

# Shock wave visualization in a 3D-printed axisymmetric critical flow venturi nozzle using Background-oriented Schlieren technique

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## ABSTRACT

The shock wave visualization and its quantitative location measurement were performed with a 3D-printed axisymmetric critical flow venturi nozzle (CFVN). One of the challenges of this research is to develop visualization techniques for the conditions under which strong refraction occurs at the wall such as gas flow in an axisymmetric flow field. The axisymmetric CFVN made of transparent material is used to demonstrate the technique. CFVNs complying with ISO 9300 is widely used as flow meters and transfer of gas flow standard. Several experimental results indicate that the supersonic resonance between shock wave and boundary layer in CFVN slightly changes the flow rate. This phenomenon limit the CFVN operation condition and a major factor that undermines convenience of the CFVN. However, shock wave behavior in the axisymmetric CFVN is one of the unsolved issues. In this paper, the characteristics of the discharge coefficient of the 3D-printed axisymmetric CFVN for the supersonic flow fields were evaluated. Subsequently, the capability to visualize the shock wave in the CFVN was confirmed. As a result, the relationship between shock wave location and pressure ratio was clarified quantitatively.

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## 1. Introduction

Axisymmetric internal flows are widely utilized in various engineering applications, including internal combustion engines, aerospace systems, and industrial plants. In such applications, a precise understanding of the structure and characteristics of internal flows is crucial for performance evaluation and optimization. However, curved walls in fluid experiments complicate accurate measurements. For example, because the sensing surface of a pressure transducer is generally flat, installing it in a supersonic flow creates a surface discontinuity that can generate shock waves, thereby complicating accurate pressure measurements. Moreover, in internal flow experiments, differences in refractive index between the flow medium and the optical window

frequently lead to image distortion, making flow visualization and understanding the flow structure challenging. Due to these difficulties, fundamental studies based on two-dimensional flow fields have been actively conducted from an engineering perspective. Representative examples include rectangular shock tubes and supersonic wind tunnels. Studies on such two-dimensional flows have contributed to the advancement of fluid dynamics.

In compressible flows, changes in cross-sectional area constitute a major factor that significantly influences the flow field. Key flow properties, such as Mach number, pressure, and temperature, cannot be simply represented by a two-dimensional flow of an axisymmetric internal flow. For this reason, studies employing axisymmetric configurations are often preferred. In such cases, however, specific considerations are required for flow visualization and measurement. For example, in measurements of supersonic flows in a cylinder, attempts have been made to mitigate optical distortions using corrective lenses (Hosseini 2010). Nevertheless, measurement techniques for axisymmetric internal flows with varying cross-sectional areas remain a significant challenge.

Critical Flow Venturi Nozzle (CFVN) is one of the devices requiring visualization of axisymmetric internal flow fields. As discussed in Chapter 2, the flow at the nozzle throat reaches critical states, and the mass flow rate of CFVN is governed by the upstream pressure and temperature. Because of its long-term stability and repeatability, CFVN is commonly employed in gas flow measurements. Mickan et al. (2016) utilize the CFVN to prove the equivalence of reference standards for gas flow of PTB (Germany), LNE-LADG (France), NIM (China) and NIST (USA) and in Japan, CFVN is operated as a national standard transfer device. The guidelines and standards in using CFVN are defined by ISO 9300:2022. ISO 9300 defines two types of axisymmetric designs, and Fig. 1 is one of them, the Toroidal-throat venturi nozzle.

