

Time-resolved particle tracking using ME and low repetition-rate imagers

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ABSTRACT

Time resolved particle tracking offers the advantage of interrogation robustness alongside the opportunity of time-domain flow analysis and pressure from PIV. In this work, multi-exposure recording is devised as a technique to *boost* the measurement frequency of PTV systems adopting imagers with a moderate frame rate. The technique is based on the recent advancements of multi-exposure particle tracking, significantly simplified by the use of regular time intervals. The working principle is discussed, with the interplay between the frequency boost factor and the cumulative particle image density. The algorithm is described as a modified version of the Shake-the-Box method, whereby individual particle images are replaced by multi-pulse traces. The method relies upon an *initialization* stage where traces are built from 2 frames. After that a *converged* stage follows, where traces in the following recording are predicted by Lagrangian time marching and corrected by image re-projection.

Experiments demonstrate the viability of the technique, for large-scale aerodynamic experiments are conducted in a low-speed wind tunnel at free stream between 5 and 20 m/s. An omnidirectional PIV system is used for the measurement of the air flow developing around a circular cylinder of 125 mm diameter ($Re = [42, 83, 125, 167] \times 1000$). The system comprises 12 sCMOS cameras of 1.25 Mpx, covering a domain of $1.1 \times 0.8 \times 0.5 \text{ m}^3$ at a framing rate of 500 Hz. Depending on the flow velocity each recording captures 1 to 4 exposures from pulsed LED arrays illumination. At the highest velocity, the system yields measurements at a rate equivalent to 2 kHz.

The time-resolved three-dimensional structure of the Kármán wake behind the cylinder is reconstructed using instantaneous Cartesian grid reduction based on vortex-in-cell data assimilation at a grid resolution of 1 vector/cm. The dynamical evolution of the vortex dominated flow is visualized with time sequences of Q -criterion iso-surfaces. A visible transition is observed of the organisation of the secondary motions between $Re = 125,000$ and $167,000$. At the highest Re arch-like vortices become more pronounced, which modulate the spanwise coherence of the Kármán rollers.

1. Introduction

Three-dimensional Lagrangian particle tracking represents the state-of-the-art for the investigation of complex flows (Schröder & Schanz, 2023). The time-resolved analysis enables a wealth of possibilities, ranging from dynamic visualization of unsteady flow phenomena to advanced data reduction that infer the instantaneous pressure field (van Oudheusden et al. 2008, among others), or time and frequency domain analyses. Not less importantly, time-resolved particle image recordings allow a significantly more robust tracking by predict-and-search algorithms, notably Shake-the-Box (STB, Schanz et al., 2016), where the particle's trajectory is modelled to anticipate its subsequent position and the corresponding projections onto the imagers.

Time-resolved measurements, however, often require high-speed illuminators and imagers because the tracers motion needs to be captured with a relatively small displacement (typically 1-2% of the field of view). At typical flow speed and optical magnification, the associated time separation Δt , as sketched in Figure 1(a) imposes a high repetition rate f both for illumination as well as for the imagers. For instance, aerodynamic experiments conducted in wind tunnels, employ high-speed PIV hardware in the range of 5-10 kHz to perform time-resolved PIV measurements in flows at velocity up to 20 m/s. For volumetric experiments conducted at large scale (field of view in the order of the metre) by use of helium filled soap bubbles (Scarano et al., 2015), the above requirements can relax to 1-2 kHz, given that a larger particle displacement is allowed between subsequent recordings.

At higher flow speeds or in the absence of high-speed PIV hardware, experiments typically revert to the most practiced double-frame (single exposure) technique, as sketched in Figure 1(b), where particles are illuminated twice within a short temporal separation and their light is recorded onto two separate frames. Operating in double-frame mode comes however at the cost of a lower measurement accuracy and reliability: the two-dots velocity estimation suffers from a lower dynamic range (Fincham and Spedding, 1997); pairing two dots in place of a multi-point trajectory is prone to more spurious pairing. Furthermore, even when successful, the Lagrangian acceleration cannot be estimated on two points basis, precluding the estimation of the pressure field.

Recent research on the use of multiple exposures in single frame (Scarano et al., 2025), as well as double frame recordings (Grille Guerra et al., 2026) has indicated that volumetric experiments can be conducted where a particle trace can be reliably detected with up to 5 pulses into a single frame. The adoption of asymmetric timing of illumination and the iterative particle reconstruction method (IPR, Wieneke, 2013) have shown that the tracers' path and velocity can be measured more accurately than the two-pulse systems. However, the problem of overlapping images of particles travelling at vanishing velocity introduces a gap in the measurement range. A general solution to said gap has not been found yet, which compromises the application of single-frame multiple exposure systems in fluid flows.

The development of compact sCMOS cameras operating in the 100 – 500 fps range and the use of high repetition rate LED arrays for volumetric illumination opens to alternative strategies towards time-resolved particle tracking, alternative to those adopting specialized high-speed lasers and kHz rated CMOS imagers. The present work proposes the combination of imagers with such moderate

repetition, the multi-exposure (ME) recording technique and with Lagrangian particle tracking (Shake-the-Box) to achieve time-resolved (TR) measurements. The resulting technique is referred to as *METR-STB*.

2. Working principle

The approach proposed in *METR-STB* is that of decoupling the imager frame rate f_{acq} from the illuminator pulse repetition rate f_{pulse} , such to achieve a *boost* of the particle position recording rate, based on the multiple exposures collected in each recorded frame.

