

Investigation of the dynamics of coherent structures downstream of delta winglet pairs using Proper Orthogonal Decomposition analysis

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ABSTRACT

The present study experimentally investigates the flow downstream of a delta winglet pair vortex generator mounted in a rectangular channel. Snapshot Proper Orthogonal Decomposition (POD) is applied to Stereoscopic Particle Image Velocimetry (SPIV) measurements in different transverse planes downstream the vortex generator at $x/h = 3$ and $x/h = 6$. Two Reynolds numbers are considered: $Re = 800$ and $Re = 12000$, representative of transitional and turbulent regimes, respectively. The analysis focuses on the modal analysis of coherent flow structures, the energy content of the POD modes, the spatial organization of the leading modes, and the spectral analysis of the associated temporal coefficients. The results show that the modal energy distribution and the leading spatial modes depend on both Reynolds number and the position of measurement planes in the downstream direction. At $Re = 800$, the cumulative energy of the first two modes E_{1+2} decreases from 31.5% at $x/h = 3$ to 18.3% at $x/h = 6$, indicating a reduced dominance of the leading coherent structures farther downstream. At $Re = 12000$, the first two modes contain a substantially lower fraction of the fluctuating kinetic energy, with $E_{1+2} = 9.8\%$ at $x/h = 3$ and 7.3% at $x/h = 6$, suggesting a broader distribution of energy among modes. In addition, The spatial modes support this trend: the transitional near-wake case exhibits the clearest low-order coherent organization, whereas the turbulent case shows less clearly paired structures. For $Re = 800$ at $x/h = 3$, the quasi-circular phase portrait and the common spectral peak of the first two temporal coefficients support the presence of a paired low-frequency oscillatory motion, consistent with coherent vortex dynamics. These results provide an insight of the modal decomposition of the coherent flow dynamics induced by delta winglet pairs.

1. Introduction

Vortex generators (VGs) are widely used as passive flow-control devices for heat-transfer enhancement in heat exchangers and channel flows (Fiebig, 1995, 1998; Jacobi & Shah, 1995). In this context, longitudinal vortex generators (LVGs) are commonly employed to generate streamwise vortical motion and modify the downstream velocity field, with typical geometries for instance, delta

wings, rectangular wings, delta winglets, and rectangular winglets (Zhu et al., 1995; Ahmed et al., 2012; Chai & Tassou, 2018).

For winglet pairs, two principal arrangements are commonly distinguished: common-flow-down (CFD) and common-flow-up (CFU) configurations (Joardar & Jacobi, 2007; Sinha et al., 2013; Ke et al., 2019). In the CFD arrangement, the transverse distance between the leading edges is smaller than that between the trailing edges, whereas the opposite arrangement defines the CFU configuration (Torii et al., 2000; Sinha et al., 2013; Ke et al., 2019). Since the relative performance of CFD and CFU arrangements is configuration-dependent (Fu et al., 2023), the comparison between these arrangement is out of scope. The present study focuses on a CFD delta winglet pair, which has been widely used to generate counter-rotating longitudinal vortices and to investigate the associated downstream wake development (Tiggelbeck et al., 1993; Biswas et al., 1994; Hamidouche et al., 2020).

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The downstream flow induced by delta winglet pairs has been examined experimentally (Eibeck & Eaton, 1987; Hamidouche et al., 2020; Younes et al., 2023, 2024) and numerically (Biswas et al., 1994; Ke et al., 2019; Raj & Datta, 2024; ?), highlighting the development of primary longitudinal vortices, secondary structures, and Reynolds-number-dependent modifications of the wake topology. These studies provide an established basis for analysing the mean flow and vortex topology. However, a complementary modal description is required to assess how the fluctuating kinetic energy is distributed among spatial structures and how this distribution evolves with Reynolds number and downstream position.

Proper Orthogonal Decomposition (POD) provides a standard framework for extracting energy-ranked modes from flow-field data (Berkooz et al., 1993). In the snapshot formulation introduced by Sirovich (1987), POD is particularly suitable for experimental datasets with a large number of spatial degrees of freedom and a finite number of snapshots. In the specific context of longitudinal vortex generators, Saha et al. (2017) applied POD to a turbulent channel flow with built-in vortex generators at $Re = 6000$, showing that the leading modes contain only a limited fraction of the total turbulent kinetic energy, with approximately 25% captured by the first 15 modes. This energetic

interpretation is particularly relevant for delta-winglet-pair wakes, where organized motions may coexist with more distributed turbulent fluctuations depending on the flow regime. Recent SPIV measurements have identified vortex meandering downstream of delta winglet pairs in the transitional regime (Younes et al., 2024). The present analysis therefore examines how the modal energy distribution and dominant spatial modes evolve with Reynolds number and downstream position, and whether low-order organization remains distinguishable at higher Reynolds number.

In the present work, snapshot POD (Sirovich, 1987) is applied to Stereo Particle Image Velocimetry (SPIV) measurements downstream of a delta winglet pair vortex generator. Two Reynolds numbers, representative of transitional and turbulent regimes ($Re = 800$ and $Re = 12000$), are considered at two downstream locations ($x/h = 3$ and $x/h = 6$, where h denotes the channel height). The objective is to analyse the modal energy distribution and the evolution of the dominant spatial modes with Reynolds number and downstream distance, while assessing whether low-order coherent motion remains distinguishable in each flow regime.

