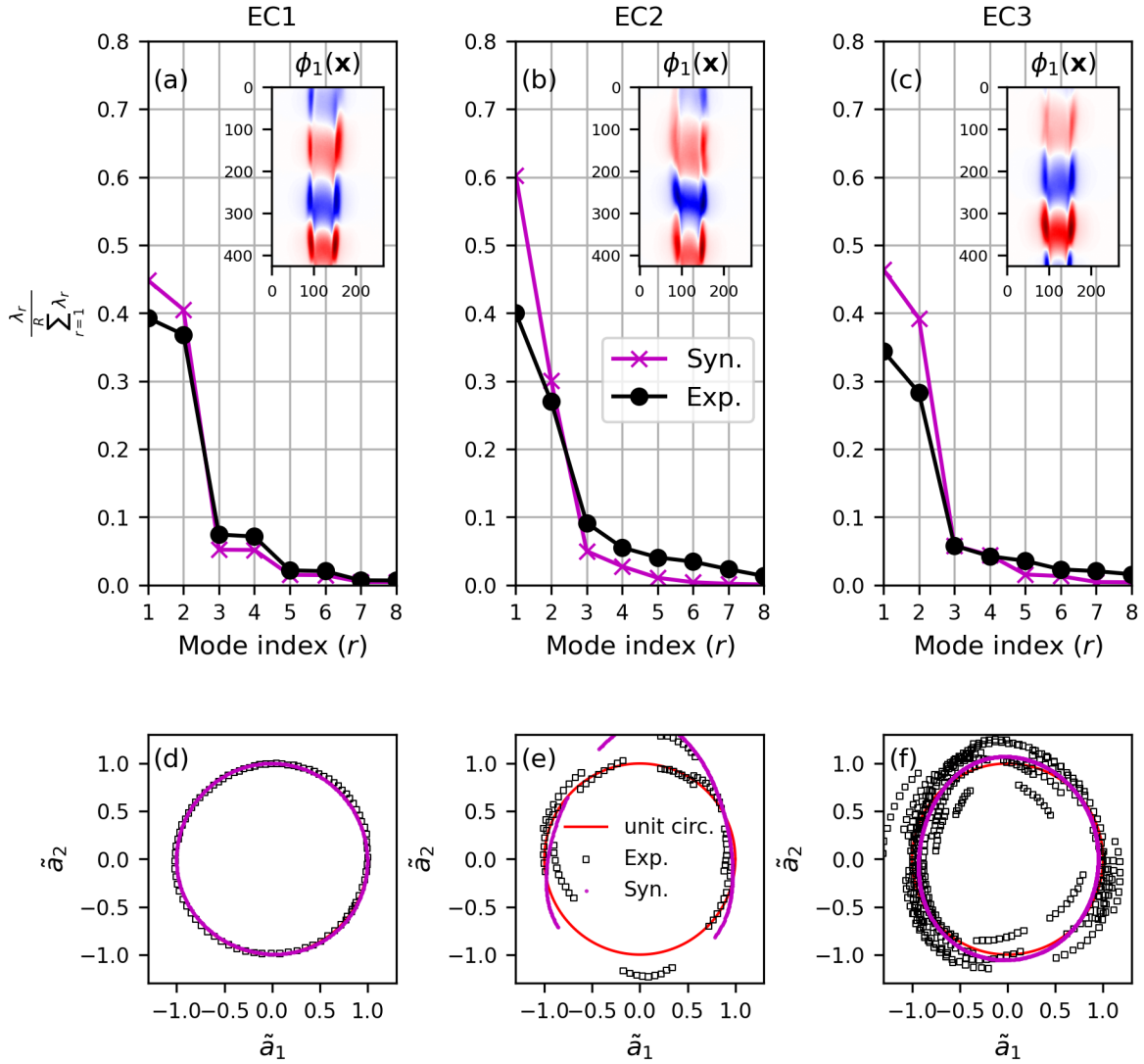


Figure 11. (a–c) Eigenvalue spectra (first eight modes) and the (d–f) corresponding phase portraits produced by POD of the experimental datasets: (a, d) EC1 ($N_B = 1$ bursts, $L_B = 440$ frames/burst), (b, e) EC2 ($N_B = 10$ bursts, $L_B = 44$ frames/burst), and (c, f) EC3 ($N_B = 50$ bursts, $L_B = 44$ frames/burst), with $f_0 = 11.4$ Hz and $f_s = 5$ kHz. In (a–c), the inset shows the leading spatial mode: $\phi_1(\mathbf{x})$.

For the first burst-acquisition case (EC2), a degraded frequency estimate $\hat{f}_0 = 10.63$ Hz (6.7% absolute error) is obtained. Based on the previous SC2 analysis, this result is expected since the burst length has been shortened without a compensatory increase in the total number of frames. The experimental eigenvalue spectrum seen in Fig. 11(b) agrees reasonably well with the synthetic one, although once again more variance is captured in the high-order modes. By contrast, the synthetic spectrum concentrates much more variance in the leading mode (~60%).

