

Simultaneous Tracking And Sizing Of Bubbles Under Ultrafast Illumination Based On Transient Scattering Statistics

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ABSTRACT

Turbid and turbulent conditions arising from high-void-fraction bubbly flows remain experimentally challenging. Occlusion and photon scattering result in information loss, which, when compounded, becomes too significant for conventional approaches to mitigate. To remove the influence of scattered photons, ultrafast optical Kerr effect gating is implemented to selectively remove damaging scattered photons. As the separation of transient scattering modes is a function of bubble size, analysis of the gated ballistic light is also used to determine size. Two experimental frameworks within a vertical bubble column facility are introduced to simultaneously track bubbles and determine their sizes using minimal spatial information and a modified particle-tracking code. At low void fractions, this proved consistent with independent calibration and previous experimental studies. In higher void fraction conditions, tracking ballistic signals appeared successful for resolving velocities, even where physical occlusion became significant. Internal ballistic scattering modes were recovered, providing a basis to obtain bubble size distributions within optically thick conditions with continued development.

1. Introduction

Although ubiquitous, bubbly flows remain perplexing. Especially at high void fractions and under strong turbulence, there remains a strong reliance on empirical relations due to the absence of a unified theory of bubble dynamics, coalescence, fragmentation, and turbulence. Instrumentation of these highly transient, highly turbid conditions remains experimentally demanding, creating gaps in collective understanding. Bubble size distributions (BSD), which describe bubble density as a function of size, are essential for characterizing bubbly flows. This is especially true in periodic and transient events, such as active wave breaking and degassing, where conditions and distributions change rapidly.

Although there have been significant improvements in optical measurements of bubbly flows, notably with multi-camera arrangements and convolutional neural network-based identification (Ma

et al., 2023; Hessenkemper et al., 2022), limits remain in their applicability at high void fractions. A key cause of such limitations is the occlusion problem created by the bubbles themselves, manifested on two fronts. Initially, bubbles create a physical blockage, which is easily seen where they overlap in a single view. Depending on the technique used, light source or other tracers may also be blocked, hindering measurements in more extensive regions beyond the bubble profile. Multiple views and backlit illumination have historically been used to overcome this occlusion (Villafañe et al., 2025; Cioncolini & Magnini, 2025; Ni, 2024). In doing so, some of the lost information can be recovered, in turn increasing confidence in measurements when constituent views are combined.

Due to their size, geometry, and change in refractive index, bubbles are also excellent photon scatterers. As light passes through bubbles, photons experience rapid deflection depending on their relative angle of incidence. This scattering can be seen in multiple ways; the most prominent is with blurring and gradient effects at the bubble's surface. Local internal scattering coupled with intense reflections at the surface obscures the true boundary of the bubble, increasing the uncertainty in bubble sizing. Moreover, when used with particle tracers, this scattering may overwhelm the appearance of tracers near the bubbles' surfaces, increasing ghost particle formation or preventing flow measurements in their vicinity.

As void fractions increase, with smaller and more numerous bubbles, these problems worsen significantly. The combined scattering of multiple tightly packed bubbles creates vast localized light and dark regions. Again, these may obscure bubbles or other features. Each scattering event will also attenuate light, creating a diffuse light field. In these cases, information is lost due to photon behavior rather than physical obstructions; hence, this can be considered occlusion caused by scattered photons. The influence of a dispersed phase and turbidity, including bubbles, on scattering and occlusion can be standardized through Optical Thickness (τ). Physically, this is the ratio of the mean photon path length to the geometric depth and is proportional to the dispersed phase volume fraction (Eq. (1)):

$$\tau = \frac{3Q_s L_v}{2d_{32}} \phi = \frac{\pi L_v}{4} \sum Q_{s,d_i} n_{d_i} d_i^2 \quad (1)$$

Where L_v is the geometric depth of the volume of interest, Q_{s,d_i} is the scattering efficiency of a bubble of diameter d , and n_{d_i} is the bubble number density.

Multiple perspectives and structured illumination are generally applicable when the optical thickness remains below 2. (Masuk et al., 2019; Belden et al., 2012). Whilst physical occlusion can be accounted for across different perspectives, photon occlusion becomes too significant to mitigate. At extremes, when τ exceeds 10, hard-field diagnostics, including x-ray densitometry, have been highly effective (Villafañe et al., 2025; Aliseda & Heindel, 2021). By employing much shorter wavelengths, scattering efficiency decreases markedly, virtually eliminating the photon occlusion problem. However, these tend to lose applicability in transitional flows, where single-phase regions

persist and rapidly change, such as during active air entrainment.

There is still a need for optical diagnostic techniques that can adequately probe turbid environments with optical thicknesses of 2-10. Essentially, this must mitigate the significant scattering and photon occlusion. With typical pulsed sources, the light's spatial extent can exceed that of scattering particles or bubbles by several orders of magnitude. That suggests that the scattering phenomena seen are instantaneous and, therefore, irremovable. However, Light-matter interactions become transient under ultrafast illumination, making ultrafast optical diagnostics a viable solution. By reducing the illuminating pulse duration to the sub-picosecond scale, the time required for component scattering modes exceeds the pulse lifetime; hence, constituent modes appear independently, and damaging high-order modes can selectively be removed.

The transient scattering phenomena with submerged air-bubbles were modeled by Chaussonnet & Bardet (2020), initially for a single bubble, and this was later extended to polydisperse bubble plumes by Chaussonnet et al. (2022). By applying Lorenz-Mie Theory (LMT) and Debye Scattering Expansion, the scattering cross-section of both bubbles and droplets were simulated as a function of time. This accounted for the size and location of bubbles, considering the inherent pulse dispersion and chirp. With a pulse duration on the order of 70 fs, constituent modes could be separated for bubbles as small as 10 μm in diameter. Due to the negative change in refractive index with air bubbles, the first two scattering modes, in accordance with the Debye expansion, appear before the unscattered light and the remaining higher-order modes. This is dubbed 'Ballistic Light,' and is seen primarily at the optical center of the bubbles. The time between these modes and the undisturbed mode was also found to depend on bubble size. This is exemplified with a 1 mm bubble in figure 1. Theoretically, it is, therefore, possible to infer the size of bubbles based solely on the ballistic light. By introducing a Monte Carlo framework and radiative transport theory, similar relations were derived for mono- and polydisperse bubble plumes with optical thicknesses up to 20. Although this shows the potential to characterize bubbly flows, a practical approach would require an equally short exposure time to determine the duration and location of the constituent modes. Given limitations of electronic speeds due to internal resistance, this would appear impossible with conventional electronics. However, all optical devices and nonlinear optical behavior provide a solution.