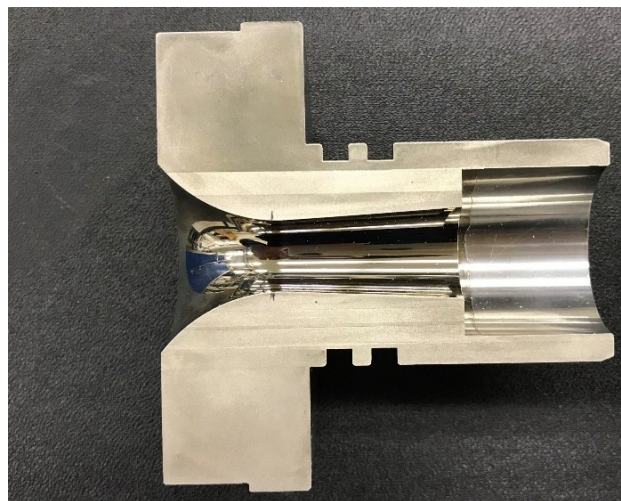


**Figure. 1** Cross-sectional photograph of a Critical flow venturi nozzle

The performance metrics of a CFVN are primarily defined by its mass flow characteristics and critical back-pressure ratio (CBPR). Mass flow characteristics are typically expressed in terms of discharge coefficients, as defined by the semi-empirical ISO 9300 formula based on the Reynolds number. CBPR is the ratio of downstream pressure to upstream pressure (Back pressure ratio: BPR) at which the mass flow rate changes due to the collapse of critical state. In contrast to the mass flow characteristics, the CBPR characteristics are strongly influenced by the nozzle geometry, such as the diffuser length. Furthermore, under certain conditions, critical state is temporarily lost and slight changes in flow rate occurred (Von Lavante et al. 2001, Takegawa et al. 2020). Therefore, ISO 9300 recommends that the BPR does not exceed 0.25 at low Reynolds numbers, which greatly detracts from the convenience of CFVN. Critical state collapse is thought to occur mainly when shock waves pass through the throat and no longer satisfy choking conditions in the throat. However, where shock waves are located under specific conditions remains an unsolved problem (Wright et al. 2016). Experiments using 2D nozzles have validated several shock wave location prediction models (Yagi et al. 2017), but studies with axisymmetric shapes have not been reported. The objective of this study is to clarify such axisymmetric internal flow fields. We had constructed a visualization experimental system for internal flow using transparent 3D-printed nozzles and Background-oriented Schlieren (BOS) technique (Yamagishi et al. 2026). In this paper, the following two investigations were conducted. First, to verify the accuracy of the 3D-printed CFVN, the Reynolds number dependence of discharge coefficient observed with the common metal CFVN specified in ISO 9300 were checked. Next, the quantitative shock wave location was measured and compared with a shock wave location prediction model.

## 2. Theory

### 2.1 Discharge coefficients

Critical flow venturi nozzle (CFVN) is a nozzle with a throat shown in Fig. 1. The flow velocity at the throat reaches the speed of sound when the BPR (i.e., the ratio of the downstream pressure to the upstream pressure) is lower than the CBPR. This condition corresponds to the critical state, and the mass flow rate  $Q_{mth}$  through the CFVN can be determined according to Eq. (1) for an ideal gas.

$$Q_{mth} = SC^* P_0 \sqrt{\frac{N_g}{RT_0}} \quad (1)$$

where  $S$  is the cross-sectional area at the throat,  $C^*$  is a coefficient called the critical flow function,  $P_0$  and  $T_0$  are stagnation pressure and temperature,  $R$  is the gas constant and  $N_g$  is the molar mass. Typically,  $Q_{mth}$  is referred to as the theoretical mass flow rate, whereas the actual flow rates  $Q_m$  measured in experiments are usually smaller than  $Q_{mth}$ , primarily due to effects such as viscosity and other non-ideal flow phenomena. The ratio of  $Q_m$  to  $Q_{mth}$  is referred to as the discharge coefficient  $C_d$ , which represents a key parameter characterizing the performance of the CFVN (Eq. (2)).

$$C_d = \frac{Q_m}{Q_{mth}} \quad (2)$$

For CFVN specified in ISO 9300, it is known that  $C_d$  can be fitted using the Reynolds number  $R_{eth}$ , as expressed in Eq. (3) (Ishibashi et al. 2000).

$$C_d \cong \alpha - \frac{\beta}{\sqrt{R_{eth}}} \quad (3)$$

where  $\alpha$  and  $\beta$  are empirical constants.

## 2.2 Normal shock wave model

In this section, we introduce the prediction model of a shock wave location with one-dimensional flow and normal shock wave assumption.

Figure 2 shows the pressure distribution in a CFVN when a normal shock is located in the diffuser. Along curve  $a-b-c$  and  $d-e$ , the flow evolves isentropically, whereas along  $c-d$  it undergoes a change across a shock wave. Please note that directly measuring the pressure at point  $e$  is difficult.

