

Combined DFD and Phosphor Thermometry for Synchronous Droplet Size, Position and Temperature Measurement

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ABSTRACT

In sprays and droplet laden gas flows, the spatial, size and temperature distributions of the droplets usually play a fundamental role in process efficiency. Examples are spray drying, spray cooling, spray combustion or flue gas scrubbing. The goal of the present study is to combine the depth-from-defocus (DFD) technique for droplet size and position with phosphor thermometry for droplet temperature. Although both techniques are well-established, several possibilities arise for their combination into a single measurement instrument. In this study a solution with a single illumination source and a dual-camera, dual focal length solution is examined. Results confirm the feasibility of this approach.

1. Introduction

This study addresses the measurement task of simultaneously measuring the size, velocity, and temperature of droplets over a finite volume. Such a measurement is particularly interesting and important in dispersed two-phase gas/liquid flows in which the gas carrier flow is typically at a temperature significantly different from that of the droplets. Examples include spray drying, supercooled large droplets (SLD) in icing wind tunnels, fuel droplets in a combustor environment, or spray cooling. In these examples, all the quantities - size, velocity and temperature of the droplets - inherently influence the underlying thermodynamic processes of the application, be it evaporation, nucleation, or ignition. Thus, their simultaneous measurement is essential for any experiments designed for model validation or to generate data for comparison to numerical simulations on a single drop or drop ensemble basis.

Numerous techniques are capable of measuring drop size, including laser diffraction (Swithenbank et al., 1976; Merkus, 2009; Dodge et al., 1987; Sijs et al., 2021), phase Doppler (Albrecht et al.,

2013), interferometric particle imaging (Damaschke et al., 2005), time-shift technique (Schäfer & Tropea, 2012), holography (Thompson, 1974; Pu et al., 2005; Wu et al., 2017; Kumar et al., 2019), direct imaging or depth from defocus (DFD) (Sharma et al., 2023; Zhou, Tropea, et al., 2020; Zhou et al., 2021). However, only the latter two also provide a time-resolved three-dimensional position measurement, as would be necessary to measure the drop velocity. Since holography requires a coherent, polarized light source, it is less amenable to illumination using a light source compatible with a temperature measurement; hence, the DFD technique has been chosen to be more suitable for further development.

Similarly, numerous techniques are available for the measurement of drop temperature, including rainbow refractometry (Anders et al., 1996; Hom & Chigier, 2002; Wu et al., 2017, 2018; Li et al., 2020), Raman scattering (Schweiger, 1990; Moritz & Schweiger, 2000), IR thermography, laser-induced fluorescence (LIF) (Lemoine & Castanet, 2013; Dunand et al., 2012; Zhang et al., 2023) and laser-induced phosphorescence (LIP) (Stiti et al., 2021; Omrane et al., 2004). In some instances these techniques acquire both temperature and drop size (Hu & Huang, 2009); however, a three-dimensional position measurement is lacking. Anticipating the necessity of combining a temperature measurement technique with the DFD technique, laser-induced photoluminescence processes are considered the most promising.

Many LIF based techniques are based on an intensity measurement; however, in practice intensity based techniques are susceptible to many external influences and ratiometric approaches are therefore necessary, involving often two or more spectral bands. In contrast, a measurement based on lifetime is more robust, although necessitating an ultrashort laser pulse for time resolved measurements. Generally, LIP is found to be considerably more sensitive to temperature variation compared to LIF (Omrane et al., 2004), which is also the motivation for its choice in the present study.

Both the DFD technique and temperature measurements using the lifetime decay of phosphorescent particles are well established techniques, and principally, they could measure independently and simultaneously in the same flow volume. However, the goal in the present study is to explore how these two techniques are best combined in a single measurement system, at best sharing optical components. In combining these two techniques, several possibilities exist, depending on the number of illuminating wavelengths, focal lengths, and sensors/cameras employed, as summarized schematically in Fig. 1. Two of these possible optical DFD+PT configurations are pictorially represented in Fig. 2, whereby the first of these configurations (one illumination wavelength, two focal lengths and two cameras, Fig. 2(a)) has been chosen for demonstration in the present study (see also Zhou, Luo, et al. (2020)).