The connection between subsequent recordings, as shown in Figure 1(c), removes the need for an asymmetric sequence of pulses. The latter was initially intended to disambiguate the direction of motion in the single frame technique. In this case the directional ambiguity is solved by “connecting dots” at the edge of each trace across subsequent frames.

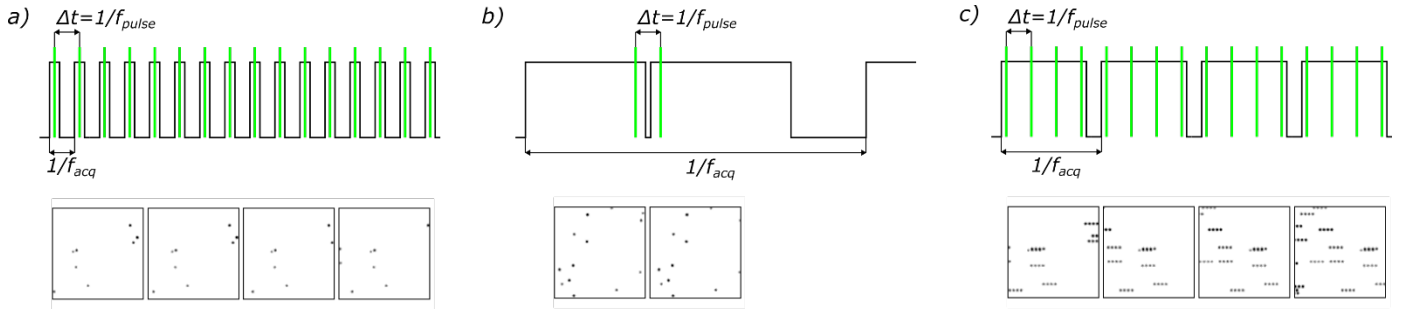


Figure 1. (top) Template of illumination (green) and camera exposure (black), together with image details (bottom row), for: time-resolved (high frame-rate) acquisitions (a); double-frame (low repetition rate) systems (b); time-resolved multiple-exposure (4 pulses/frame) approach (c).

The discussion herein focuses exclusively on three-dimensional measurements such as those obtained by Lagrangian particle tracking. The method relies largely upon the most practiced algorithms for 3D particle reconstruction (IPR, Wieneke, 2013) and tracking (STB, Schanz and Schröder, 2016).

Furthermore, as also discussed in the experimental section, the measurement technique assumes digitally programmable illumination, using LED arrays. The large scale size of the measurement domain is achieved using neutrally buoyant tracers (HFSB, Bosbach et al. 2009). Finally a multitude of digital cameras surrounding the region of interest and the object therein form a large aperture 3D PIV system also referred to as omnidirectional-PIV (Hendriksen et al. 2025).

2.1 Frequency boost

In the present context the attention is placed on the frequency of image acquisition f_{acq} and that of the pulsed illumination f_{pulse} . In a conventional, time resolved system each frame includes a single illumination pulse, thus $f_{\text{acq}} = f_{\text{pulse}}$ and each frame yields a sample of the particle position. When the

repetition rate of the pulsed illumination is set at a multiple N of the frame rate, each frame will capture N images of the same particle (viz. exposures), separated by an equal time interval $\Delta t = 1/f_{\text{pulse}}$. As a result, the sampling frequency of each tracer along its trajectory becomes N -times larger than that of the image recording device.

The condition to be met for time-resolved particle tracking is that the samples of the particle along its trajectory are sufficiently close for a reconstruction algorithm to advance the trajectory to the next particle position with a high probability of success. Such condition is practically fulfilled when the particle displacement $|\Delta \mathbf{x}|$ is small with respect to the scales of interest in the flow. The latter condition depends on the specific properties of the flow and is therefore difficult to evaluate *a priori*. A practical approach is to consider a displacement within 1% of the measurement domain L , which corresponds to approximately 10 pixels in image space when using a 1 Mpx sensor. The latter may be further relaxed ($|\Delta \mathbf{x}| > 10$ pixels) when an estimate (prediction) of the particle position at the successive time step is available (Saredi et al., 2020) or for particularly low values of the seeding density (e.g. $\text{ppp} < 0.01$). The displacement is proportional to the flow velocity V and the pulse time separation Δt . Given the above, a time resolved system (with a single exposure in each frame) needs to obey a relation of following type:

$$f_{\text{acq}} \geq \frac{V}{0.01 L} \quad (\text{single exposure}) \quad (1)$$

Considering a measurement domain of 50 cm length, imagers with a frame rate of 100 Hz, the corresponding maximum measurable velocity based on eq. 1 is $V_{\text{max}} = 2$ m/s. When performing the same experiment with $N = 5$ exposures in each frame, the maximum velocity can be increased to 10 m/s and the condition in eq. 1 modifies into:

$$f_{\text{acq}} \geq \frac{V}{0.01 L \cdot N} \quad (N \text{ exposures}) \quad (2)$$

The factor N can be regarded as a factor that boosts the measurement frequency, based on multi-exposure recording.

From the above discussion it emerges that the effect of multiplying the exposures potentially increases the temporal resolution of the system.