The paper is organized as follows. Section 2 describes the experimental setup, the SPIV arrangement, and the measurement conditions. Section 2 describes the experimental facility and the SPIV measurement conditions. Section 3 first presents the mean-flow topology. It then introduces the POD procedure and discusses the modal energy content, spatial mode evolution, and temporal behaviour of the leading coefficients. Section 4 summarizes the main findings and outlines perspectives for further analysis.

2. Experimental setup

The experimental setup, previously described in (Hamidouche et al., 2020; Younes et al., 2024), consists of a closed-loop installation with a rectangular test channel of cross-section $175 \times 50 \text{ mm}^2$ and length 1240 mm, as shown in Fig. 1a. The test section is equipped with a delta winglet pair vortex generator, as illustrated in Fig. 1b. The coordinate system is defined such that x is the streamwise direction, y is the wall-normal direction, and z is the spanwise direction. Stereo Particle Image Velocimetry (SPIV) is used to measure the three velocity components in transverse ($y-z$) planes located at $x/h = 3$ and $x/h = 6$, where $h = 50 \text{ mm}$ denotes the channel height. Measurements are performed for two Reynolds numbers, $Re_{D_h} = 800$ and $Re_{D_h} = 12000$, defined as $Re_{D_h} = U_0 D_h / \nu$, where U_0 is the bulk velocity, ν is the kinematic viscosity, and $D_h = 2bh/(b+h)$ is the hydraulic diameter of the rectangular channel of width b and height h . These two Reynolds numbers correspond to two flow rates controlled by the fan speed, as shown in Fig. 1a.

The flow is seeded using a smoke generator and illuminated by a Nd:YAG Litron Nano L50-100 laser, delivering laser pulses of 60 mJ at 532 nm, with a maximum repetition rate of 100 Hz. The images are recorded using two FlowSense CX 12M-160 CMOS cameras arranged in an angular stereoscopic configuration, each equipped with a 60 mm lens. Each camera has an effective resolu-

tion of 4096×3072 pixels and provides 8-bit image digitization. Due to the inclined viewing angles inherent to the stereoscopic arrangement, Scheimpflug adapters are used to maintain focus over the full measurement plane and to limit image distortion. This stereoscopic arrangement allows for the reconstruction of the three velocity components in the transverse measurement plane.

For each Reynolds number and downstream position, 4096 instantaneous velocity fields are acquired at a sampling frequency of $f_s = 80$ Hz. This sampling frequency can be compared with the inverse of the advective time scale based on the channel height, U_0/h . For $Re = 800$, with $U_0 = 0.15 \text{ m s}^{-1}$ and $h = 0.05 \text{ m}$, $U_0/h = 3 \text{ Hz}$, so that $f_s/(U_0/h) \simeq 26.7$. For $Re = 12000$, with $U_0 = 2.29 \text{ m s}^{-1}$, $U_0/h = 45.8 \text{ Hz}$, giving $f_s/(U_0/h) \simeq 1.75$. Measurement uncertainties are estimated following the statistical approach of (Benedict & Gould, 1996) giving a confidence level of 99%.

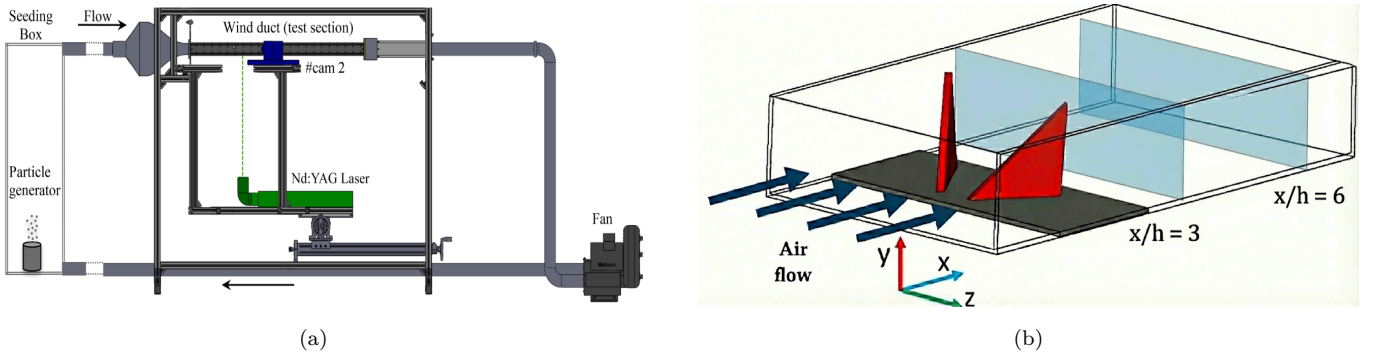


Figure 1. Measurement setup (Hamidouche et al., 2020) (a) and VG with orientation axis and measurement planes (b)

3. Results

3.1. Mean-flow topology

The mean velocity fields in the transverse $y-z$ plane are shown in Fig. 2 for $Re = 800$ and $Re = 12000$ at $x/h = 3$ and $x/h = 6$. The background contours represent the nondimensional mean streamwise velocity $\bar{U}/U_0$, while the vectors indicate the nondimensional mean in-plane velocity components $(\bar{V}, \bar{W})/U_0$. For all cases, the mean fields exhibit the characteristic signature of a counter-rotating longitudinal vortex pair generated by the delta winglet pair, with two cross-plane circulations and a non-uniform streamwise velocity distribution (Hamidouche et al., 2020). Localized low-speed regions are observed near the vortex cores, while higher values of U/U_0 are present farther from the two vortex centers.