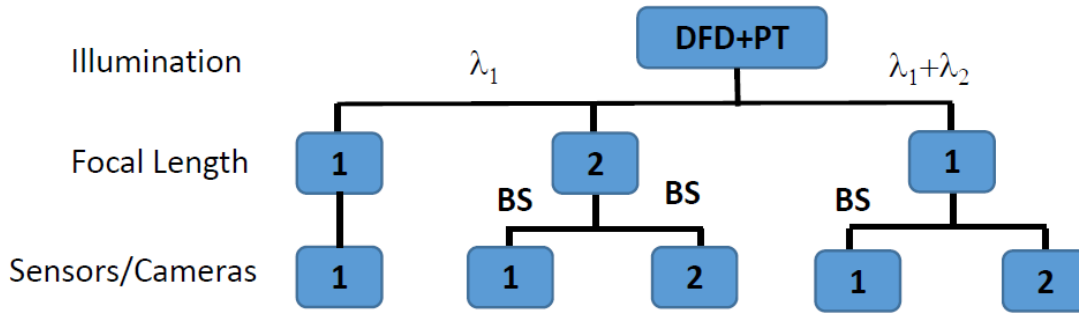


Figure 1. Schematic representation of possible configurations for combining Depth from Defocus (DFD) with lifetime based Phosphor Thermometry (PT). (BS - Beam splitter)

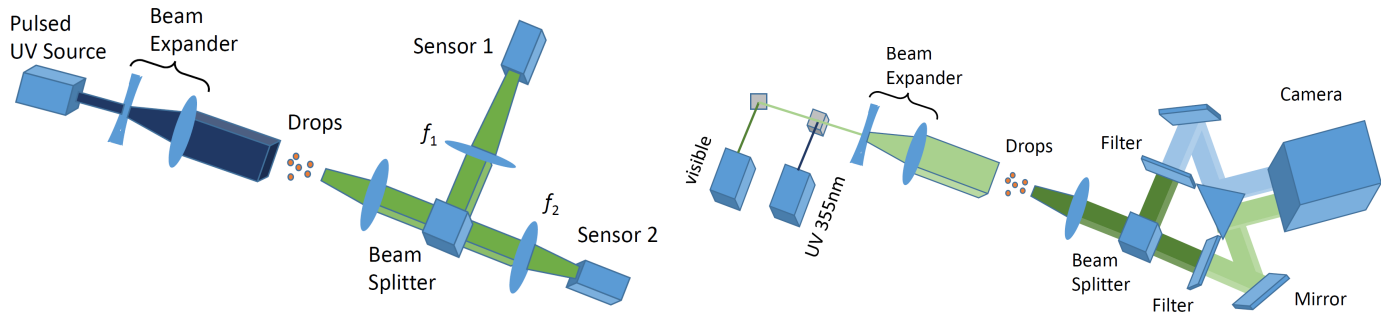


Figure 2. Schematic representation of example optical DFD+PT configurations: (a) using one illumination wavelength, two focal lengths and two cameras; (b) using two illumination wavelengths, one focal length and one camera with a split screen.

2. Measurement Principles

2.1. Phosphor Thermometry

Upon excitation, the emission intensity of the phosphorescent molecules follows approximately a first-order exponential decay law.

$$I(t) = I_0 \exp(-t/\tau). \quad (1)$$

The decay constant τ is calculated from the ratio of intensity integrals over two adjacent exposure periods, where τ is a time constant dependent on temperature and must be calibrated for any chosen dye molecule, resulting in a relationship between temperature and lifetime expressed as:

$$T = f(\tau), \quad (2)$$

whereby decay times are shorter for higher temperatures. Depending on the phosphor, decay times can be in the millisecond range or below; hence, a detector with high temporal resolution is required. Typically a double frame camera as used in PIV (CCD or CMOS) is appropriate for planar measurements. Details of image processing procedures can be found in Yi & Kim (2014).