2.2 Limits and shortcomings

A limit to the frequency boost is dictated by the seeding density often expressed in particles per pixel (ppp). Following Grille Guerra et al. (2026) a distinction is made between ppp (seeding density for a single exposure) and the cumulative appearance of particle images when N pulses are included in a frame cPPP (cPPP = $N \cdot \text{ppp}$). However, ME recording introduces three main shortcomings: 1) the seeding density is increased by a factor N and with it the probability of ghost particles formation; 2) multiple exposures at uniform time separation form traces with directional ambiguity; 3) at vanishing velocity, the exposures of a tracer overlap and form a single blob. The following section describes the *METR-STB* algorithm and the way how it addresses the mentioned aspects.

2.3 METR-STB algorithm

The analysis of time-resolved multi-exposure image sequences is based on the single- and double-frame ME-methods described in Grille Guerra et al. (2026), with an additional time-marching scheme that accounts for the temporal information from frame to frame. The algorithm is illustrated here taking the case of two exposures per image ($N = 2$) as example, but the discussion is easily extended to $N > 2$.

For a sequence containing K frames (frame index $k = 0, 1, 2, \dots, K-1$) each particle is captured $2K$ ($N \times K$) times in the measurement domain. We refer to the sample time instant (i.e. time of the illumination pulse) as t_n with $n = 0, 1, \dots, 2K-1$. Particle images are triangulated from multiply exposed recordings, yielding their position in 3D space by the iterative particle reconstruction algorithm (IPR, Wieneke, 2013). The motion analysis method is sequential in its nature and it follows two stages: the *initialization* produces the first estimate of particle traces; once traces are initialised the *convergence* analysis follows, as a time marching method.

Traces Initialization

The first two recordings of the sequence are considered, containing two traces. Their combination yields a four-points trace spanning the two frames, where the four samples are equally-spaced in time. We introduce the temporal template $\tau = \{t_0, t_1, t_2, t_3\}$. A marked difference from the single frame recording technique with asymmetric timing, is that the time instants in τ are not fully unknown: samples “0” and “1” belong to the first frame and “2” and “3” to the second one.

From the set of particle positions, possible candidate traces are built using the 4-pulse double-frame scheme described in Grille Guerra et al. (2026). The procedure is briefly described as follows: first, for all particles in the first frame, all possible matching particles in the second frame within the allowed velocity range are searched for, calculating a displacement dx_{12} between t_1 and t_2 . Next, extrapolating the particle position from t_2 toward t_3 using dx_{12} , matching particles within the allowed acceleration range are taken. Finally, the particle position at t_1 is extrapolated backward toward t_0 using $-dx_{12}$, again matching particles within the allowed acceleration range are searched for, collecting all possible traces related to all particles in the first frame.

A description of how a candidate trace is selected from four detected particles is given in Figure. 2.

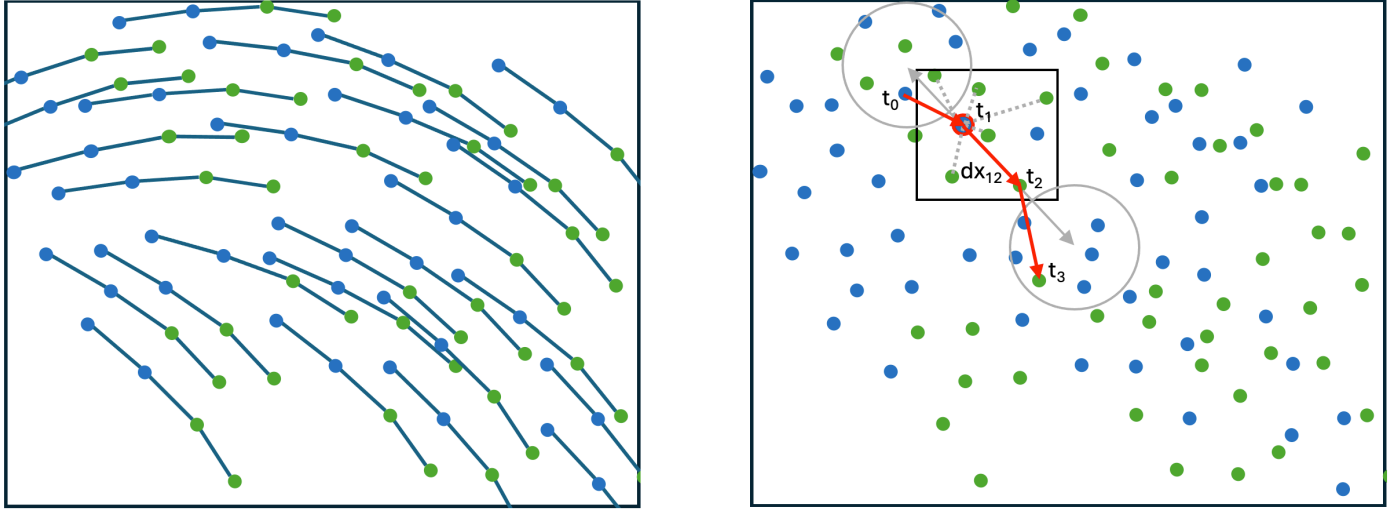


Figure 2. Left: actual traces (4-point) built from the first (blue particles) and second (green) frame. Right: a trace candidate starting at the red particle in the first frame and taking a matching particle in the second frame within the rectangular allowed velocity range is considered. Candidate matching particles at t_3 and t_0 are found using $\pm dx_{12}$ and the allowed acceleration range (grey circles). In the present case such candidate is a spurious trace.