At $Re = 800$, the mean topology at $x/h = 3$ shows two relatively compact vortical regions located on either side of the channel centreline. Farther downstream, at $x/h = 6$, the cross-plane motion remains visible, but the streamwise velocity deficit becomes more spatially spread and the

vortex signature appears less compact. At $Re = 12000$, the time-averaged vortex-pair topology is still identifiable at both downstream locations, although the velocity field appears more spatially distributed than in the lower-Reynolds-number case. These observations are limited to the mean flow and therefore do not describe how the fluctuating kinetic energy is distributed among the unsteady spatial structures. This motivates the use of POD to analyse the modal energy content and the evolution of the dominant spatial modes with Reynolds number and downstream distance.

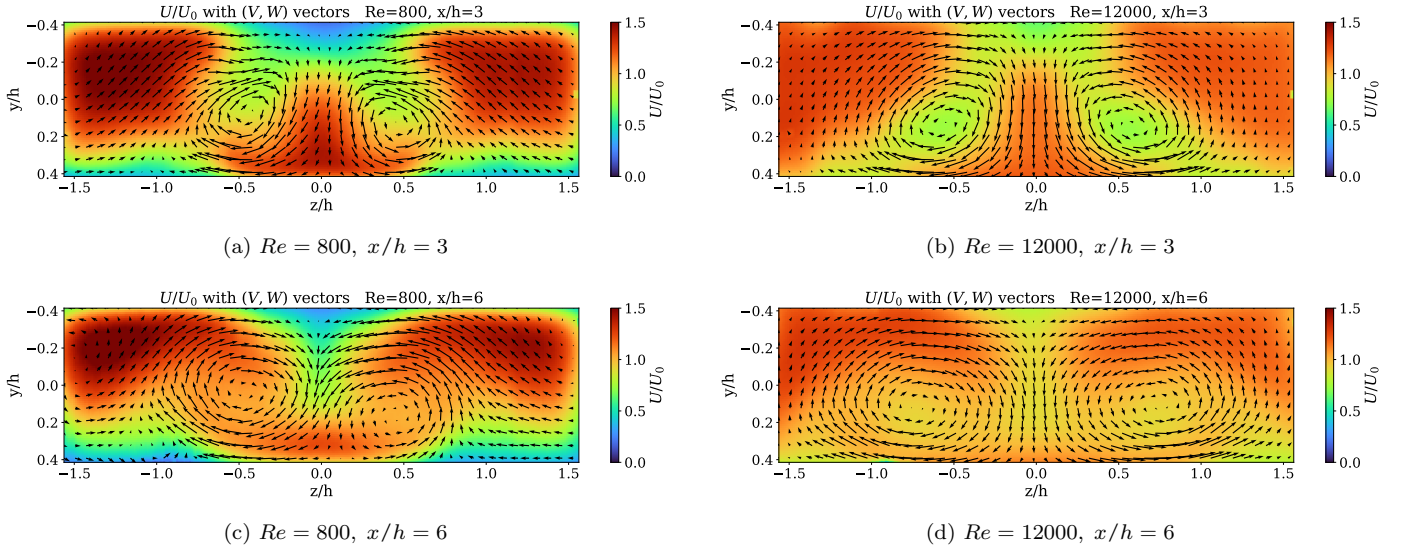


Figure 2. Mean velocity fields in the y - z plane. The background shows the nondimensional streamwise velocity U/U_0 , and the arrows represent the in-plane velocity vectors $(V, W)/U_0$. Results are shown for $(Re = 800)$ and $(Re = 12000)$ at $(x/h = 3)$ and $(x/h = 6)$.

3.2. Snapshot POD procedure

Snapshot Proper Orthogonal Decomposition is applied to the three-component velocity fluctuation fields obtained from the SPIV measurements. For each downstream plane, the instantaneous velocity field is decomposed as

$$\mathbf{u}(y, z, t_n) = \bar{\mathbf{u}}(y, z) + \mathbf{u}'(y, z, t_n), \quad (1)$$

where $\mathbf{u} = (U, V, W)$, $\bar{\mathbf{u}}$ is the time-averaged velocity field, and $\mathbf{u}' = (u', v', w')$ denotes the fluctuating velocity field. The POD analysis is performed on the fluctuation snapshots $\mathbf{u}'(y, z, t_n)$, so that the extracted modes describe the energetic organization of the unsteady part of the flow rather than the mean topology.