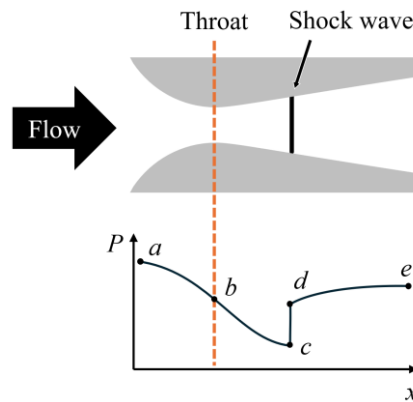


**Figure. 2** Pressure distribution in a critical flow venturi nozzle

In most studies on CFVNs compliant with ISO 9300, the wall static pressure in the downstream piping is measured instead. Therefore, it should be note that misalignment of the pressure sensor may lead to errors in the measured pressure due to the velocity distribution.

$$\frac{A_c}{A_b} = \frac{1}{M_c} \left[ \frac{(\gamma - 1)M_c^2 + 2}{\gamma + 1} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (4)$$

Using the area ratio relative to the throat (point *b*), the Mach number at point *c* is given by Eq. (4).

Furthermore, the pressure and Mach number at point *d* are given by Eq. (5) using the Mach number at point *c* and the stagnation pressure  $P_0$ .

$$\frac{P_d}{P_0} = \frac{2\gamma M_c^2 - (\gamma - 1)}{\gamma + 1} \left[ 1 + \frac{\gamma - 1}{2} M_c^2 \right]^{-\frac{\gamma}{\gamma-1}} \quad (5)$$

$$M_d^2 = \frac{(\gamma - 1)M_c^2 + 2}{2\gamma M_c^2 - (\gamma - 1)} \quad (6)$$

Then, along *d–e*, the flow is isentropic, and the Mach number and pressure are given by Eqs. (7) and (8).

$$\frac{A_e}{A_d} = \frac{M_d}{M_e} \left[ \frac{(\gamma - 1)M_e^2 + 2}{(\gamma - 1)M_d^2 + 2} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (7)$$

$$\frac{P_e}{P_d} = \left[ \frac{2 + (\gamma - 1)M_e^2}{2 + (\gamma - 1)M_d^2} \right]^{-\frac{\gamma}{\gamma-1}} \quad (8)$$

The relationship between the shock wave location and the BPR  $P_e/P_0$  can be obtained by numerically solving Eqs. (4)–(8). This model simplifies the internal flow in the nozzle and neglects the pressure loss at the exit. Nevertheless, the fundamental behavior of the shock can be explained using this model.

### 3. Experiment

In this paper, two experiments were conducted. One is the measurement of the  $C_d$  of 3D-printed CFVN by nozzle-to-nozzle test to validate the accuracy of 3D-printed CFVN, and the other is the shock wave visualization using BOS technique to clarify the quantitative shock wave location.

### 3.1 Nozzle-to-nozzle test

Nozzle-to-nozzle test method is the basic method to evaluate the characteristics of CFVN (Takegawa et al., 2020). Figure 4 shows the setup of the nozzle-to-nozzle test. 3D-printed CFVN was placed downstream, while a reference CFVN with throat diameter of 10 mm was positioned upstream. The temperature sensors (T1 and T2) and the pressure sensors (P1 and P3) were employed to measure the stagnation temperature and pressure of Eq. (1) and determine the mass flow rate of CFVNs. The discharge coefficient of the reference CFVN was known. During the experiment, flow was generated in the test line by applying sufficient differential pressure to the CFVNs using compressed air from a high-pressure chamber and a vacuum pump. Since the mass flow rate in the test line was constant, the mass flow rate of the downstream CFVN was determined from that of the upstream reference CFVN. The test line shown Fig. 3 was installed in a temperature-controlled room ( $23^{\circ}\text{C} \pm 1 \text{ K}$ ). In all tests, the measurement was performed after the temperature and pressure were stable. The upstream pressure of the reference CFVN was set at 6 points between about 105 kPa and 600 kPa by changing the chamber pressure. During the measurements under each condition, the upstream pressure variation was up to 0.2 kPa.