The following step reduces the multitude of candidate traces to the single, most likely one, for each particle detected in the first frame. Here, a cost function built on a composite criterion of kinematic similarity (Scarano et al., 2025) together with a neighbourhood consistency criterion in the trace velocities, typically taking into account a neighbourhood of 10 to 20 particles. Finally, inconsistencies of traces sharing the same particles need to be resolved. A competitive scheme eliminates the trace with the poorest score of the kinematic/neighbourhood cost function. The latter often resolves also the directional ambiguity, i.e. whether a particle in the first frame belongs to t_0 or t_1 . The same applies for the second frame and t_2 and t_3 .

Once a consistent set of traces has been assigned (inner loop) the process is iteratively improved by using the particle location of all detected traces as a start input for IPR in the next iteration (outer loop). This approach allows eliminates ghost particles and ghost traces using as additional constraints velocity and acceleration bounds along with the kinematic similarity. Convergence involves typically 3-5 loops of IPR and tracking.

2.4 Overlapping particle images within a trace

In the low-velocity regime, particle image pairs at t_0 and t_1 , and also at t_2 and t_3 , come to overlap. Unless more elaborate interrogation techniques are adopted, this condition poses a hard limit to the single frame multiple-exposure technique. The *METR-STB* approach is based on multiple frames and the case of overlapping particle images is solved as follows. We assume that when the observed displacement $|\Delta \mathbf{X}|$ (i.e. its projection orthogonal to the line of sight) is smaller than the particle image diameter (typically based on optical diffraction) a single, higher intensity peak is produced instead of multiple peaks. As a result, IPR detects a single particle sample in each frame. The centroid of such peak is expected to occur at the middle distance between the two positions in each image, i.e. $\mathbf{x}_{0,1} =$

$\frac{x_0+x_1}{2}$ and $x_{2,3} = \frac{x_2+x_3}{2}$ as shown in Figure 3. The distance between them is $dx = x_{2,3} - x_{0,1}$, corresponding to the distance travelled during two time intervals (for the present example). This condition is recognized by the absence of admissible particle samples in the neighbourhood of the considered positions $x_{0,1}$ and $x_{2,3}$ and their peak intensity exceeding that expected from neighbouring traces. Once this condition is recognized, the 2 blobs are converted into a 4-samples trace with approximate positions according to the following expression:

$$(x'_0, x'_1, x'_2, x'_3) = (x_{0,1} - \frac{dx}{4}, x_{0,1} + \frac{dx}{4}, x_{2,3} - \frac{dx}{4}, x_{2,3} + \frac{dx}{4}) \quad (3)$$

The equation is slightly more complex for more than two exposures. Furthermore, the treatment of intermittent peak detection (mixed dots and streaks) is not discussed here for sake of conciseness.

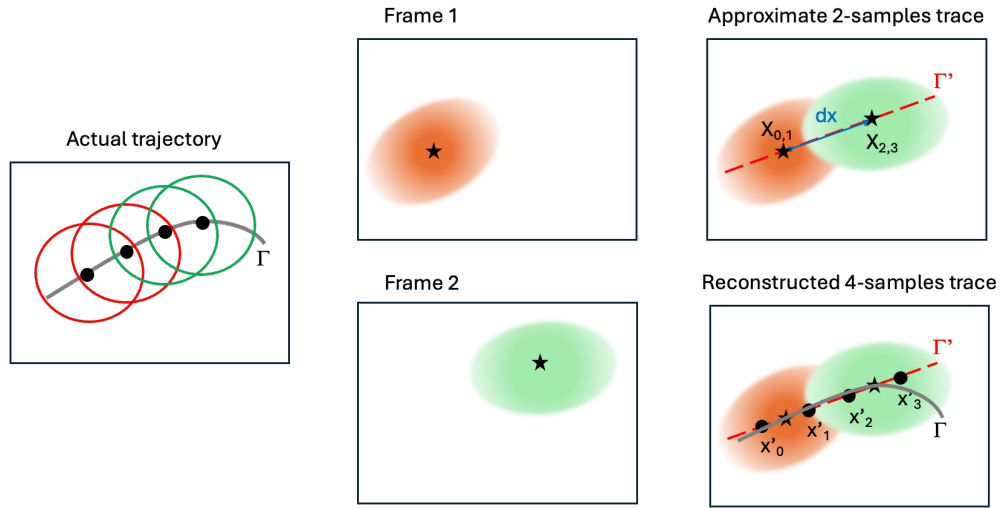


Figure 3. Two-frame analysis of a tracer producing overlapping images in first (red) and second (green) image. The circles in the left image indicate the particle image diameter.

A shortcoming of the method is that the trajectory is linearized and the particle acceleration is neglected. The above situations, however only occur rarely and their effect on the overall velocity field measurement can be considered negligible. The experimental section discusses them, confirming their very limited occurrence.

2.5 Time-marching (convergence phase)

Once the first two images are analysed, the spatial distribution of tracers as well as their velocity and acceleration is determined. Under these circumstances a time marching algorithm advances the information to the next frame with predict-and-search. In this case the particle tracers' position in physical space is predicted (by trajectory extrapolation), in turn yielding a prediction of their appearance in the images. The Shake-the-Box algorithm here is slightly modified to account for the double exposure, double-frame of second and third image.