Using the snapshot formulation of POD (Sirovich, 1987), the fluctuating field is approximated as

$$\mathbf{u}'(y, z, t_n) \simeq \sum_{k=1}^N a_k(t_n) \phi_k(y, z), \quad (2)$$

where $\phi_k(y, z)$ is the k -th spatial POD mode and $a_k(t_n)$ is its associated temporal coefficient. The modes are ranked according to their modal energy λ_k , and the relative energy contribution of each mode is defined as

$$E_k = \frac{\lambda_k}{\sum_{j=1}^N \lambda_j}. \quad (3)$$

The cumulative energy of the first m modes is given by

$$E_{1:m} = \frac{\sum_{k=1}^m \lambda_k}{\sum_{j=1}^N \lambda_j}. \quad (4)$$

3.3. POD results

The POD analysis is applied to the three-component velocity fluctuation fields (u', v', w') . The convergence of the snapshot POD basis is assessed from the evolution of the modal energy with the number of snapshots. The energy fractions of the two leading modes become nearly insensitive to the number of snapshots beyond 2048, indicating that the 4096 available snapshots provide a sufficiently converged basis for the present analysis. Temporal correlation analysis also shows low correlation levels between snapshots, with values below 0.25, supporting the statistical representativeness of the dataset (Kushwaha et al., 2025; Saha et al., 2017).

The POD energy distribution is summarized in Table 1. Here, E_1 and E_2 denote the energy fractions of the first two modes, E_{1+2} their cumulative contribution, and n_{50} and n_{70} the number of modes required to capture 50% and 70% of the total fluctuating kinetic energy, respectively. At $Re = 800$, E_{1+2} decreases from 31.5% at $x/h = 3$ to 18.3% at $x/h = 6$, while n_{50} increases from 12 to 176 modes. This indicates that the low-order modal contribution becomes less dominant far from the vortex generator. At $x/h = 3$, increasing the Reynolds number from $Re = 800$ to $Re = 12000$ reduces E_{1+2} from 31.5% to 9.8%, while n_{50} increases from 12 to 44. The fluctuating kinetic energy is therefore more broadly distributed among POD modes at higher Reynolds number. At $x/h = 6$, the first two modes contribute less than 20% of the total fluctuating energy for both Reynolds numbers, indicating that the low-order representation is less dominant farther downstream.

The spatial structure of the first six POD modes is shown in Figs. 3–6. At $Re = 800$ and $x/h = 3$ Fig. 3, the leading modes are spatially organized around the region occupied by the mean vortex pair. Modes 1 and 2 show large-scale fluctuation patterns distributed on both sides of the centreline, with in-plane velocity fluctuations that remain closely related to the cross-plane motion observed in the mean field. Their comparable energy levels and spatial organization suggest a low-order modal pair that will be discussed further in this section. Higher modes at the same location retain recognizable spatial structures, but they introduce more localized regions of positive and negative streamwise fluctuation.

Table 1. Summary of POD energy distribution for SPIV measurements at two downstream locations. E_1 and E_2 denote the energy fractions of the first two POD modes, E_{1+2} their cumulative contribution, and n_{50} and n_{70} the number of modes required to capture 50% and 70% of the total fluctuating kinetic energy.

Re	x/h	E_1 [%]	E_2 [%]	E_{1+2} [%]	n_{50}	n_{70}
800	3	17.98	13.55	31.53	12	391
12000	3	5.37	4.47	9.85	44	135
800	6	10.86	7.41	18.27	176	846
12000	6	3.76	3.57	7.33	49	144