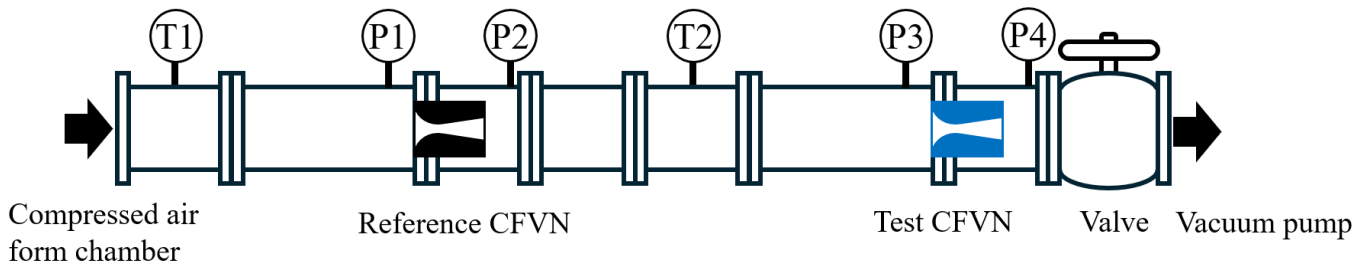


**Figure. 3** Setup of the nozzle-to-nozzle test (T: temperature sensor; P: pressure sensor)

### 3.2 3D-printed optical CFVN

In this study an optical CFVN was fabricated using 3D printer (Form 4, Formlabs) with clear resin (Clear v5, Formlabs). Despite its excellent transparency, the surface layer marks resulting from the printing process significantly impede optical access. Therefore, we polished the nozzle as shown Fig. 4 (a). Since the nozzle was polished manually, the cross-sectional area at throat is not accurate. Measuring the precise shape of the optical CFVN is the future work. Also, additional investigation is needed for long-term stability.

Figure 4 (b) shows the design of the optical CFVN. The CFVN consists of two parts. Section A-B-C forms the inlet arc, and point B corresponds to the throat, which has the smallest cross-sectional area. The throat diameter  $d$  was 20 mm. Section C-D serves as the diffuser. It is tangentially

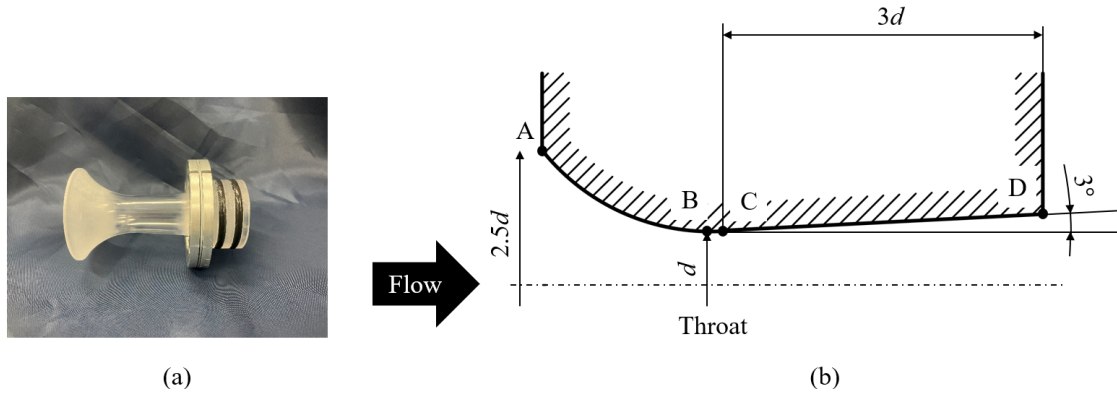


**Figure. 4** 3D-printed optical CFVN; (a) Photograph, (b) Design

connected to the inlet arc, with a length three times the throat diameter and an angle of  $3^\circ$ . According to ISO 9300, the diameter of the tube connected to point A is specified. However, during the visualization experiments, point A was left open to the atmosphere in order to reduce the load on the nozzle.

### 3.3 Setup of the shock wave visualization experiments

Visualization experiments were performed by opening the optical CFVN to the atmosphere. All components upstream of the test CFVN in Fig. 3 have been removed. The ratio of pressure measured in P4 to atmospheric pressure was calculated as BPR. Figure 5 shows the setup of the visualization experiment. Flow visualization was performed using Background-oriented Schlieren (BOS) technique. The camera is synchronized with a pulsed LED light source and has an exposure time of approximately  $3 \mu\text{s}$ . The images were taken with a digital camera (EOS R7, Canon) with a 200 mm focal length lens. A random dot printed on an OHP sheet was used as the background image. Due to the roughness of the optical CFVN, the random dot size was about 16 pixels per dot, which is larger than the typical BOS experiment. The valve in Fig. 3 was utilized to control the BPR.