Existing studies on phosphor thermometry focus mainly on phosphor selection, system calibration, and fluorescence decay constant calculation, while few investigations target phosphor-doped droplet imaging. The present work explores the inherent imaging characteristics of luminescent droplets, and preliminary experimental results are presented in Fig. 3. Even for droplets accurately positioned at the object plane, the luminescent images are inherently blurred. In this experiment, suspended droplets doped with phosphorescent dyes were excited by a 280 nm LED and images were acquired at different times after illumination. Each frame corresponds to a further 1 ms interval after illumination. In the image shown in Fig. 3(a) two bright spots appear on the upper portion of the drop; these are induced by the needle on which the drop is suspended, but do not affect the intensity distribution measured on the mid-plane diameter, marked with a red line. Fig. 3(b) presents the horizontal mid-plane intensity distribution of droplets at different decay moments in steps of 1 ms, with Gaussian fitting of raw data. The results normalized with the maximum intensity in Fig. 3(c) demonstrate that the Gaussian standard deviation remains constant during the luminescence decay. This standard deviation will be designated σ_p , the subscript 'p' standing for 'particle'.

Traversing the suspended drop away from the object plane and out of focus again results in Gaussian intensity distributions, exhibiting increasing standard deviations with increasing degree of out-of-focus (Fig. 3(d)). In this graph, each defocus step represents a movement of 3 mm further away from the object plane. These preliminary experiments were each repeated three times, yielding a linear increase in σ_p with z , as shown in Fig. 3(e).

2.2. Depth from Defocus (DF)

In the imaging principle of the DFD technique, the grayscale blurred image $g(x, y)$ of spherical particles (or droplets) under backlight illumination can be characterized via convolution between the perfectly focused sharp particle image $f(x, y)$ and the blur kernel $h(x, y)$.

$$g(x, y) = f(x, y) \otimes h(x, y) \quad (3)$$

where $f(x, y)$ is a normalized (binarized) intensity image of the in-focus particle on the image plane

$$f(x, y) = 0 \quad \text{if outside particle contour} \quad (4)$$

$$= 1 \quad \text{if inside particle contour} \quad (5)$$

and the blur kernel is assumed to be Gaussian:

$$h(x, y) = \frac{1}{2\pi\sigma_z^2} e^{-\frac{x^2+y^2}{2\sigma_z^2}} \quad (6)$$