Hunter & Bardet (2024) demonstrated the sizing of single bubbles and those within densely crowded plumes, based on their experimentally obtained transient scattering response, using Optical Kerr Effect Gating (OKG). This marked the first use of OKG with bubbly flows, expanding on its use in dense droplet sprays (Sedarsky et al., 2013; Schmidt et al., 2009). This is a third-order nonlinear optical effect that can produce picosecond exposure times when incorporated into an external shutter. Given the much shorter separation between ballistic modes in droplets, previous iterations still relied on visual droplet sizing, although OKG removed damaging scattering modes and proved useful for optical sectioning. Bubbles, with their much larger size requiring substantial

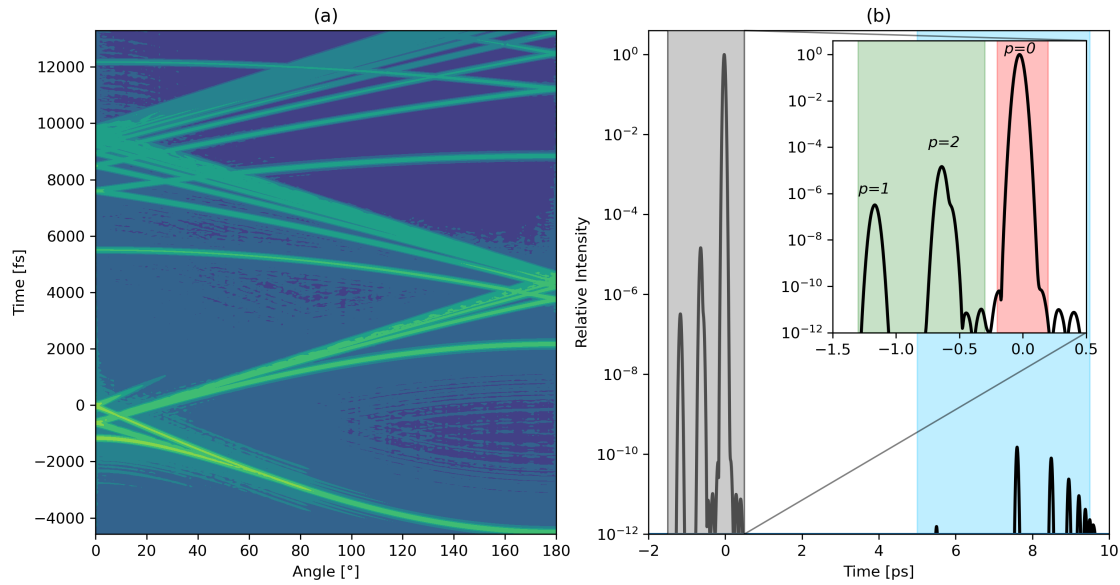


Figure 1. (a) The intensity of light scattered as a function of viewing angle and time, in femtoseconds, for a 1 mm air-bubble submerged in water. (b) The component modes when viewed at 0° (180° from the initial angle of incidence). Key features are highlighted with the ballistic light (green), undisturbed light (red), and the corrugating snake photons (blue). Produced using the method of Chaussonnet & Bardet (2020)

spatial information to faithfully resolve size, provide a stronger use case for scattering-based sizing, as the separation between scattered modes is quantifiable. As ballistic light emerges primarily from the center of bubbles, significantly less spatial information is necessary, allowing sizing even when physical blockages remain.

The investigations of Hunter & Bardet (2024) focused on static bubbles; however, the full benefits and uses of OKG would require demonstration in dynamic scenes. Unlike static bubbles, dynamic bubbles would exist within a field of view for much shorter times, limiting the amount of temporal information obtainable. In addition, the ballistic light emerging from the bubbles needs to be tracked between frames in space to properly account for changes in intensity, as required for sizing.

This work will describe modifications to the framework of Hunter & Bardet (2024) to simultaneously track bubbles whilst recovering their transient scattering modes for sizing. First, the working principle of Optical Kerr Effect Gating will be explained, introducing a new solid-state Kerr cell that reduces exposure times to approximately 1.8 ps. Following this, a new experimental modular bubble channel will be introduced, including two initial test cases to develop processing methods for extracting the position and intermodal position of ballistic signals. After discussing the current results and the state of development, the necessary future steps will be outlined.

2. Optical Kerr Effect Gating

The refractive index of a material is typically considered in terms of its influence on the speed of light. This may be considered as 'Optical Resistance,' describing the impedance to light, or electromagnetic wave propagation. The polarization state of the incident electromagnetic wave is also influenced by this impedance, especially with birefringent media, where the refractive index changes with direction. Typically, birefringence will rotate the polarization plane, after first converting linear polarization to circular polarization. Wave plates and retarders operate on this principle: controlling the strength and duration of birefringence determines the extent of the induced rotation. However, the refractive index is not constant and can be readily changed in response to excitation. Multiple factors may alter the refractive index of a material, for example, thermal gradients.