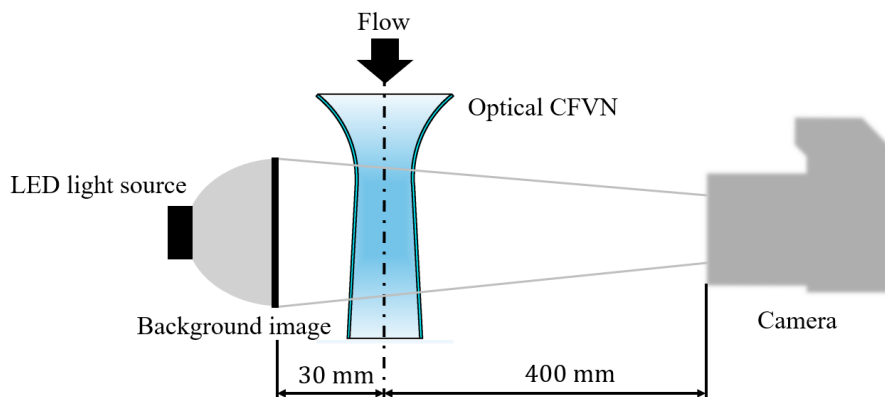


**Figure. 5** Setup of the visualization experiments

## 4. Result and discussion

### 4.1 Discharge coefficient of 3D printed CFVN

Figure 6 shows discharge coefficient  $C_d$  of the 3D printed CFVN obtained by nozzle-to-nozzle test method. Exhibits high stability can be seen under each pressure condition. The dash line in Fig. 6 indicates the fitted line using Eq. (3) and the fitting error was less than 0.05 %. This result demonstrates that the 3D-printed CFVN possesses characteristics similar to those of a machined CFVN. On the other hand, the values of  $\alpha$  and  $\beta$  in Eq. (3) were not exactly same as those of the machined CFVN. This is the effect of surface roughness and dimensional error. Based on these results, although some errors are present, the 3D-printed CFVN produces a flow field similar to that of a machined CFVN.

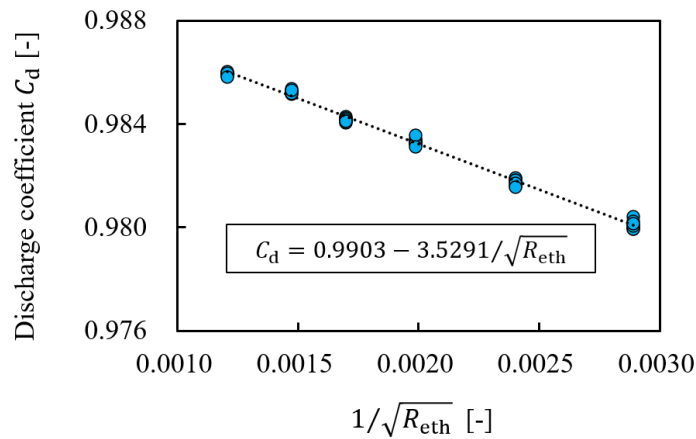


**Figure. 6** Discharge coefficient of the 3D printed critical flow venturi nozzle and the fitted line  
( $R_{eth}$ : Reynolds number)

### 4.2 Shock wave visualization results

Figure 7 shows examples of the displacement fields obtained by Background-oriented Schlieren technique. The BPR was 0.797, 0.819, 0.828 and 0.847. Regions that were not visualized due to limitation of the optical access are masked. The density change that occurs in the flow fields is measured as displacement in the BOS technique. According to the results, we successfully visualized the shock wave in a 3D-printed axisymmetric nozzle. Figure 8 shows the variation of the distance between shock wave and throat along nozzle axis. The shock wave position is taken as the point where the displacement attains its maximum. From the results, it can be confirmed that the shock wave approach throat when the BPR rises.