The influence of burst-to-burst variability is clearly illustrated in Fig. 11(e), where the trajectories taken by the experimental bursts show dispersion in the radial direction. This feature is absent in the synthetic case, where bursts in similar regions collapse onto a single path. This experimental dispersion likely reflects intermittent fluctuations, slow drifts, and spatial asymmetries that were captured from the real flame. Even so, the experimental and synthetic results show consistent qualitative behavior with respect to the elliptical distortion and gaps in phase coverage.

For the second burst-acquisition case (EC3), an improved frequency estimate $\hat{f}_0 = 11.28$ Hz (1.06% absolute error) is recovered. The paired structure for the eigenvalue spectrum in Fig. 11(c) is only partially recovered, which suggests that the accumulation of real-flame variability can have a considerable impact on the representation of the dynamics. Again, the radial dispersion of the experimental trajectories shown in Fig. 11(f) is substantial when compared to the synthetic result. A clear non-uniform sampling effect is also observed, with certain phases of the oscillation sampled more heavily than others. Even so, the trajectories remain close to circular and exhibit only mild distortions from both real-flame effects and the discontinuities introduced by burst-mode sampling.

These findings are consolidated in Fig. 12 to highlight the overall agreement between the experimental results and the synthetic MC predictions ($n_{MC} = 500$) across the different acquisition configurations. Under favorable acquisition conditions (EC1), both the experimental and synthetic estimates remain close to the reference frequency with a low level of predicted variability (standard deviation of ~0.09 Hz). For EC2, the downward shift in the estimated frequency is well predicted, suggesting that the estimator performance is governed primarily by the sampling structure, even though the real flame contains additional variability that is not reproduced in the synthetic model. For EC3, good estimator performance is recovered, although a larger amount of variability is predicted when compared to EC1 (standard deviation of ~0.6 Hz), again indicating the influence of the acquisition configuration. Overall, the strong agreement between the experimental measurements and the synthetic predictions demonstrates that the simplified flame model captures the essential dynamics governing the flickering H₂/air flame for the oscillation and acquisition conditions of this work.

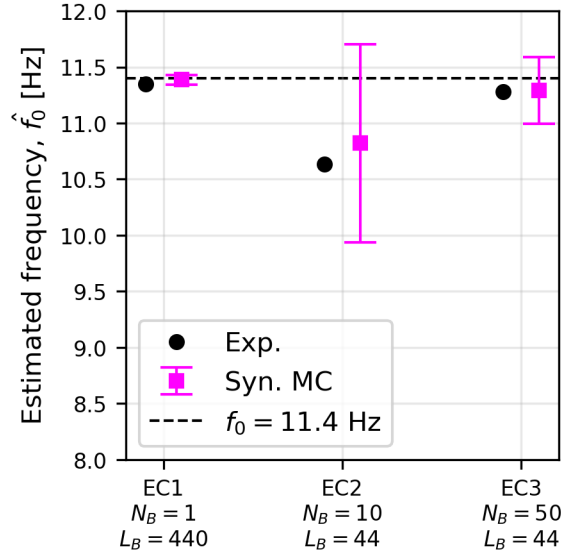


Figure 12. The estimated frequency $\hat{f}_0$ for the three experimental configurations (EC1–EC3), comparing the experimental measurements (black circles) with the synthetic Monte-Carlo predictions (magenta squares). The error bars span one standard deviation.

4. Conclusions and perspectives

This study demonstrates that POD provides a suitable and effective framework for representing the dynamics of flickering H₂/air flames. Through the analysis of experimental and synthetic datasets under a variety of acquisition configurations, it is shown that reliable estimates of the flickering frequency can be recovered even when individual bursts are much shorter than the oscillation period. The accuracy of the frequency estimate depends on L_B through a critical temporal coverage mechanism, with the dominant error reduction occurring when $C_B = L_B f_0 / f_s = 1$. The accuracy also depends on N_B through a stochastic phase coverage mechanism, with an approximate N_B^{-1} scaling identified for $C_B \approx 0.6$ – 0.8 . Therefore, frequency-estimation accuracy on the order of $\sim 1\%$ can be readily obtained with appropriate choices of L_B and N_B . For future studies, it is of interest to extend this methodology to turbulent flames and to incorporate laser-based diagnostics such as OH-PLIF for improved spatial resolution and species selectivity. With modern multi-leg burst-mode laser sources, coupled diagnostic setups can also be arranged, for example including OH-PLIF, NO-PLIF, and PIV. With a suitable decomposition-based approach for simultaneously captured burst-mode data, it may be possible to correlate the relevant flow, flame, and pollutant formation structures and their dynamic evolution.

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