Traces recovery

Unassigned particles in the second image (t_2 and t_3) and traces (up to t_3) are stored on a temporary set. IPR is applied to the third image alone, and the previously computed traces are preferentially continued by the newly found particles in the third image (t_4 and t_5). Still unused particles at t_2 and t_3 are matched with new particles at t_4 and t_5 to find possible newly starting traces. All possible continued old traces and newly found traces are then subjected to the composite criterion for acceptance and the set of traces is determined based on cost function maximization.

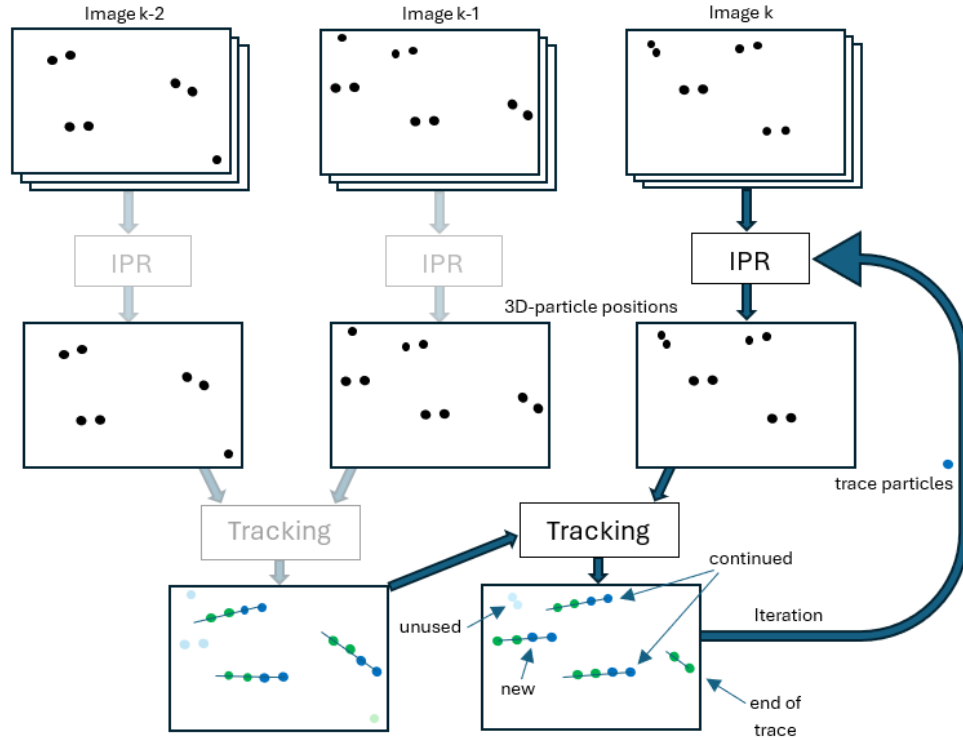


Figure 4. Time-marching *METR-STB* procedure.

Two to three iterations of IPR on third frame, followed by tracking yield the most likely set of traces. Conversely, ending traces are stored in the database, while continuing and new ones (together with the unused particles at t_4 and t_5) are retained for further advancements in the following frames. This time-marching algorithm is schematically depicted in Figure 4 for an image set at index k .

3. Experiments

Aerodynamic experiments are conducted in the Open Jet Facility (OJF) at the Faculty of Aerospace Engineering of Delft University of Technology. A 2-meter long circular cylinder, with a diameter $D = 12.5$ cm is immersed in the free stream stream at speed of 5, 10, 15 and 20 m/s ($Re = [42, 83, 125, 167] \times 1000$). Large-scale volumetric PIV is employed, using helium-filled soap bubbles released by an in-house seeding rake placed ahead of the wind tunnel contraction. Five *LED-Flashlight 300* arrays from LaVision GmbH provide pulsed illumination from multiple directions, and 12 CMOS cameras (*Imager CX3-5* from LaVision GmbH) are arranged in triplets (Figure 5) onto four antennas, following the approach of omnidirectional PIV (Hendriksen et al., 2025). The cameras are equipped with 12 mm

focal length objectives and the f-number is set to $f_{\#} = 5.6$ in order to achieve the desired depth of field. The cameras are operated at an effective resolution of 1.25 MPx (5.5 μm pixel pitch after pixel binning), allowing for an acquisition frequency of $f_{\text{acq}} = 500$ Hz. For the given configuration, this results in a digital image resolution of approximately 1 px/mm. A measurement volume of 9D (x), 7D (y) and 4D (z) is obtained, or approximately 0.5 m³.