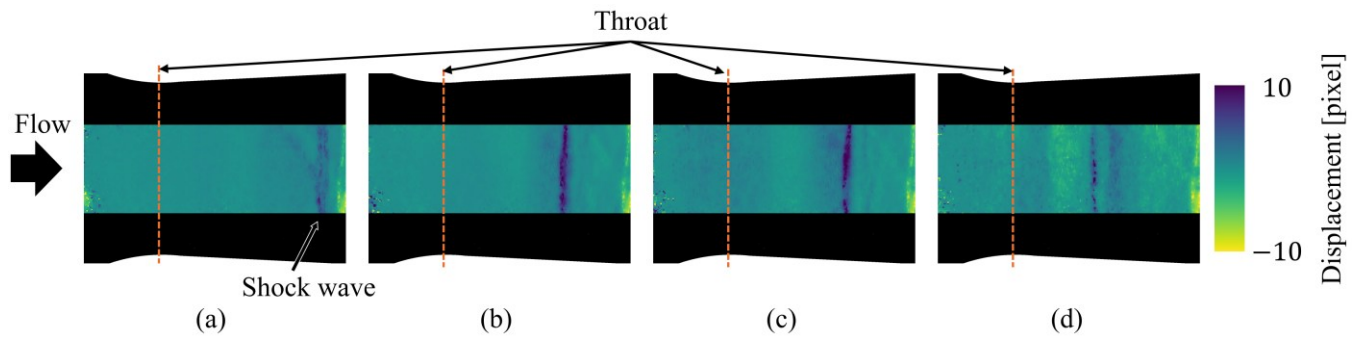


**Figure. 7** Shock wave visualization results for different BPR: (a) 0.797, (b) 0.819, (c) 0.828 and (d) 0.847

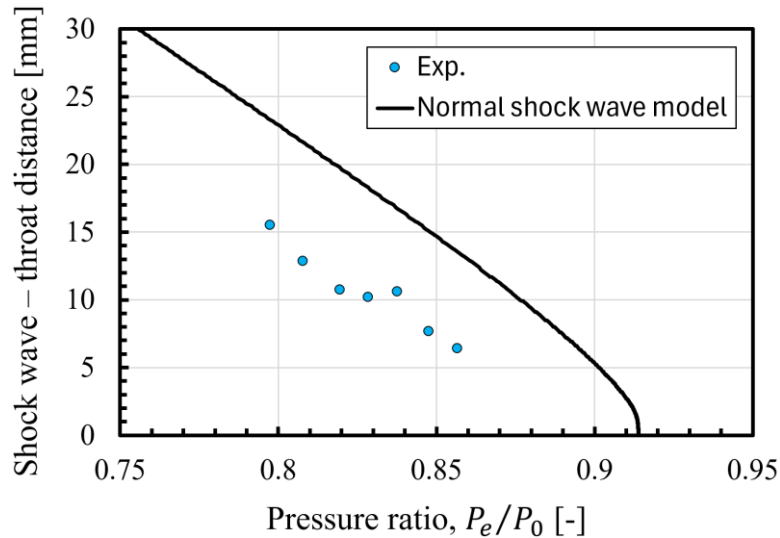


**Figure. 8** Distance between the shock wave and the throat along the nozzle axis

The shock wave location was located closer to the throat than that prediction by the normal shock wave model. This discrepancy is caused by pressure losses due to wall friction and pressure measurement error due to misalignment of the pressure transducer. Nevertheless, the qualitative trend agrees well with the theoretical prediction. A slight fluctuation in the shock wave location was also observed, suggesting that the shock wave may have oscillated during the measurements; however, further investigation is required to clarify this behavior.

## 5. Conclusion

In this study, the mass flow rate measurement and shock wave visualization was performed with a 3D-printed critical flow venturi nozzle (CFVN). The discharge coefficient of the 3D-printed CFVN was well fitted using the approximation curve described in ISO 9300. This indicates that the discharge coefficient of the 3D-printed CFVN has a Reynolds number dependence similar to that of the machined CFVN. From the visualization results, the shock waves in axisymmetric nozzle were

successfully visualized using Background-oriented Schlieren (BOS) technique. And the quantitative relationship between the shock wave location and the back pressure ratio was investigated. These results demonstrated that using a 3D-printed CFVN makes it possible to visualize the internal shock wave in axisymmetric nozzles and to investigate the characteristics of axisymmetric flow.

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