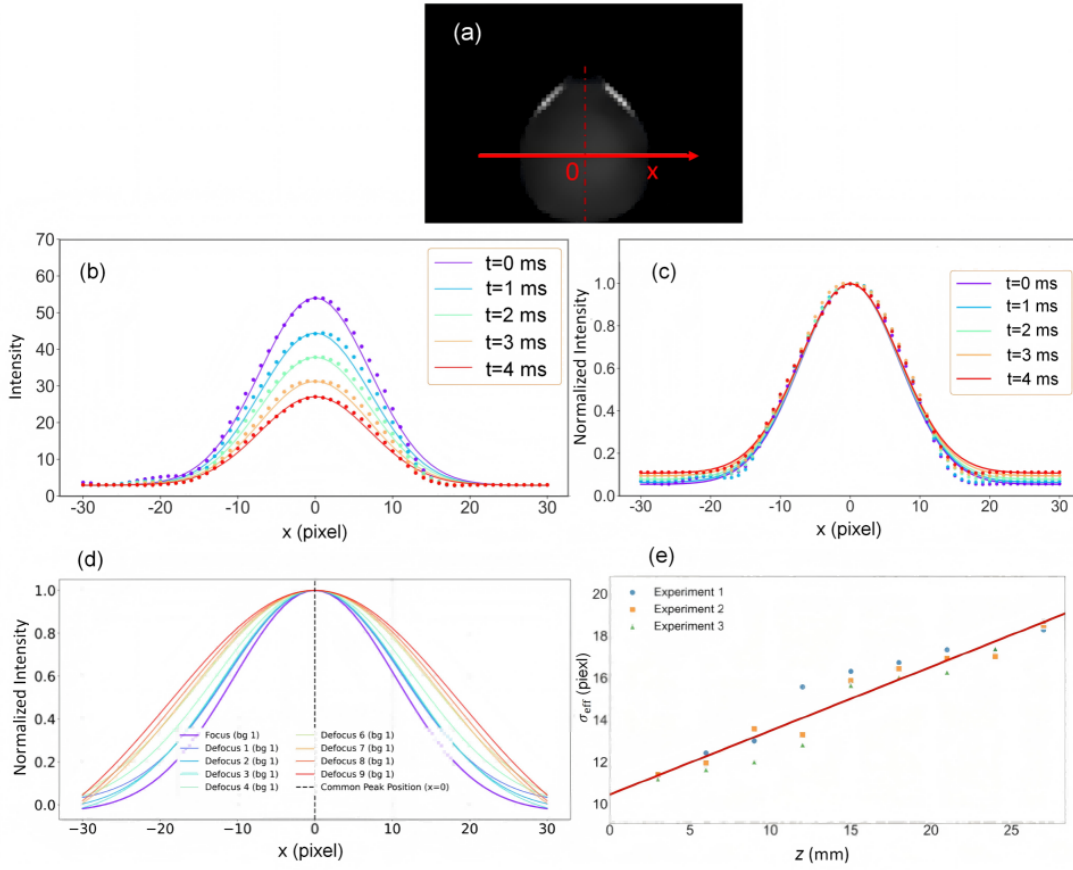


Figure 3. (a) Image of a water drop (diameter 2 mm) doped with $1 - BrNp \cdot M\beta - CD \cdot ROH$ phosphor dye and excited with an LED at wavelength 280 nm; (b) Intensity profile through a horizontal mid-plane slice of image fitted to Gaussian curves; (c) data from diagram (b) normalized using the respective maximum amplitude. Time between frames 1 ms, starting at Frame 1 immediately after excitation ($t = 0$). (d) Gaussian curve fitting for different depth positions, with a step size of 3 mm (e) Changes in the standard deviation width obtained by Gaussian fitting versus depth position

with σ_z being its width. This width is related to the distance of the particle from the object plane (Δz) through various optical parameters of the system and is given by the linear relation (Rao et al., 2024)

$$\sigma_z = \frac{ADM}{2f} |\Delta z| = \beta |\Delta z| \quad (7)$$

A is an experimental constant for the imaging system, D is the aperture diameter, f is the focal length and M is the magnification. Since A , D , M and f are invariant for a given DFD measurement system, these terms are replaced with a single constant β . Further details of the DFD measurement principle and adaptation to various applications can be found in previous studies by the authors (Sharma et al., 2023; Zhou, Tropea, et al., 2020; Zhou et al., 2021; Rao et al., 2024).

2.3. DFD+PT

From these preliminary remarks it becomes evident that images of phosphor-doped droplets excited by a UV LED will exhibit an inherent blur quantified by σ_p and an additional blur if the drop is out of focus. The working hypothesis is that the measured degree of blur, denoted as σ_{eff} can be expressed as

$$\sigma_{\text{eff}} = \sqrt{\sigma_z^2 + \sigma_p^2} \quad (8)$$

which is simply a consequence that the convolution of a Gaussian curve with a Gaussian curve is also a Gaussian curve. In practice, σ_{eff} can be measured, but without knowing either the size or position of the droplet, the portion attributable to σ_z or σ_p cannot be determined, thus, a solution for the two unknowns, d_0 and Δz , is not immediately possible.. To tackle this problem, the first step is to quantify the dependencies of σ_z and σ_p as a function of droplet size and position. This is performed using calibrations, which require a dedicated experimental facility as described in section 3. Nevertheless, the principle way forward is outlined in the following.

The first calibration aims to quantify the out-of-focus blur (σ_z) as a function of droplet position. Droplet position is expressed as Δz in millimeters away from the object plane. This calibration determines the β in Eq. (7) and was performed not with a droplet but with a calibration plate and a reticle of fixed diameter. One such example calibration curve is illustrated in Fig. 4(a). This is a linear relation, albeit, not passing through the origin. There is an offset, known from Rao et al. (2024) and originating from the finite resolution of the camera when the particle/droplet becomes so small that the image is only a few pixels large. For subsequent analyses this linear dependence is expressed as:

$$\sigma_z = \alpha_i + \beta_i \Delta z \quad (9)$$

Next, a calibration relation between σ_p and drop size is required. This is obtained holding phosphor doped droplets of varying size in the object plane, illuminating them with UV light and measuring the resulting intensity standard deviation. Since $z = 0$, the measured σ_{eff} corresponds directly to σ_p . Such a calibration curve is shown in Fig. 4(b), and can also be represented using a linear fit:

$$\sigma_p = k + l d_0 \quad (10)$$

Finally, in the present experimental facility, telecentric lenses were not available, thus the magnification also varies with z position. This must be quantified, since the magnification will directly influence any measured value of σ_{eff} . As pictured in Fig. 4(c), the magnification can be expressed using the linear equation:

$$M_i(z) = a_i + b_i(z) \quad (11)$$

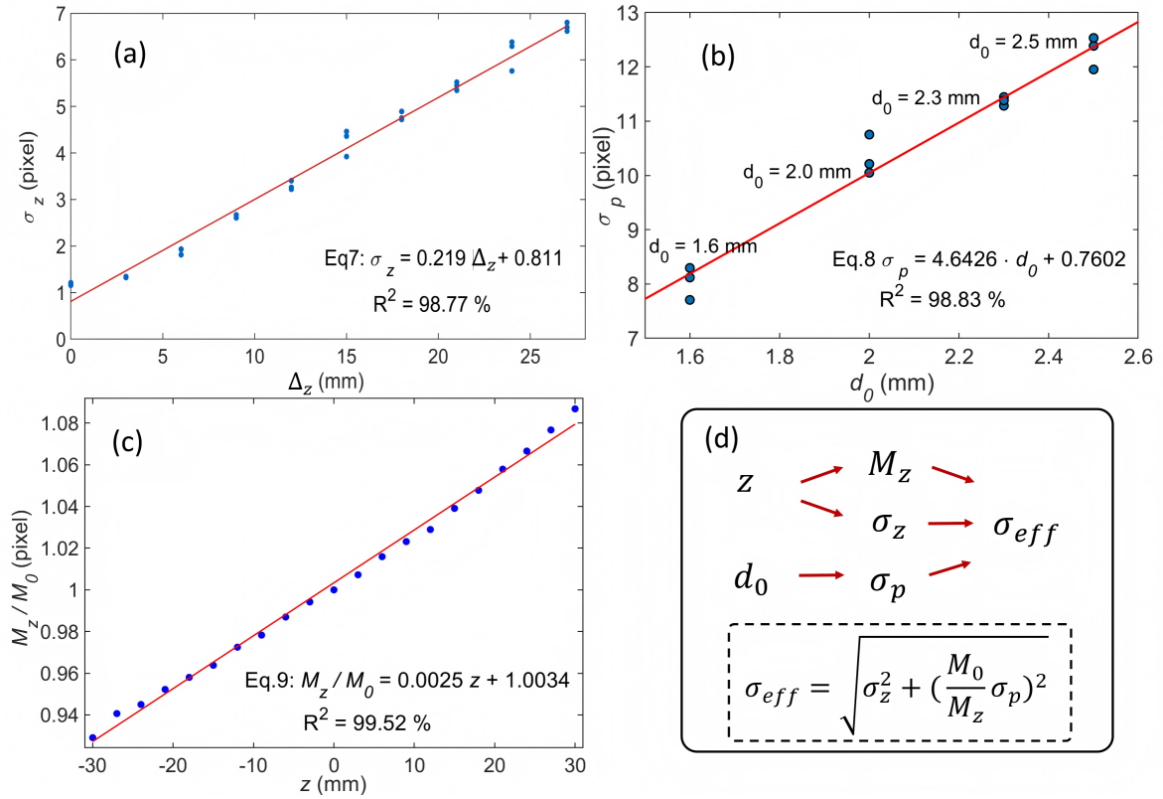


Figure 4. (a) Calibration relationship between Δz and σ_z ; (b) Calibration relationship between d_0 and σ_{eff} ; (c) Calibration relationship between $|z|$ and magnification (d) schematic illustration of the relationships among various parameters