The Optical Kerr Effect describes a third-order nonlinear optical phenomenon in which the refractive index is instantaneously and temporarily changed under the action of intense optical excitation. These changes may be expressed simply in Eq. 2, where $\chi^{(3)}$, the third order non-linear susceptibility term, and $\langle \tilde{E}^2 \rangle$, the pulse averaged field strength of the incident light squared (Boyd, 2008):

$$n = n_0 + \frac{3\chi^{(3)}}{4n_0} \langle \tilde{E}^2 \rangle \quad (2)$$

Depending on the duration and energy of excitation, multiple physical mechanisms may be triggered to cause such a change in the refractive index, ranging from disturbances in electron orbits, semiconductor transitions, and heat deposition. The duration of induced changes in the refractive index depends on the mechanisms triggered; electron disturbances, for example, typically occur on the femtosecond scale, whereas induced thermal effects may persist for several milliseconds to seconds. Ultrafast laser pulses contain very high temporal energy densities, with peak intensities on the order of 10 GW/cm^2 and beyond. Due to thermal diffusivity limitations, this energy may be transferred to an active medium without depositing significant heat. Therefore, ultrafast excitation provides sufficient energy to induce a measurable change in the refractive index without triggering slower thermal responses. Furthermore, excitation with a polarized pulse, where there is a preference in direction, promotes production of birefringence and, by extension, alterations to the output polarization state.

Induced changes in the polarization state via the Kerr effect can be observed by confining an optically excited medium between two perpendicular polarizers. This creates an external ultrafast shutter or gate, henceforth referred to as an optical Kerr gate or Kerr cell, in which the effective exposure time equals the lifetime of the induced birefringence. The central media within the Kerr cell will also exhibit dominant causal mechanisms that can drastically alter the duration of birefringence. Hunter & Bardet (2024) used Carbon Disulfide (CS_2) as the central active media, in line with

previous experiments in droplet sprays (Linne et al., 2005; Rahm et al., 2016). Molecular reorientation dominates the Kerr response of CS_2 (Shapiro & Broida, 1967). Spheroidal, non-polar molecules physically rotate to align with the exciting polarization direction, creating a long-duration molecular order, giving rise to birefringence. The lifetime of this response is determined by the time taken for isotropy to be restored, which follows an exponentially modified Gaussian profile. The characteristic decay constant of CS_2 's induced birefringence is quoted between 1.8 and 2.1 ps (Mourou & Malley, 1975). Considering it takes at least five decay time scales for the system to go back to its ground state, this results in an effective exposure time on the order of 10 ps. In previous experiments, given the pulse energies employed, the effective exposure time was measured between 5 and 20 ps (Paciaroni et al., 2018; Idlahcen et al., 2009; Purwar et al., 2014). While this is short enough for optical sectioning, exposures of this length are much less effective for determining inter-modal times.

To reduce the Kerr gate response time and improve the fidelity of sizing and the determination of scattering lifetimes, CS_2 has been replaced with a solid, wide-band-gap semiconductor. Being solid with a covalent macro-structure, molecular rotation is restrained, limiting the influence of reorientation. This leaves electronic disturbances being dominant in the Kerr response, producing a response on the same order as the exciting pulse duration. Outside of fluid mechanics, solid-state Kerr cells have previously been used in ultrafast gated fluorescence spectroscopy. Gundlach & Piotrowiak (2008) investigated semiconductor carrier dynamics using a Kerr-gated confocal microscope, with an effective exposure time of 200 fs using a thin solid Kerr media. The new Kerr Cell is depicted in the optical arrangement used in this investigation, shown in figure 2.

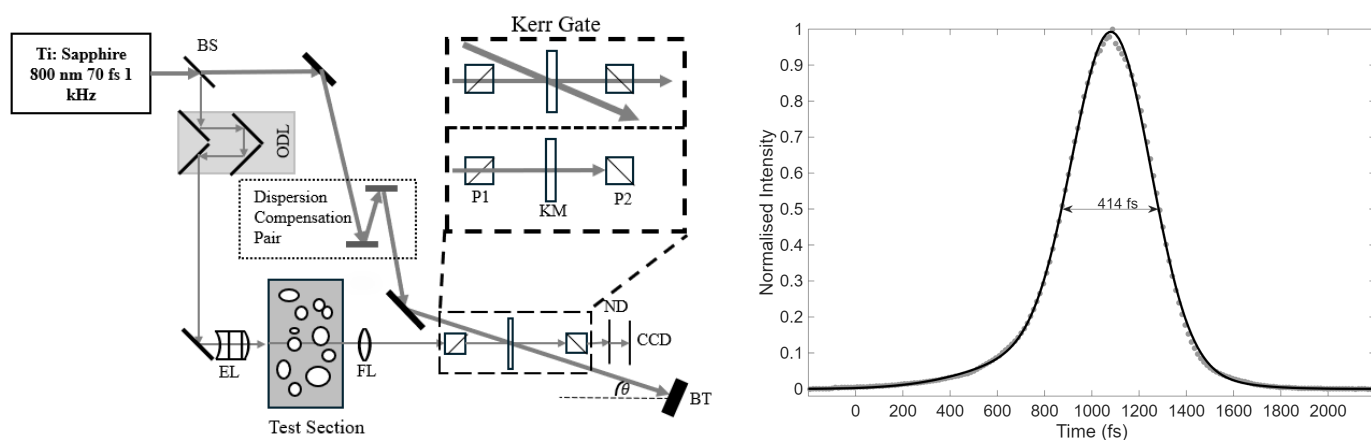


Figure 2. (Left) Simplified diagram of the optical arrangement used for Optical Kerr Gating. The working principle of the Kerr gate is highlighted, showing that transmission occurs when the Kerr media (KM) is optically excited. (Right) The measured effective exposure response of the new solid-state Kerr gate.

To synchronize the generated transient scattering with the triggering of the Kerr cell, a single ultrafast Ti:Sapphire laser (Coherent© Astrella) is used, emitting 800 nm, 70 fs pulses at 1 kHz. These characteristics are consistent with the simulations of Chaussonnet & Bardet (2020). The total pulse

power can be adjusted as needed, with a maximum of 7 mJ per pulse; however, 2.8 mJ is typically used in the current configuration. After the laser head, a 50:50 thin-plate beam splitter produces the two necessary imaging and excitation paths. Along the imaging path, a secondary beam splitter further reduces the power, in turn reducing the likelihood of additional nonlinear phenomena such as self-focusing and nonlinear refraction (Boyd, 2008). An optical delay stage is then used to control the timing between the two pulses, with a resolution of 0.5 μm , corresponding to 33 fs. Altering the path lengths during acquisition offsets the exposure delay, thereby making different scattering modes visible and selectively removing undesired modes. The delayed image pulse is then expanded through a telescope section before entering the test section. Light emerging from the test section is collected in a similar objective telescope section before entering the Kerr cell.