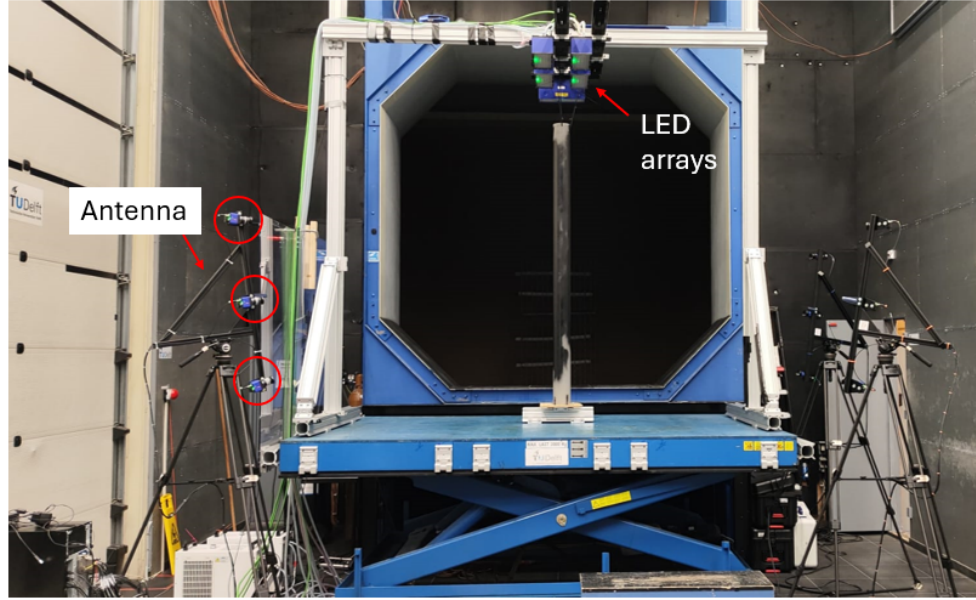


Figure 5. Experimental setup in the wind tunnel outlet: LED arrays provide high-speed illumination and 4 antennas holding 12 CMOS cameras. One antenna is red circled.

For this large-scale PIV system, eq. 1 is satisfied up to a freestream velocity of $V=5$ m/s. At that speed, particles travel approximately 10 px between frames, and therefore a conventional single-exposure time-resolved acquisition can be used. However, higher values of the wind tunnel velocity require the *METR-STB* approach, adding more exposures per frame to maintain the 10 px particle displacement between pulses. The following conditions have been explored: 10, 15 and 20 m/s, using $N=2, 3$ and 4 pulses per frame respectively. Image details in the above conditions are shown in Figure 6. Since the bubble production rate remain constant even when varying the wind tunnel speed, the spatial concentration of tracers is inversely proportional to the latter and it decreases when the velocity is increased. The result is that the cumulative seeding density does not vary with the flow speed and it remains relatively constant among conditions, at $c_{\text{ppp}} = 0.03$. Instead the concentration of traces declines with the wind tunnel speed as it is associated to the number of particle tracers present in the measurement volume.

4. Results

For the single-exposure time-resolved recording at 5 m/s, 40,000 particles are tracked on average every frame. Considering the measurement volume, the above translates into a spatial particle concentration of 0.1 particles/cm³. An instantaneous visualization of particle trajectories over nine

consecutive frames, coloured by the associated streamwise velocity, is given in Figure 7 (top-left). To ease the inspection, only a slab of 10 cm (out of 80 cm) in the spanwise direction is depicted. An analogous visualization is also included in Figure 7 for the *METR-STB* cases at higher speed. As expected, the number of particle images remains constant, the number of trajectories decreases linearly with the wind tunnel velocity, in agreement with the hypothesis of a constant generation of tracers transported at increasing velocity (Caridi et al., 2016). The proposed tracking method proves effective in the stagnation region in front of the cylinder and also inside the recirculation region close to the back of it, despite the limited tracer displacement causing more a frequent overlap of particle images (see section 2.4).

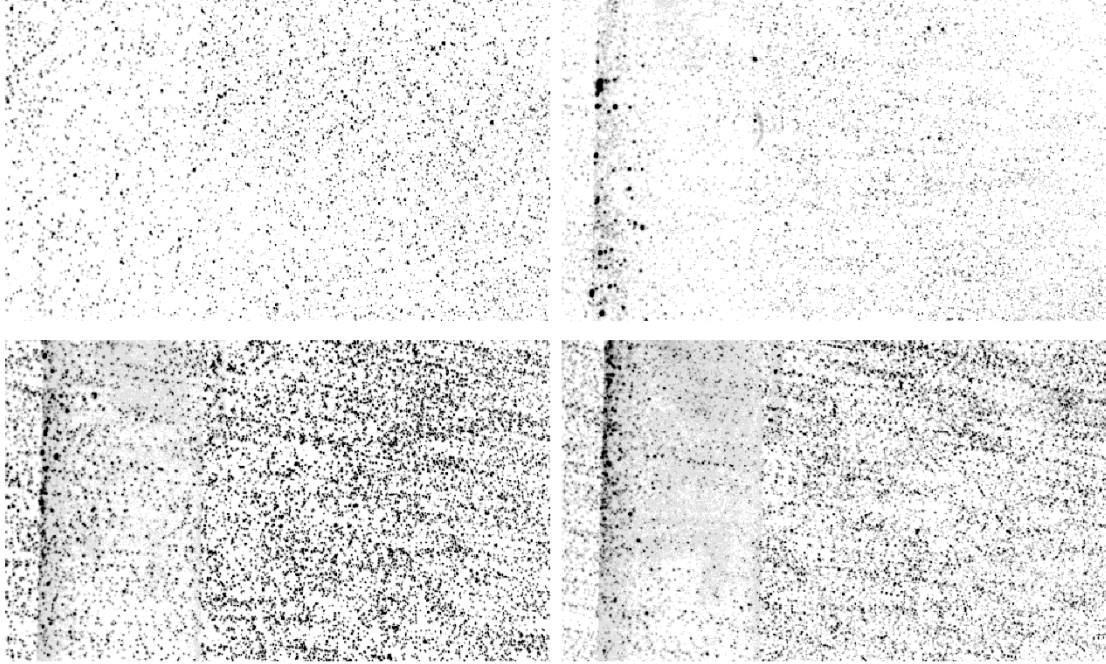


Figure 6. Image details of the flow around a circular cylinder for: single exposure at 5 m/s (top-left), $N=2$ exposures at 10 m/s (top-right), $N=3$ exposures at 15 m/s (bottom-left) and $N=4$ exposures at 20 m/s (bottom-right). Flow is approximately from left to right.