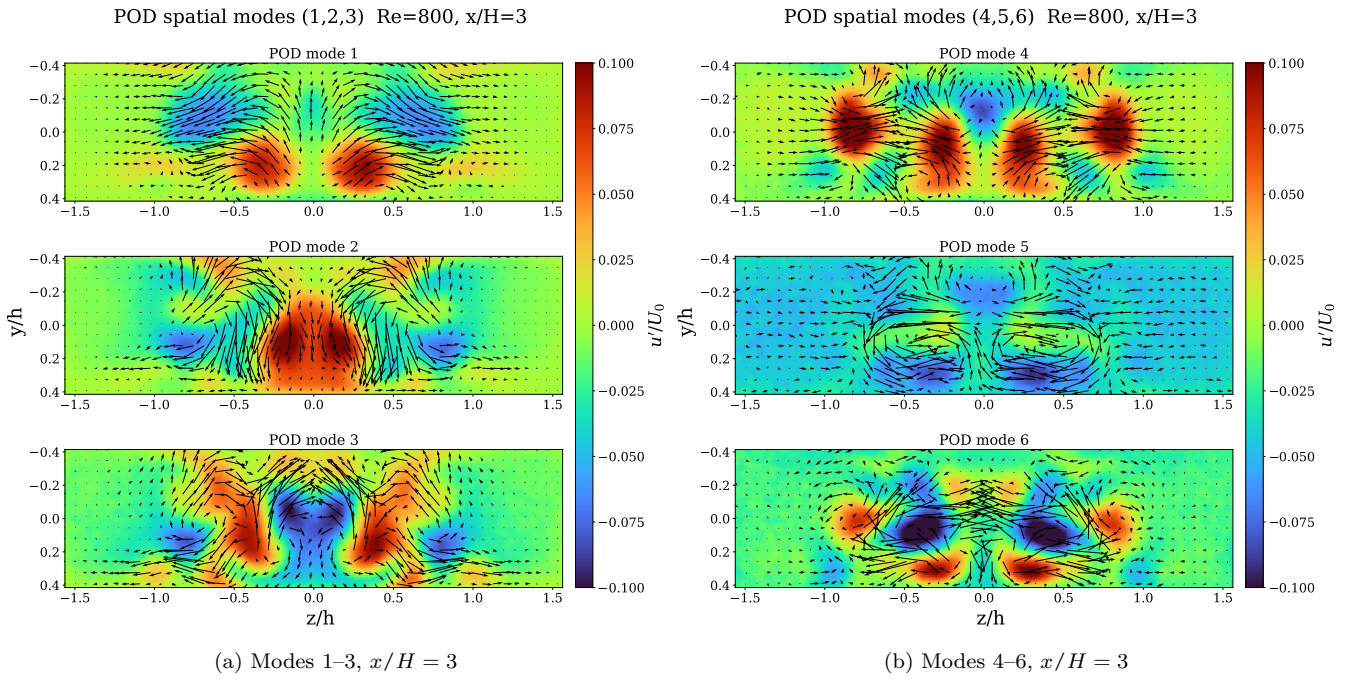


Figure 3. First six POD modes at $Re=800$ and $x/H = 3$. The background shows the nondimensional streamwise velocity fluctuation u'/U_0 , and the arrows represent the in-plane velocity fluctuation vectors $(v', w')/U_0$.

At $Re = 800$ and $x/h = 6$, the leading modes contain less energy than at $x/h = 3$, with E_{1+2} decreasing from 31.5% to 18.3%. Nevertheless, their spatial distribution, shown in Fig. 4, still exhibits a relatively organized and approximately symmetric structure with respect to the channel centerline. Compared with the case at $x/h = 3$, where two more distinct modal cores can be identified, the structures at $x/h = 6$ appear more horizontally spread, particularly near the lower channel wall, where the two sides of the modal pattern become less clearly separated. This suggests that the downstream evolution affects not only the amount of energy captured by the leading modes, as quantified in Table 1, but also the spatial arrangement of the dominant structures.

At $Re = 12000$ and $x/h = 3$ in Fig. 5, the leading POD modes contain a smaller fraction of the fluctuating kinetic energy than in the $Re = 800$ case. Their spatial patterns are not symmetric and

do not show the same clear modal pairing. Although localized energetic structures are still visible in the streamwise and in-plane fluctuation fields, the first two modes do not represent a vortex pair. This is consistent with the reduction of E_{1+2} to 9.8%, indicating a broader distribution of energy among POD modes.

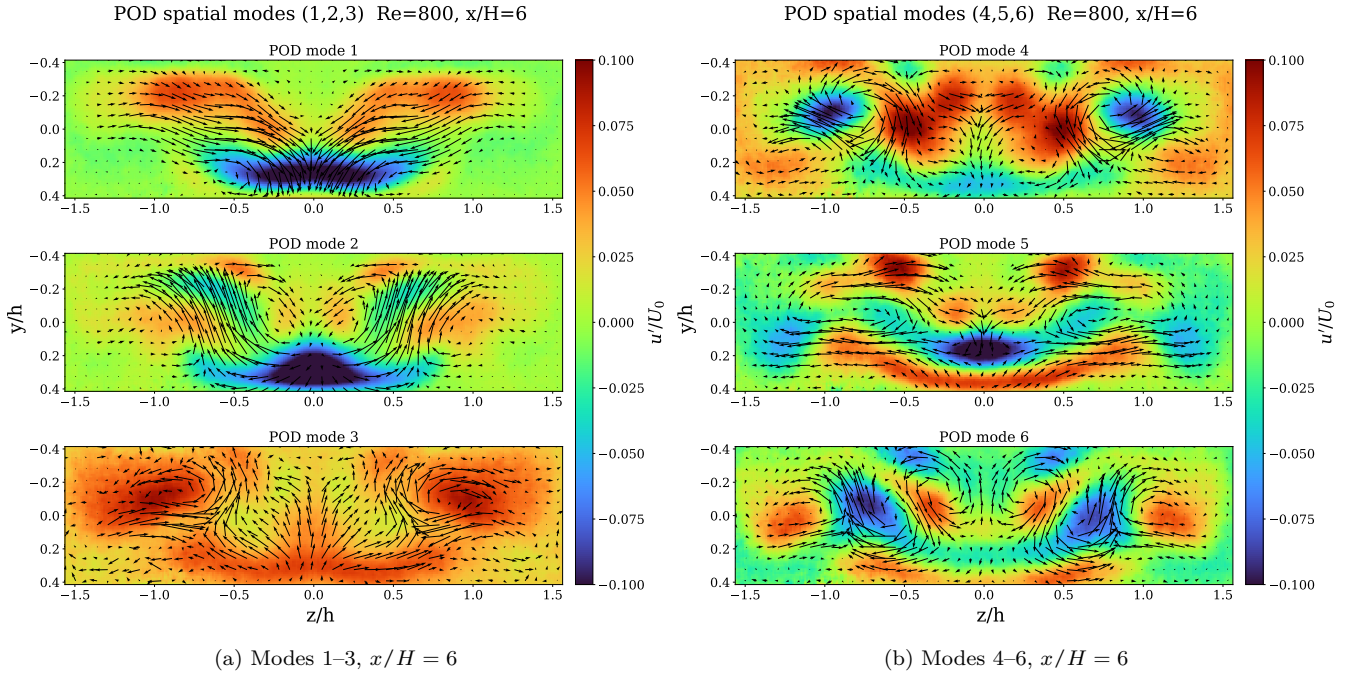


Figure 4. First six POD modes at $Re=800$ and $x/H = 6$. The background shows the nondimensional streamwise velocity fluctuation u'/U_0 , and the arrows represent the in-plane velocity fluctuation vectors $(v', w')/U_0$.