However, a single set of relational expressions is insufficient to directly solve for the two unknown parameters, droplet size d_0 and position Δz . To address this, a dual-camera DFD measurement system is proposed, as schematically pictured in Fig. 2(a). The two cameras with two different focal lengths; hence, two different object planes and magnifications, measure respectively $\sigma_{eff,1}$ and $\sigma_{eff,2}$. Each optical system has its own calibration relations, (9), (10) and (11), expressing respectively σ_z to Δz , σ_p to d_0 and magnification. Realizing that there is a known Δz_{12} difference in the object planes of the two detector paths, a system of equations for the two unknowns z_i and d_0 can be

written using Eq. (8) as a basis

$$\begin{aligned}
\sigma_{\text{eff},1} &= \sqrt{\sigma_{z_1}^2 + \sigma_p^2} = \sqrt{(\alpha_1 + \beta_1 z_1)^2 + \left(\frac{M_{z_1}}{M_0} \sigma_p\right)^2} \\
&= \sqrt{(\alpha_1 + \beta_1 z_1)^2 + \left(\frac{a + b(z_1)}{M_0} (k + l d_0)\right)^2} \\
\sigma_{\text{eff},2} &= \sqrt{\sigma_{z_2}^2 + \sigma_p^2} = \sqrt{(\alpha_2 + \beta_2 z_2)^2 + \left(\frac{M_{z_2}}{M_0} \sigma_p\right)^2} \\
&= \sqrt{(\alpha_2 + \beta_2 z_2)^2 + \left(\frac{M_{z_2}}{M_0} (k + l d_0)\right)^2} \\
&= \sqrt{(\alpha_2 + \beta_2 (z_1 + \Delta z_{12}))^2 + \left(\frac{a + b(z_1 + \Delta z_{12})}{M_0} (k + l d_0)\right)^2}
\end{aligned} \tag{12}$$

whereby the known Δz_{12} has been used to express $\sigma_{\text{eff},2}$ in terms of d_0 and Δz_1 in the last equation.

By means of the dual imaging method, $\sigma_{\text{eff},1}$ and $\sigma_{\text{eff},2}$ are extracted from the acquired Image 1 and Image 2. Prior calibration of the two imaging systems supply all the coefficients in equations (9), (10) and (11). In addition, Δz_{12} is known, hence $z_2 = z_1 + \Delta z_{12}$. On this basis, a system of two equations with respect to the two unknowns z_i and d_0 are available.

This solution procedure is illustrated in Fig. 5. Specifically, the decay curve required for lifetime calculation is fit to Eq. (1) from a sequence of 100 recorded frames to implement the temperature measurement using Eq. (2). The unknowns Δz_1 and d_0 are solved using the first frame of the decay image sequences captured synchronously by the two independent imaging channels.

3. Experimental Method

To investigate the dependence of σ_z and σ_p on d_0 and Δz , an experimental calibration facility was assembled, which allowed d_0 and Δz to be systematically varied. The setup is schematically illustrated in Fig. 6. With this arrangement, images of single drops, with or without phosphor, at varying distances from the object plane can be captured. The position of the drop, i.e., the depth Δz , is controlled using a translation stage, and pendant droplets are generated using a syringe.