The obtained long particle trajectories allow for the use of advanced data assimilation algorithms which employ not only the scattered velocity sampled at the particles' locations, but also Lagrangian acceleration. Following this rationale, the vortex-in-cell data assimilation method is applied that assimilates simultaneously velocity and acceleration (Schneiders and Scarano, 2016). The VIC# implementation of the method, available in the DaVis 11 software (Jeon et al. 2022), has been applied on a grid of 1 cm pitch, resulting in a representation of the unsteady flow field on a Cartesian grid. An instantaneous visualization of the 3D flow is given in Figure 8, using iso-surfaces of the Q -criterion to visualize vortical structures in the cylinder wake. The iso-surfaces are selected such that $\frac{Q D^2}{v^2} = 2$, and coloured by spanwise vorticity to ease distinguishing among the counter-rotating vortices.

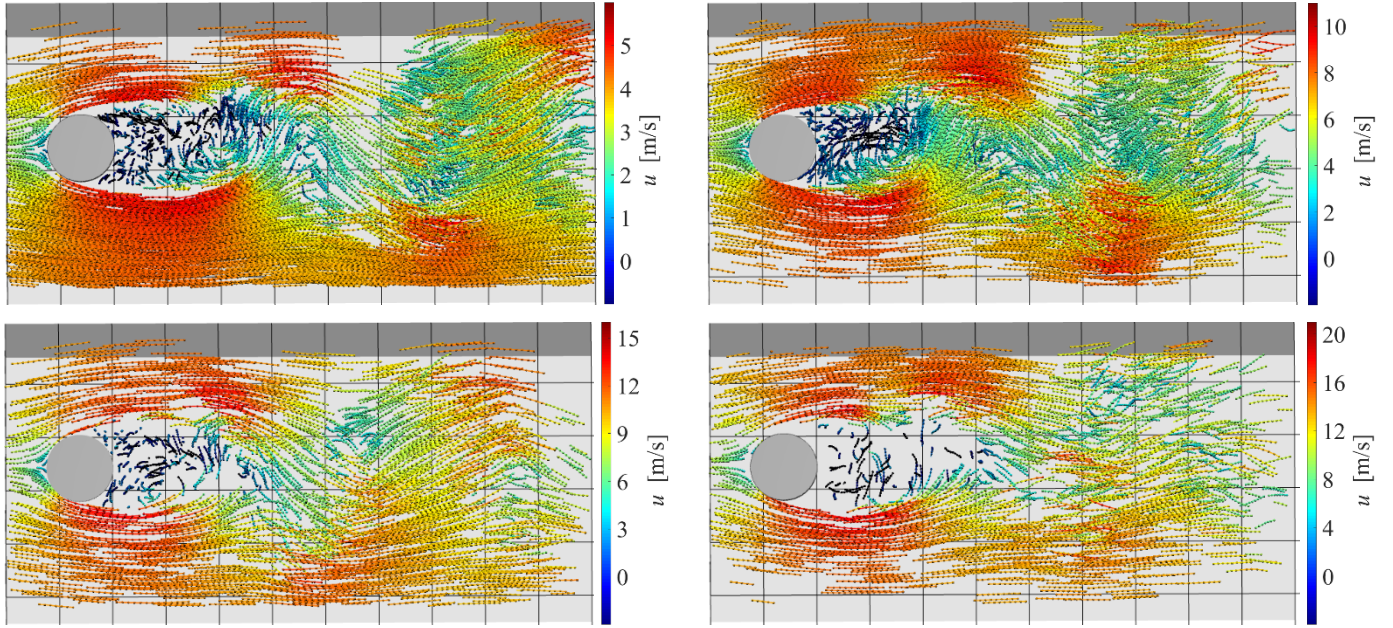


Figure 7. Particle traces around the cylinder from nine consecutive pulses, coloured by streamwise velocity. Traces selected from a 10 cm spanwise slab sub-volume. Single-exposure ($N=1$) and time-resolved measurements at 5 m/s (top-left), $N=2$ exposures at 10 m/s (top-right), $N=3$ exposures at 15 m/s (bottom-left) and $N=4$ exposures at 20 m/s (bottom-right). The color-coding range is scaled to the free-stream velocity.

The illustrations reveal the presence of spanwise coherent vortices (Williamson 1996), shed with an alternating sign, and also thinner, streamwise-oriented vortical structures interconnecting the main rollers. Comparable vorticity patterns were reported in literature for measurements at lower Reynolds numbers ($Re = 5,000$; Scarano and Poelma, 2009). Since all four measurements belong to the sub-critical regime ($Re_D < 2 \cdot 10^5$, Roshko, 1961), the results are qualitative similar, although the spanwise coherence of the main rollers for the case at 20 m/s ($Re_D = 1.67 \cdot 10^5$ for this case) is more visibly weakened by the occurrence of arch-like vortices, presumably due to proximity to the critical regime.