At $Re = 12000$ and $x/h = 6$, in Fig. 6, the leading modes display a more diffuse spatial organization than in the lower-Reynolds-number cases. The positive and negative regions of u'/U_0 are distributed over the transverse plane, and the associated vector fields vary substantially from one mode to another. Modes 1 to 5 do not form a clearly identifiable low-order pair, although mode 6 exhibits a more symmetric spatial pattern with two vortical structures. This suggests that some organized spatial features remain present, but they are not concentrated in the first two most energetic modes. This is consistent with the low value of $E_{1+2} = 7.3\%$, indicating that the fluctuating kinetic energy is distributed over a broader set of modal structures rather than being dominated by a single low-order pair. Overall, the spatial modes are consistent with the trends obtained from the modal energy distribution. The clearest low-order organization is observed at $Re = 800$ and $x/h = 3$, where the first two modes contain a relatively large fraction of the fluctuating kinetic energy and retain a spatial structure connected to the mean vortex-pair region.

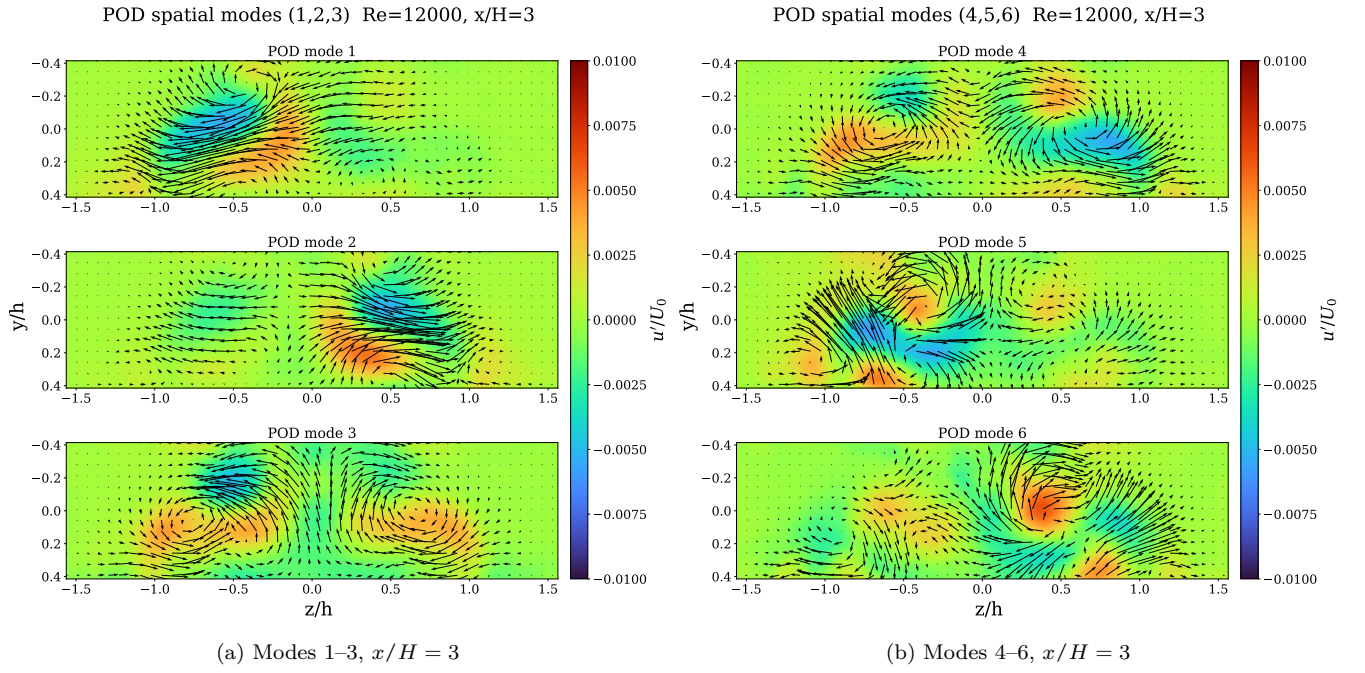


Figure 5. First six POD modes at $Re=12000$ and $x/H = 3$. The background shows the nondimensional streamwise velocity fluctuation u'/U_0 , and the arrows represent the in-plane velocity fluctuation vectors $(v', w')/U_0$.