Simultaneously, the more intense trigger pulse is directed towards the Kerr gate via a pair of dispersion-compensation mirrors. These correct the pulse elongation due to non-linear refraction or chirp, thereby maintaining the 70 fs initial pulse length. As this is a monochrome arrangement, a slight angle between the trigger pulse and the image is required to prevent the trigger pulse from being captured by the camera sensor. Therefore, the trigger pulse occurs in the Kerr media at approximately 10° with respect to the image pulse, with vertical polarization. The image pulse is linearly polarized at approximately 55° from the vertical, as it enters the Kerr cell. This is selected to approximately offset the polarization states of the two pulses by the 'Magic Angle,' of 54.7° (Tan et al., 2005), so that no artifacts of molecular reorientation after excitation remain. The second polarizer, aligned at 35° , transmits only the gated section of the pulse, which is retarded by induced birefringence. The section of the image pulse transmitted by the second polarizer is captured by a NIR-enhanced CMOS camera (Teledyne DALSA GenieTM Nano GigE M1930-NIR), after passing a neutral density filter.

Before use with bubbly flows, exposure was first measured with an empty test section. This shows a predominantly Gaussian response (Fig. 2), with a full width at half maximum (FWHM) of 414 fs and a full response time of 1.8 ps. Due to the 10° angle of incidence, there are some spatio-temporal variations within the transmitted image. This is well documented (Idlahcen et al., 2009; Rahm et al., 2015; Hunter & Bardet, 2024) and can be taken into account and corrected in processing. With the Gaussian exposure response, this correction is simplified.

3. Experimental Methodology

In the absence of a pre-existing framework, the primary aim of the current work is to establish a suitable methodology that resolves both dynamics and sizes using the obtained transient scattering modes. Unlike the static bubbles of Hunter & Bardet (2024), where acquisition focused on scattering timescales, in dynamic scenes the characteristic flow timescales must also be accounted for. To this end, a modular vertical bubble channel (Fig. 3a) is used to provide simple and re-

peatable conditions. Using calibrated and highly repeatable injectors (YLEC Diffusion LAMYLEC L1000 and CIMYLEC), in which the production rate and bubble sizes are controllable, the optical conditions can be tightly controlled, allowing optical thicknesses up to 16 without modification. As this is based on a turbulent boundary-layer channel with a 1:10:100 aspect ratio and a depth of 12.5 mm, bulk-flow features may also be introduced, allowing studies under strong turbulence.

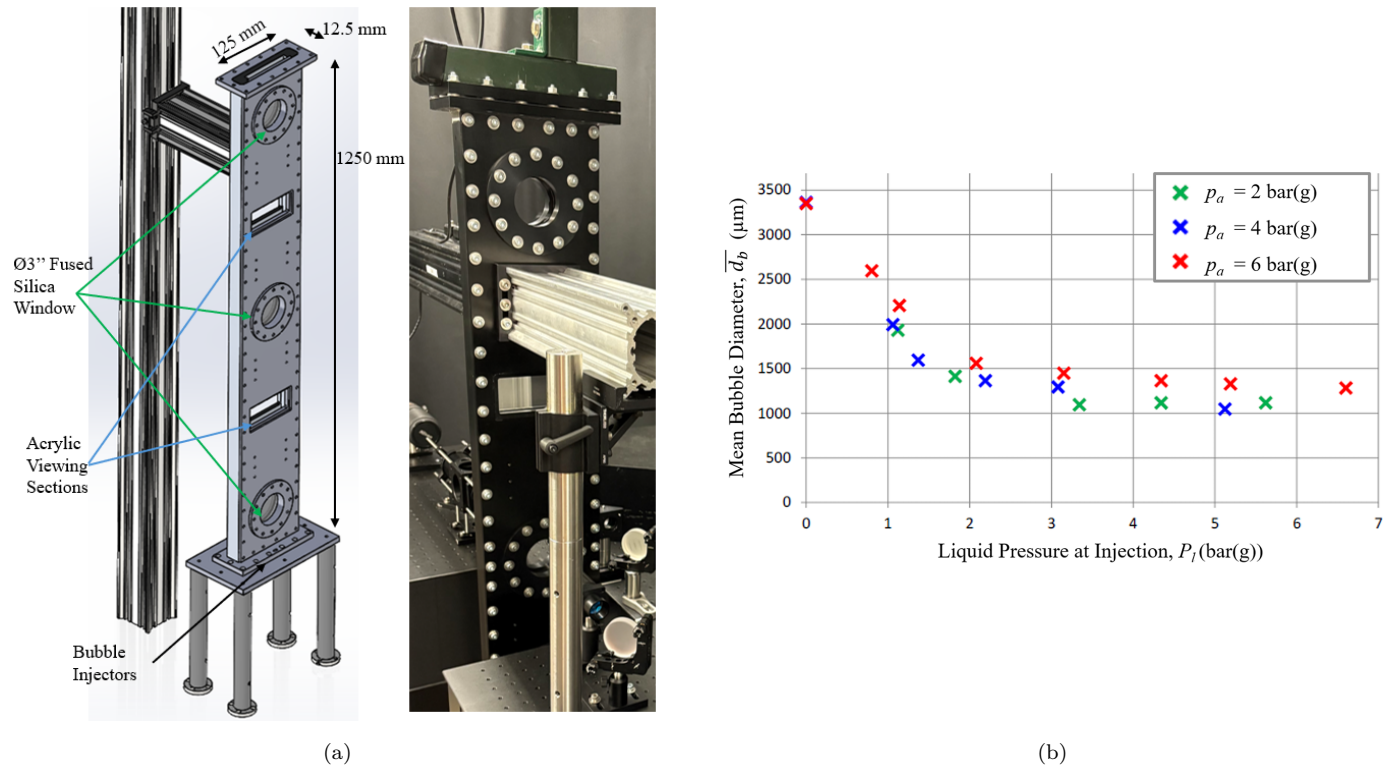


Figure 3. (a) A rendering of the modular bubble channel facility, with relevant dimensions and features alongside the physical facility built with supports and instrumentation. (b) Calibration data of injected bubble size as a function of input liquid and gas pressure for the LAMYLEC L1000 Injector used in the bubble column (provided by YLEC Diffusion).