5. Conclusions

An extension of the Shake-the-Box method to multi-exposure recordings is presented with the purpose of increasing the measurement rate of 3D particle tracking and achieve time resolved measurements with imagers of moderate frame rate. The *METR-STB* method is based on identifying traces of the multiply exposed particle images in each frame and connect them in the recorded sequence equivalent to single-exposure time resolved measurements. Specific conditions deserve attention: 1) particles at low velocity produce a streak (or blob) rather than a sequence of dots; 2) the maximum number of exposures needs to account for the cumulative seeding density.

Time-resolved experiments are presented in a large-scale low-speed wind tunnel that demonstrate the applicability of the *METR-STB* principle. The flow around a circular cylinder in a stream up to 20 m/s is captured using omnidirectional PIV, using imagers operated at a frame rate of 500 Hz and with

pulse frequency of 2 kHz. The unsteady flow organization of the Kármán wake with its street of counter-rotating rollers is captured along with the secondary instabilities that involve streamwise and transverse vorticity.

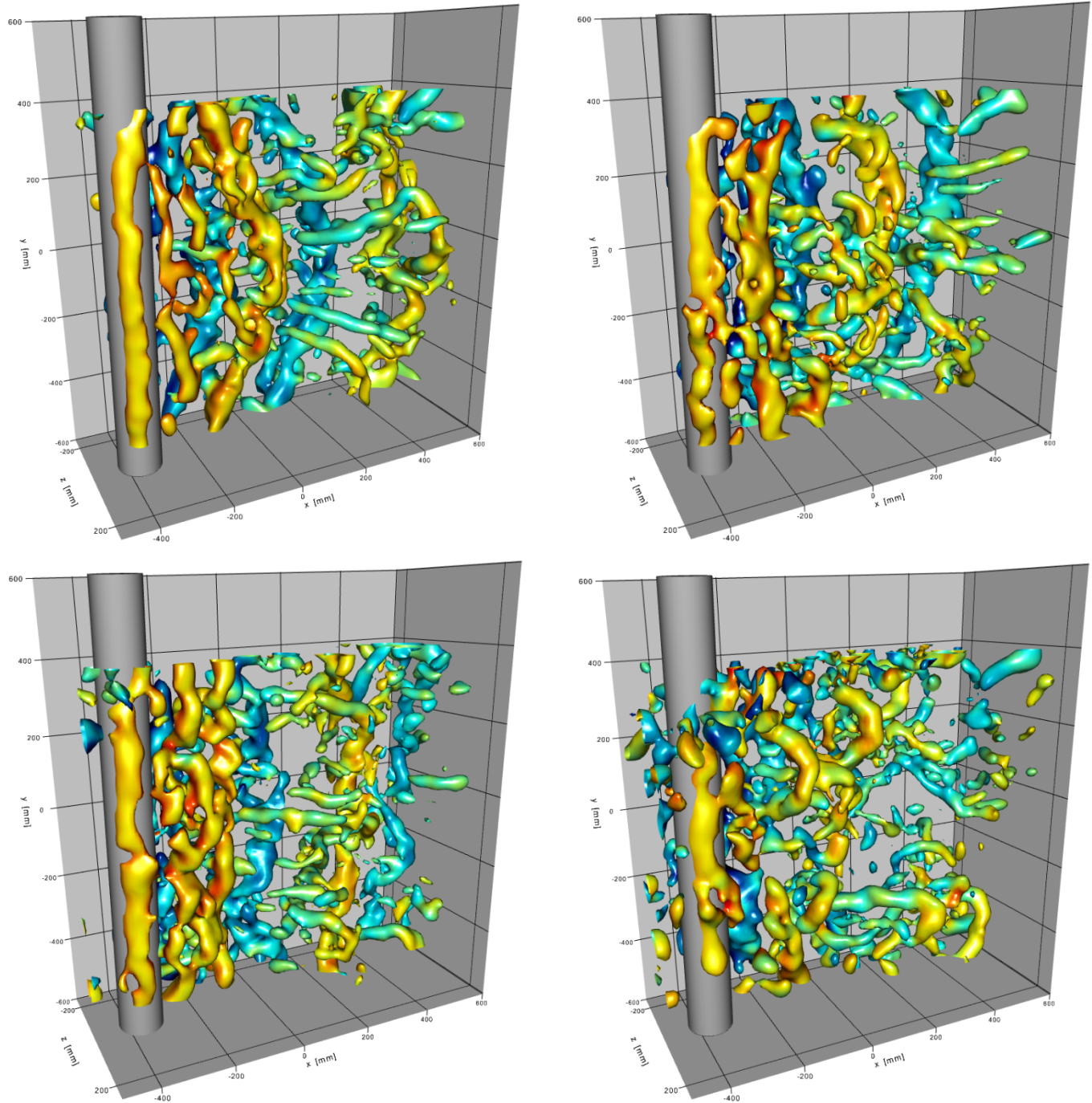


Figure 8. Instantaneous visualization of vortical structures around the cylinder Q -criterion iso-surface, color-coded by spanwise vorticity. $V = 5$ m/s (top-left), $V = 10$ m/s (top-right), $V = 15$ m/s (bottom-left) and $V = 20$ m/s (bottom-right).

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