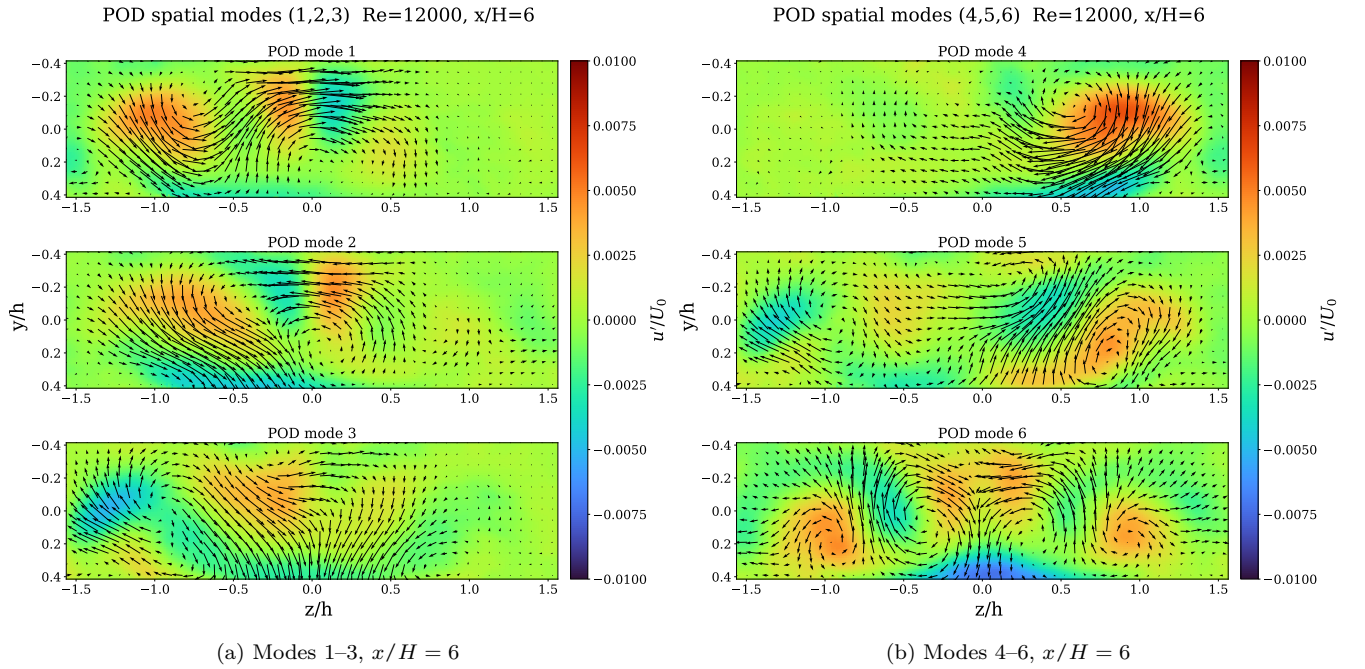


Figure 6. First six POD modes at $Re=12000$ and $x/H = 6$. The background shows the nondimensional streamwise velocity fluctuation u'/U_0 , and the arrows represent the in-plane velocity fluctuation vectors $(v', w')/U_0$.

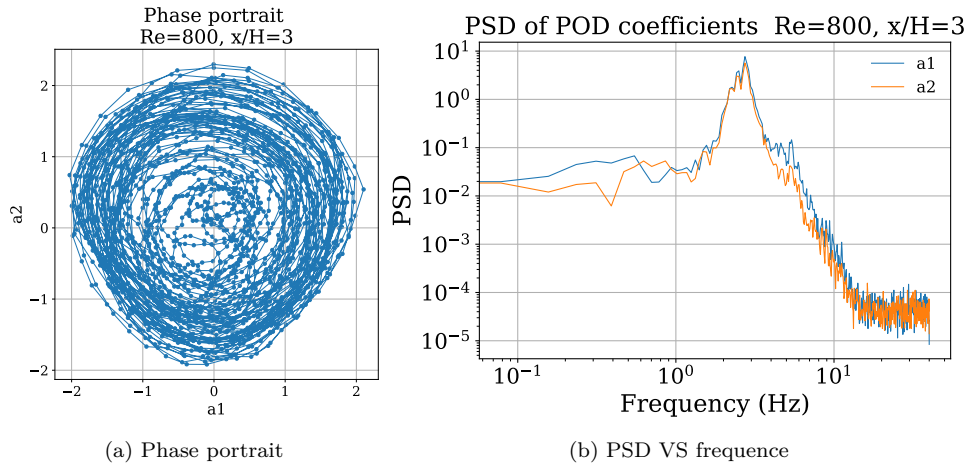


Figure 7. First two modes Pairing , (a) Power Spectral Density PSD with frequency, (b) phase portrait

The first two temporal coefficients, a_1 and a_2 described in 2, form a quasi-circular phase portrait, at $Re = 800$ and $x/h = 3$, as shown in Fig. 7b. This phase organization is commonly associated with a paired POD representation of an oscillatory coherent motion, for which two spatially orthogonal modes with phase-shifted temporal coefficients are required to describe the displacement of the structure (Sieber et al., 2016; Vanierschot et al., 2020; Riches et al., 2018). In addition, the corresponding spectral analysis exhibits the same dominant frequency peak at $f = 2.7$ Hz for both coefficients, as shown in Fig. 7a. This frequency corresponds to a Strouhal number $St = fh/U_0 = 0.9$, further supporting the interpretation of a paired low-order oscillatory motion at $Re = 800$ and $x/h = 3$.

This interpretation remains limited by the use of classical space-only POD, whose modes are ranked by energy but are not constrained to isolate a single frequency (Towne et al., 2018). The common spectral peak observed in both a_1 and a_2 therefore supports the existence of a dominant time scale associated with the paired modes, without implying a complete separation of the oscillatory dynamics from the broadband turbulent fluctuations.

Farther downstream, at $x/h = 6$, the reduced contribution of the leading modes suggests a weakening of the low-order modal organization. This is consistent with the energy redistribution observed in Table 1, where E_{1+2} decreases from 31.5% at $x/h = 3$ to 18.3% at $x/h = 6$. The downstream evolution therefore indicates that the fluctuating kinetic energy is less concentrated in the first two modes and becomes distributed over a larger number of modal structures.