For these initial investigations, two test cases are used: single bubbles rising in a quiescent flow at a constant production rate, and, secondly, a polydisperse stream of bubbles in a quiescent bulk flow. This first case is simple, and has been investigated extensively by conventional methods (Ellingsen & Risso, 2001; Tomiyama et al., 2002; Legendre et al., 2012; Aoyama et al., 2016; Cioncolini & Magnini, 2025). Replicating similar conditions provides a basis for validating tracking methods. In addition, conventional sizing methods may be used simultaneously to validate sizing. However, before installation, the injectors used have been calibrated and characterized by the manufacturer through pulsed-LED shadowgraphy (Fig. 3b). The second case is more challenging due to the increased number of bubbles to track and the inevitability of occlusion. Again, similar conditions have been investigated (Belden et al., 2012; Masuk et al., 2019; Ma et al., 2023), aiding validation of tracking. This is also more representative of the optically thick conditions that OKG imaging is

being developed to withstand. Polydisperse plumes are generated by a single repeatable sparger. The produced bubble diameters are assumed to follow a Gaussian distribution, where the Sauter mean and maximum diameters increase with the fraction of the void at injection. In all cases, the Bond and Morton numbers remain low, so all bubbles remain spherical, maintaining consistency with the simulated bubbles of Chaussounet & Bardet (2020) and Chaussounet et al. (2022).

Each case also utilizes different acquisition methods. Both, however, utilize the same acquisition rate of 1 kHz and the same underlying tracking code. A modified particle-tracking code, based on that of Crocker & Grier (1996), models the light emerging from the centers of the tracked bubbles as defocused particles. Images are initially smoothed either with a flat-field correction as described by Seibert et al. (1998) or, if the background signal is sufficiently weak, with a spatial Gaussian filter. A multistage thresholding and filtering function first identifies potential bubble locations, then removes the influence of the background. Bypass filtering then highlights, extracts, and emphasizes the light emerging from the optical center of potential bubbles. This contains the ballistic light, and provided the spherical shape, the optical center will be at the bubble's centroid. The bypass filter may result in false detections, especially in regions with partial background leakage due to the finite extinction ratio of the polarizers. Once potential ballistic light locations are identified, secondary thresholding removes likely ghost ballistic signals. Tracks are then constructed from the remaining potential locations. Crucially, this approach does not require views of the complete bubble profile and can utilize significantly less spatial information. Furthermore, there is a much lower dependence on the phase discrimination scheme for sizing bubbles. Sizing is performed by analyzing the ballistic light, making it unnecessary for segmentation to fully resolve the bubble profile.

The approaches differ for the two test cases in the way in which the scattering statistics are accumulated. Hunter & Bardet (2024), continually increased the time delay between the Kerr trigger pulse and image, producing a gating offset between consecutive frames on the femtosecond scale. This was effective as the bubbles imaged were invariant and appeared in all frames within a single stack. The same idea has been applied to the polydisperse case, assuming statistical invariance. The signal location within the gated stack is then compared with that of the undisturbed background signal to determine the temporal offset of the tracked light relative to the peak intensity. Although the trigger delay, as determined by the optical delay stage, is a single value, the effective trigger delay varies across the recorded images due to the angular separation between the trigger and image pulses (Idlahcen et al., 2009). The spatial-temporal variations induced by the Kerr cell are corrected at this stage by taking into account the position of the tracked signal within the field of view. The scattering modes are then reconstructed by combining the temporal offset with the tracked intensity. Given the number of bubbles visible, it is possible to gain sufficient scattering statistics within a single long-duration trigger delay scan. The same cannot be said for the first case with a solitary bubble.

Because ballistic signals are much less likely to become visible and reach peak intensity within a single trigger-delay scan, it is impractical to apply the same approach when only one bubble is visible. Instead, a fixed-trigger approach is employed, in which the trigger delay is held constant, and a series of frames is acquired. The offset of the trigger delay by a fixed and known amount between each series, in turn, offsets the scattered modes visible in the optical center brightness and the location of the undisturbed signal. Tracking the undisturbed background and optical center intensity across constituent series enables reconstruction of scattering statistics, assuming that all tracked bubbles are of equal size. The size determination is then performed using these reconstructed modes.

4. Results and Discussion

4.1. Solitary Rising Bubble with Fixed Gating Delay

The bubble tracks obtained using the modified particle tracking code are shown in figure 4a. These correspond to a third of all bubbles visible within an 8×8 mm field of view, 625 mm from injection across 66,000 frames. These are divided into smaller subsets, where the trigger delay is fixed, with a 330 fs offset between each subset. At injection, air is supplied at 0.4 MPa, with hydrostatic pressure at the water inlet. This produces bubbles in the 3.0-3.5 mm diameter range at an average rate of 4.8 bubbles per second.

As expected in the quiescent conditions with all motions buoyancy-driven, the flow is predominantly vertical, with a mean vertical velocity of 203 mm/s. This appears consistent with the expected terminal velocity given the diameters of the bubbles. Taking into account the distance between the injection site and the field of view, it is assumed that the terminal velocity is achieved without a deformation of the shape. As tracking has used minimal spatial information and does not track changes in shape, a spherical shape is assumed for the duration of the bubble's lifetime. This assumption is called into question because the expected size borders on the ellipsoidal and oscillatory regimes. However, a high degree of spheroidicity is indicated by the appearance of light in the center of the bubble. Fluctuations in the bubble profile would be indicated by rapid changes in the internal visible light or extinction. There is, therefore, a lower likelihood of detecting non-spherical bubbles; hence, this assumption retains merit.