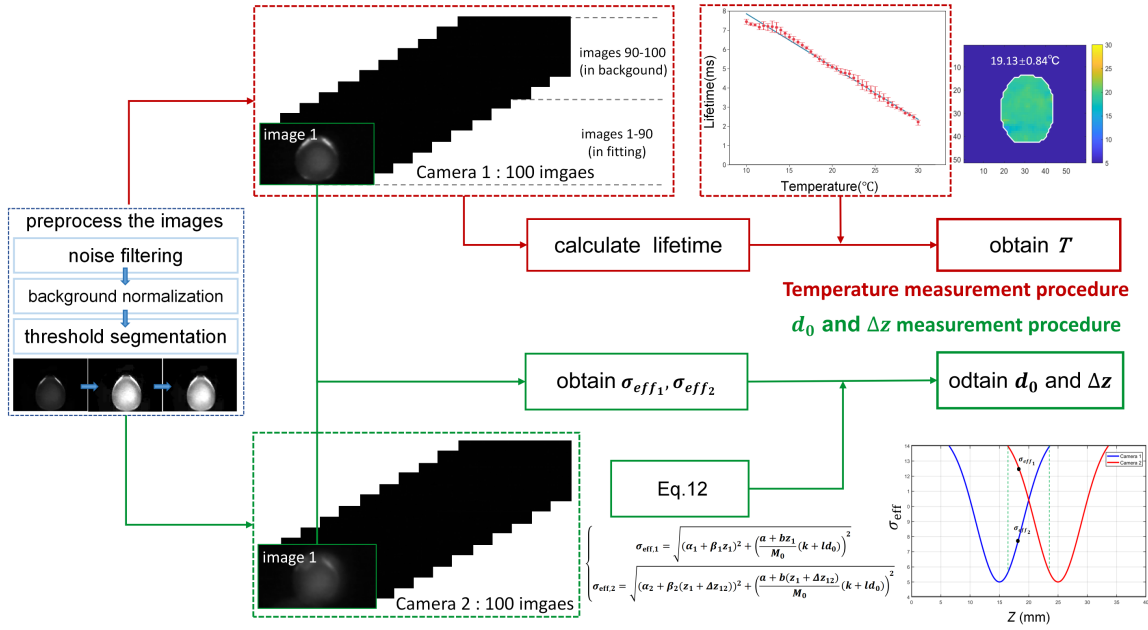


Figure 5. The measurement workflow comprising two modules: phosphorescence lifetime based temperature measurement, and a DFD based droplet measurement for σ_z and Δz_1 .

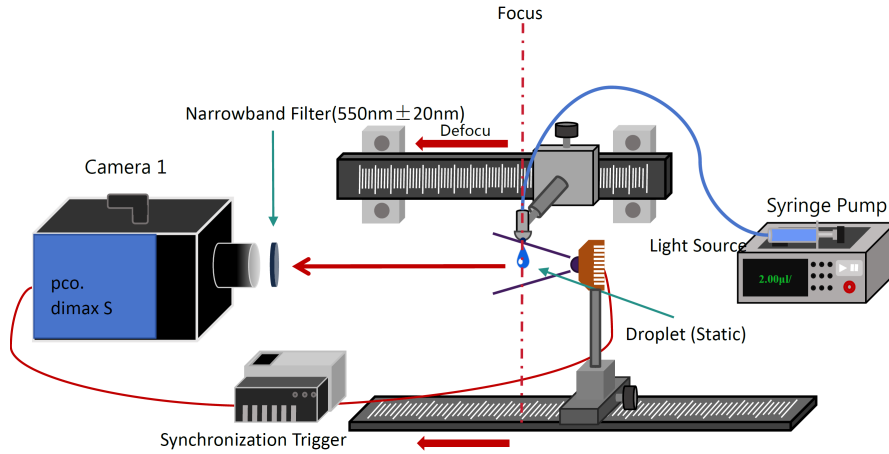


Figure 6. Experimental apparatus for generating droplets, illuminating them with a 285 nm LED and traversing the droplet out of focus in the z direction. Alternatively a calibration plate with reticle can be traversed. A droplet without phosphor doping will appear only as a shadow.

This configuration includes an LED ultraviolet (UV) excitation source and a single camera imaging system. The imaging system comprises a pco.dimax S high-speed camera, paired with a Nikon AF NIKKOR lens, and achieves synchronous control with the LED for image acquisition. A narrow bandpass filter with a center wavelength of 550 nm (consistent with Hu & Koochesfahani (2003)). The above dual-camera DFD method is verified using this experimental setup. A single imag-

ing system is adopted to excite and record images of the identical droplet twice, with an axial working distance offset of Δz_{12} between the two measurements. Identical timing sequence control and excitation intensity are maintained throughout the test, which equivalently simulates the data acquisition procedure of the dual-camera DFD technique.

4. Results

A single camera was used to capture images of the same stationary excited droplet at two different working distances (effectively a dual-camera configuration). The solutions of d_0 and Δz resulted from the measured $\sigma_{\text{eff}1}$ and $\sigma_{\text{eff}2}$ values, as presented in Fig. 7. Most data points agree well with the true values, while larger errors are observed at several points. These test results demonstrate that combining σ_{eff} from dual images with calibration data principally enables the simultaneous measurement of d_0 and Δz .