For $Re = 12000$, the less clearly paired spatial structures, together with the lower energy content of the leading modes, indicate a broader distribution of energy among POD modes rather than a single dominant low-order oscillatory organization. A possible concern could be that the acquisition frequency is not sufficient to resolve the faster turbulent dynamics at high Reynolds number. However, high-frequency LDA measurements reported by Younes et al. (2023) showed

that no dominant spectral frequency is observed for Reynolds numbers higher than $Re = 3000$. In contrast, the presence of a marked frequency peak appears characteristic of transitional Reynolds numbers, where organized large-scale motion can coexist with smaller-scale turbulent fluctuations.

The present results therefore highlight two main aspects: the downstream evolution of the vortex-induced flow away from the vortex generator, and the influence of Reynolds number on the modal energy distribution. The most distinct low-order dynamics are observed in the transitional case, $Re = 800$, in the near wake at $x/h = 3$. This observation is consistent with the vortex meandering previously reported by Younes et al. (2024) for the same vortex-generator configuration and channel flow.

Although these results provide useful evidence of Reynolds-number-dependent modal organization, they also suggest several perspectives for further investigation. First, additional Reynolds numbers within the transitional range would help clarify how the flow evolves from organized low-order motion toward a more distributed turbulent state. Second, additional measurement planes would allow the spatial development of the coherent motion to be followed more precisely, from its formation closer to the vortex generator, for example between $x/h = 1$ and $x/h = 3$, to its downstream evolution and possible breakdown farther downstream, for example beyond $x/h = 6$. Finally, for cases where a clear dominant frequency is identified, a triple decomposition framework (Hussain & Reynolds, 1970) could provide further insight by separating the phase-locked coherent contribution from the residual fluctuations, thereby revealing dynamics that are not accessible from classical time-averaged quantities alone.

4. Conclusion

The present study investigates the flow downstream of a delta winglet pair vortex generator using snapshot Proper Orthogonal Decomposition applied to SPIV measurements. Two downstream positions, $x/h = 3$ and $x/h = 6$, and two Reynolds numbers, $Re = 800$ and $Re = 12000$, were considered. The analysis focused on the modal energy distribution and on the spatial organization of the dominant POD modes.

The results show that the low-order modal contribution depends on both Reynolds number and downstream position. At $Re = 800$, the cumulative energy of the first two modes decreases from $E_{1+2} = 31.5\%$ at $x/h = 3$ to 18.3% at $x/h = 6$. At $Re = 12000$, the first two modes contain a lower energy fraction, with $E_{1+2} = 9.8\%$ at $x/h = 3$ and 7.3% at $x/h = 6$. This indicates that the leading modes are most dominant in the transitional near-wake case and become less dominant farther downstream and at higher Reynolds number.

The spatial modes confirm this trend. The case $Re = 800$, $x/h = 3$ shows the clearest low-order organization, with modes 1 and 2 connected to the mean vortex-pair region. Farther downstream,

the structures remain organized but become more spatially spread. At $Re = 12000$, the leading modes are less clearly paired, suggesting a broader distribution of fluctuating kinetic energy among modal structures.

For $Re = 800$ and $x/h = 3$, the quasi-circular phase portrait of a_1 and a_2 , together with a common spectral peak at $f = 2.7$ Hz corresponding to $St = 0.9$, supports the interpretation of a paired low-order oscillatory motion. This behaviour is consistent with previous observations of vortex meandering downstream of the same vortex-generator configuration.

These results provide an experimental modal analysis of the delta-winglet-pair-induced flow and complement previous mean-flow and numerical studies. Future work should include additional Reynolds numbers and downstream planes to clarify the formation, evolution, and possible breakdown of the coherent motion. For cases with a clear dominant frequency, triple decomposition or phase averaging could provide further insight into the phase-locked dynamics and their contribution to turbulent kinetic energy production.

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Nomenclature

x	Streamwise coordinate [m]
y	Wall-normal coordinate [m]
z	Spanwise coordinate [m]
b	Channel width [m]
h	Channel height [m]
D_h	Hydraulic diameter [m]
U_0	Bulk velocity [m s^{-1}]
ν	Kinematic viscosity [$\text{m}^2 \text{s}^{-1}$]
Re_{D_h}	Reynolds number based on the hydraulic diameter [-]
f	Frequency [Hz]
f_s	Sampling frequency [Hz]
St	Strouhal number, $St = fh/U_0$ [-]
a_k	Temporal coefficient of the k -th POD mode
E_k	Relative energy contribution of the k -th POD mode [-]
E_{1+2}	Cumulative energy contribution of the first two POD modes [-]

n_{50}	Number of POD modes required to capture 50% of the fluctuating kinetic energy
n_{70}	Number of POD modes required to capture 70% of the fluctuating kinetic energy
t_n	Time instant of the n -th snapshot [s]
λ_k	Eigenvalue associated with the k -th POD mode
ϕ_k	Spatial structure of the k -th POD mode
$\bar{\mathbf{u}}$	Time-averaged velocity field [m s^{-1}]
$\mathbf{u}'$	Fluctuating velocity field [m s^{-1}]

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