Although no bulk flow was induced, the tracks show the formation of coherent structures. In the tracks shown, this is especially evident around the (4, 3.5) position, where tracks coalesce before rapidly diverging. Horizontal fluctuations in velocity are expected due to the production rate, expected size, and induced local turbulence. Furthermore, all bubbles are created from a single point, increasing the likelihood that bubbles become entrained in the wake of preceding bubbles. As a final remark, the measurements were acquired over an extended period, providing sufficient

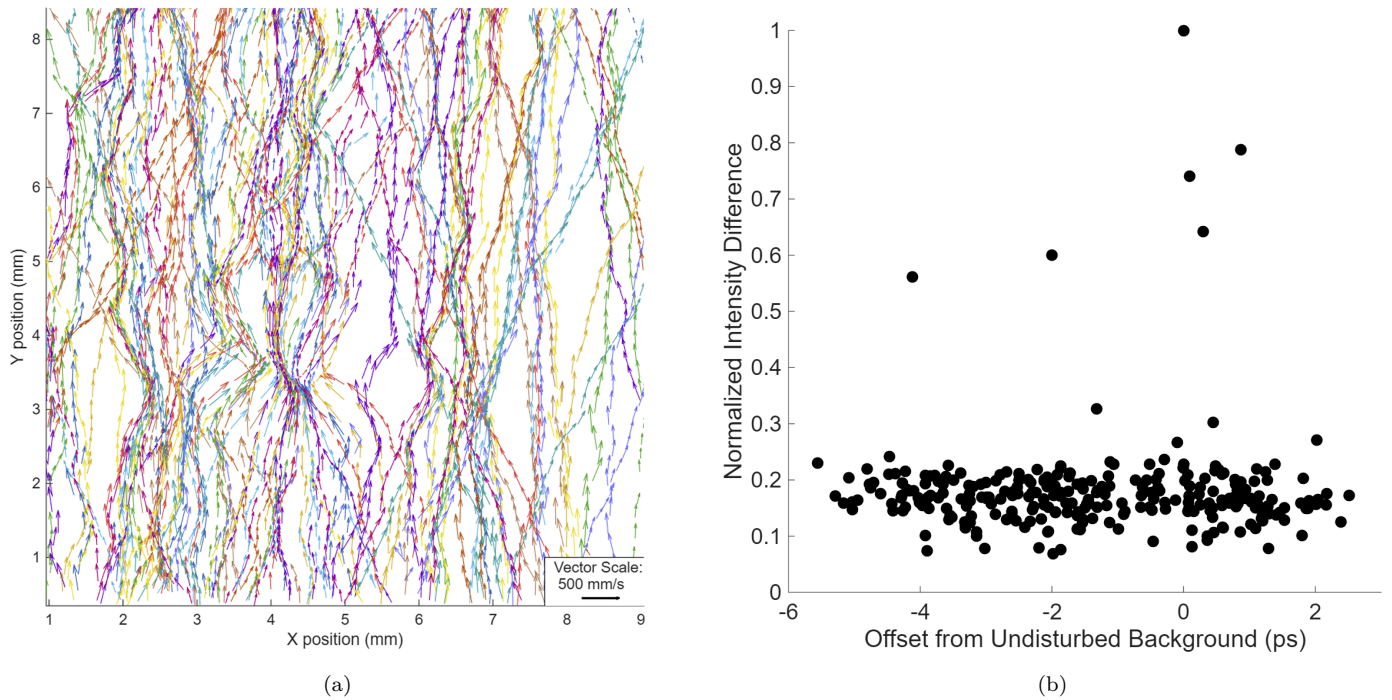


Figure 4. (a) Bubble tracks and velocity vectors for a third of all bubbles tracked across all frames acquired in progressive fixed gating delay analysis. (b) Normalized maximum optical center intensity of each bubble tracked as a function of the relative offset from the undisturbed background.

time for residual bubble-induced turbulence to develop into larger structures.

Through analysis and aggregation of intensity and position data, the transient scattering history has been inferred (Fig. 4b). Here, the temporal offset refers to the time delay between the measured and tracked central signal and the undisturbed background peak intensity. To simplify the analysis, the maximum intensity of each bubble tracked has been initially considered. This initially shows local fine peaks in the region of -4, -2, and 0 ps, with the last peak being the most intense. This would be in agreement with the simulations of Chaussonnet & Bardet (2020). From the reconstructed modes, the 3 most prominent peaks are seen at -4.12 ps, -2.01 ps, and at 0 ps offset. These are assigned as the first, second, and zeroth modes, respectively, and are compared to simulations of Chaussonnet & Bardet (2020) in figure 5.