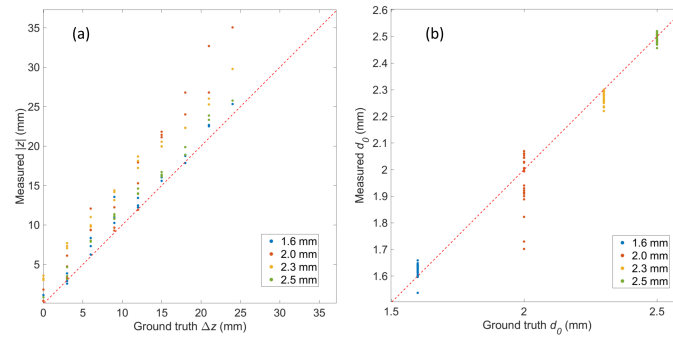


Figure 7. Calculation results of the single-camera dual-imaging method. The measurement range is 0–27 mm. (a) Comparison between ground truth and measured values of Δz ; (b) Comparison between ground truth and measured values of d_0

The temperature measurements were then performed at varying positions for droplets of three different diameters: 2 mm, 2.3 mm, and 2.5 mm. The results of these measurements are shown in Fig. 8. It is evident from these results that the lifetime temperature measurement is insensitive to the droplet position and in good agreement with the reference temperature for all droplet sizes. All measured temperatures are within $\pm 1^\circ \text{C}$ of the thermocouple reference value. The associated values of σ_{eff} are shown in Fig. 8(b), indicating the scatter obtained from repeated measurements. These results then support the feasibility of combining the DFD technique with the temperature measurements using phosphorescence lifetime.

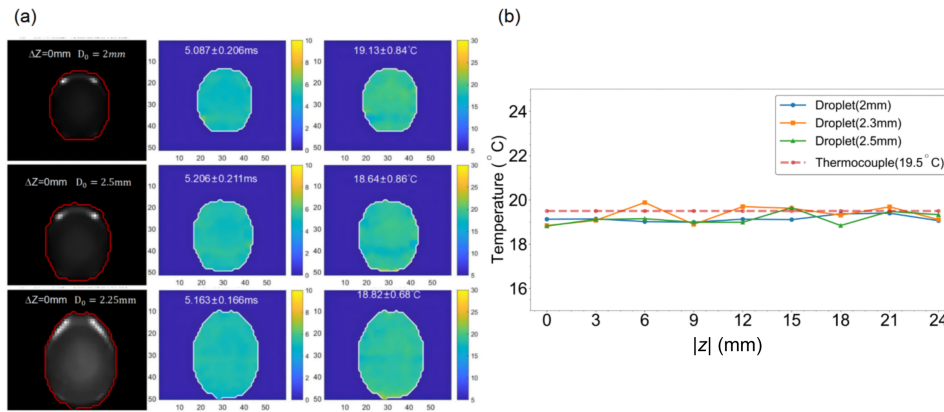


Figure 8. (a) Measured mean temperature of different size droplets at varying Δz positions and comparison to reference temperature (19°C) from the thermocouple. (b) variation of measured standard deviation of intensity profile (σ_{eff}) with Δz for droplets of four different sizes, compared with the standard deviation of a reticle on the calibration plate (σ_z).

5. Conclusions

This study outlines one method of combining the DFD measurement technique for droplet size and position with phosphor thermometry based on lifetime decay. Key conclusions include:

1. The blur of modal images σ_{eff} of excited droplets can be decomposed into an intrinsic phosphorescence blur, σ_p , and a defocus blur, σ_z . The standard deviation σ_p is directly related to the particle size and σ_z varies with distance of the droplet from the object plane of the imaging optics.
2. The degree of blur of phosphor doped droplets, quantified by σ_p , remains unchanged during the lifetime decay of the emitted intensity.
3. DFD position and size measurement and phosphorescence lifetime thermometry can be implemented synchronously on excited droplets, i.e. the phosphorescence decay does not alter the gray-scale distribution for the DFD image processing.

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