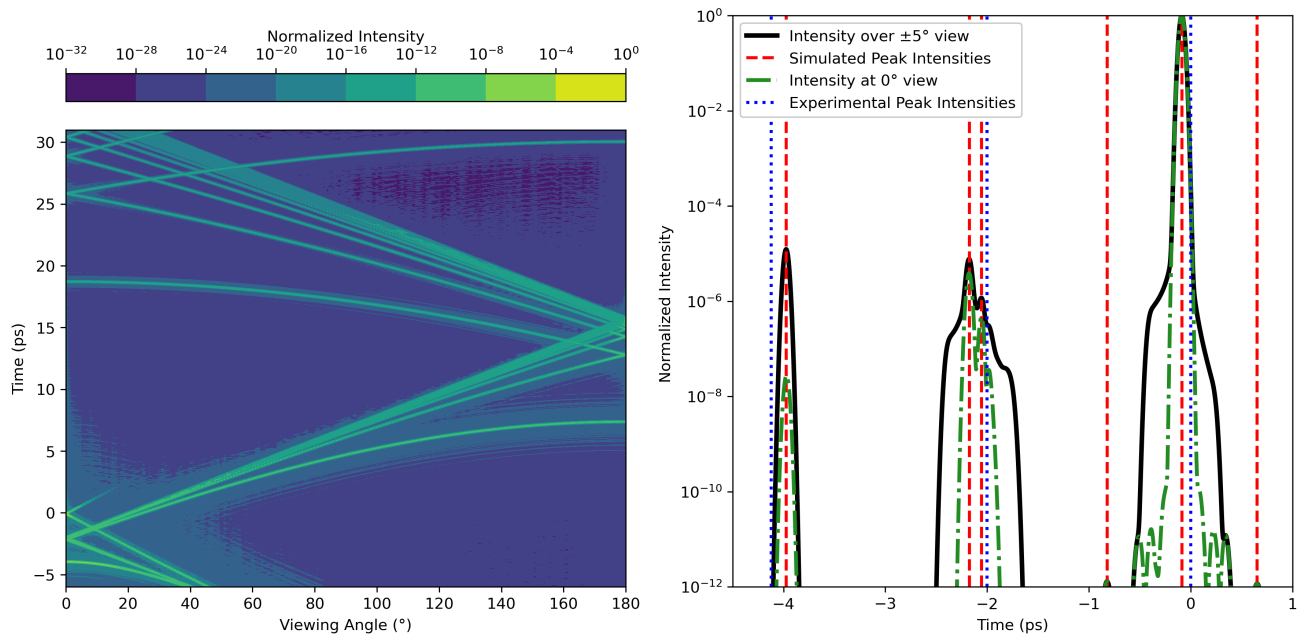


Figure 5. (Left) The intensity light scattered as a function of viewing angle and time, in picoseconds, for a 3.4 mm air-bubble centered within a 12.5 mm section of water. (Right) The ballistic modes emerging from the bubble, when viewed over a 5° range centered at 0° . The experimentally derived peaks of Figure 4b are overlaid. Produced using the simulations of Chaussonnet & Bardet (2020)

Figure 5 shows the simulated transient scattered intensity for a bubble of 3.4 mm in diameter as a function of time and viewing angle, whereas the peaks plotted correspond to the visible first, second and zeroth modes, over a 5° range centered at 0° , to represent the influence of the imaging system's numerical aperture. For ease of comparison, the locations of the experimentally derived modes have been overlaid. Using all three identified sizing modes, there is strong agreement between the zeroth and second modes, with only a slight difference of 0.11 ps for the first mode. Considering the characteristic inter-modal time between the first and zeroth of approximately 0.11 ps per 100 μm bubble diameter, this would indicate a mean bubble diameter slightly greater than 3.4 mm. This aligns with the expectations from calibration. Whilst this gives an indication of the mean bubble size, a true mean cannot be determined at this stage. Progress is now needed in analyzing the obtained scattering data and confidently determine sizes to move beyond the current qualitative comparison.

4.2. Polydisperse Bubble Plume with a Continuous Gating Offset

Using the same field of view and frame rate as with the solitary case, a continuous polydisperse plume was created from a single sparger, with a 20.3% void fraction at injection. Theoretically, the maximum bubble diameter produced in these conditions should be between 2 and 2.5 mm. Similar to Hunter & Bardet (2024), images were obtained whilst the optical delay stage moved

at a constant rate of $150 \mu\text{m/s}$. This offset gating trigger delay by approximately 1 fs per frame. Optical centers were enhanced by shifting the focal plane towards the test section exit window. This blurs the bubble's profile, hindering simultaneous sizing verification with standard visual methods. Given the lensing effect that bubbles create and the space needed to separate far-field and near-field scattering (Davis, 1955; Chaussonnet et al., 2022), this was necessary to ensure that ballistic light is captured and free of the influence of higher-order modes. Tracks from one of the described long-duration trigger delay scans are shown in figure 6a, captured over 12.5 s. Not all bubbles appearing in the field of view were tracked, as tracking was only possible when the optical centers were visible, either because they were unoccluded or the trigger offset was aligned with their constituent modes.

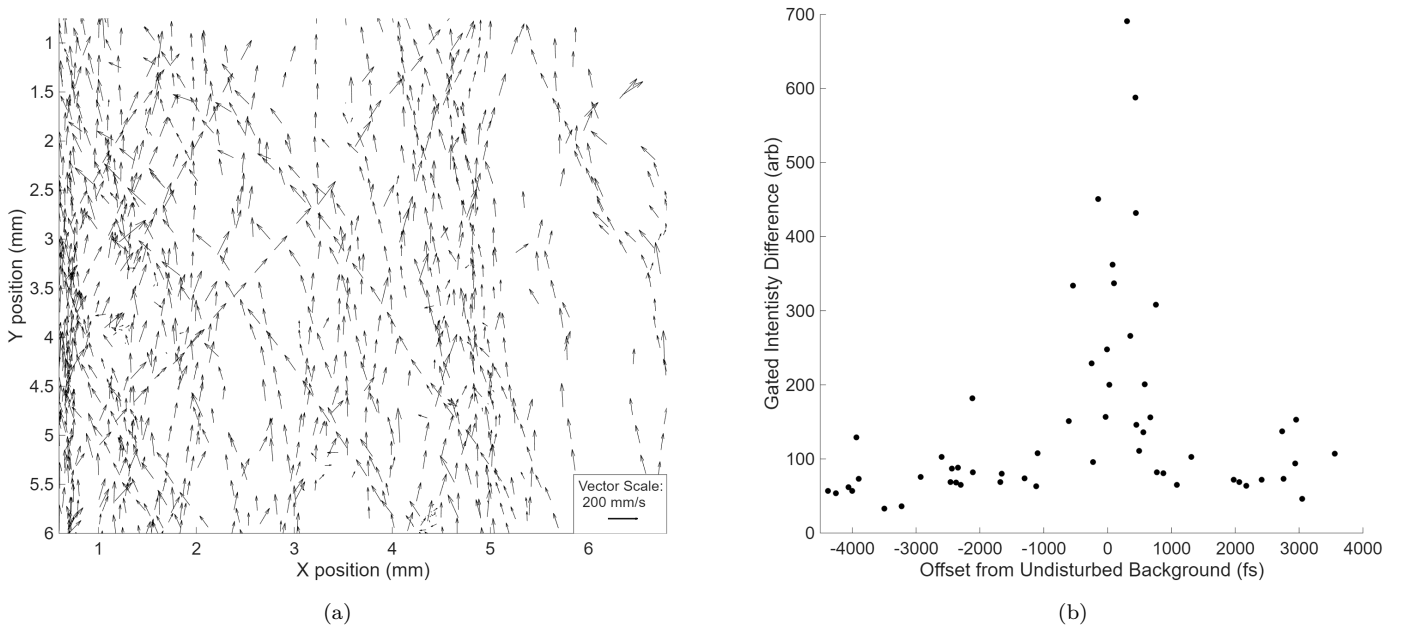


Figure 6. (a) Bubble tracks and velocity vectors for a third of all bubbles tracked within one continuous trigger delay scan. (b) The peak-intensity difference for each bubble tracked within a single trigger-delay scan, as a function of the gating offset from the undisturbed background.

Several of the tracks appear to cluster towards the edges of the field of view, which could be due to multiple factors. As the field of view is aligned with the injection location, bubbles may not have sufficient horizontal orientation before being captured, resulting in a locally greater optical thickness. In addition, a greater amount of background leakage appears towards the edges, which may influence the appearance of the optical centers. However, when reconstructing the scattering modes, the central intensity is normalized by the background intensity to remove any bias introduced by leakage. The rms vertical velocity is 267 mm/s , which again is consistent with the terminal-velocity.

The transient scattering history from the tracked optical centers in the plume is reconstructed in figure 6b. Here, each plotted point represents the peak intensity of each tracked bubble. Two

ballistic peaks appear for a single diameter, and given the range of expected sizes, it is not clear whether the peaks correspond to the first or second modes. With the current processing and acquisition method, these peaks indicate the presence of bubbles of a particular size. However, this does not quantify the number of bubbles of a given size. Although the number of peaks indicates the number of bubbles present, it remains unclear whether this is representative of the entire population. Further investigations with the previously used fixed trigger may improve the statistical significance of these measurements. When looking at peak intensity, some peaks may be assigned to scattering modes; for example, the peak at 2100 fs offset would appear consistent with the first mode of a 1.8 mm-diameter bubble. However, no second-mode peak is observed, as expected near 1000 fs. That said, the appearance of a peak at 4000 fs could indicate the first mode of a 3.4 mm bubble, in which case 2100 fs peak may correspond to the second mode. A 1.87 mm diameter would be consistent with the maximum theoretical size within the plume; however, 3.4 mm would not. It is also possible that the peaks at offsets beyond 3000 fs correspond to photons traveling through multiple bubbles.

Intensity again reaches a peak near the 0 ps offset, but appears significantly wider than in the single-bubble case. However, given the significant range in expected sizes, this may correspond to ballistic signals of sub-millimeter-scale bubbles compounded with influence of the undisturbed background. Taking into account the exposure (Fig. 2), a FWHM of 414, and the full 1.8 ps response, the constituent modes of bubbles on the order of 0.5 mm and below will potentially be enveloped by the zeroth mode, thereby suppressing the true ballistic peaks. Qualitatively, the overall shape of the recovered intensity history appears to agree with the transient simulations of Chaussonnet et al. (2022). Therefore, there is still confidence that, with the development of processing techniques and repeated datasets, meaningful statistics and measurements may be obtained.

5. Conclusions

Following on the work of Chaussonnet & Bardet (2020); Chaussonnet et al. (2022) and Hunter & Bardet (2024), optical Kerr effect gated (OKG) imaging has been expanded to dynamic bubbly flows in both simple bubble and high void fraction conditions. With the aim of developing a new approach to optically thick conditions, all tracking and sizing have been carried out using minimal spatial information. This marks not only the first successful use of OKG gating to dynamic bubbly flows, but also, more significantly, the first time experimentally obtained scattering modes have been used to size dynamic objects.

Looking first at the solitary rising bubble, the derived tracks are consistent with previous studies. These provide evidence of wake entrainment between successive bubbles and the development of coherent structures created by repeated bubble-induced turbulence. Size determination using transient light-intensity statistics, although qualitative at this stage, are also consistent with inde-

pendent calibration data. This proves the viability of ballistic signal analysis as a method to derive bubble size. Further experimental validation with more complete dataset are still needed.

The application of a particle-tracking model to the optical center of bubbles also appeared to be successful at higher optical thicknesses. Even when the bubbles were not fully visible, optical center tracking was still attempted, so that ballistic signals could be used for sizing. Continuous optical trigger-delay scanning with a polydisperse plume was also successful in sampling sections of the scattering modes. However, the interpretation of these scattering modes remains a point of contention. The sizes could not yet be determined confidently, and it remains unclear whether the sampled bubbles were representative of the population within the plume. Therefore, these long-duration trigger scans alone would not be sufficient to produce the required bubble-size distributions. Implementing the same fixed-trigger methodology as in the solitary bubble case may prove useful, especially for establishing the number of bubbles present and, in turn, the density. This will therefore be the focus of future work in the continued development of OKG Imaging.

While there have been improvements in the Kerr cell's gating time, this has still introduced some uncertainty, especially when sizing bubbles smaller than 1 mm in diameter. However, this uncertainty could be reduced by addressing some of the issues raised, including background leakage and the need to defocus images to ensure that ballistic signals were obtained. However, these could be addressed with simple changes, such as optimizing the intensity ratios between image and switch pulses, to avoid overpowering the Kerr cell. Overall, significant progress has been made towards making OKG imaging ideal for optically thick flows.

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Nomenclature

d_i	bubble diameter [mm]
d_{32}	bubble Sauter diameter [mm]
$\langle \tilde{E} \rangle$	Pulse averaged Electric Field strength [W m^{-2}]
L_v	Geometric transmission length [mm]
n	Total refractive index
n_0	Weak-field refractive index
n_d	Bubble number density [m^{-3}]
Q_s	Scattering Efficiency ratio
τ	Optical thickness
$\chi^{(3)}$	Third-order nonlinear susceptibility [$\text{W}^2 \text{m}^{-4}$]

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