

Infodynamic Patterns of Effervescent Sprays

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ABSTRACT

Effervescent atomizers are twin-fluid atomizers that internally mix liquids and gases, and represent a promising strategy for atomizing viscous liquids in food processing and other industrial applications. The atomization process and subsequent spray development are governed by internal two-phase flow, primary atomization, gas–liquid momentum exchange, and interactions with the surrounding environment. Although conventional spray characterization based on droplet size and velocity provides essential information on liquid volume, interfacial area, and momentum-related quantities, mean quantities alone do not fully express the diversity and indeterminate characteristics of atomization processes. This study introduces an infodynamic framework for effervescent spray characterization based on informature, differential informature, the Activation Factor, and the Representation Index $R_{(j)}$. Informature quantifies the amount of information associated with number-, area-, and volume-weighted droplet size distributions, whereas differential informature provides an intensive description of the spray informational structure that is independent of the measurement alphabet (i.e., statistical classes). The Representation Index quantifies the invariance of the Activation Factor when regular and variable size classes are used, revealing whether the number, interfacial area, or liquid volume is carried by coherent or multiple size populations. Application of Phase Doppler Interferometry measurements of an effervescent atomizer at different gas-to-liquid ratios revealed an “info-hollow-cone” structure, with distinct spatial regions where the droplet number, interfacial area, and volume/mass act as different information carriers. This infodynamic characterization layer provides a physically grounded description of spray indeterminacy and offers a complementary basis for interpreting hydrodynamic mechanisms and validating numerical models.

1. Introduction

Effervescent atomizers, namely twin-fluid atomizers with internal mixing, can be a viable option in some industrial processes requiring the atomization of viscous fluids, well-atomized sprays with a large interfacial area, or fine droplets with controllable momentum. These requirements are encountered in applications in the food processing industry, combustion systems, spray scrubbing,

spray coating of various surfaces, and many others. Compared to single-fluid atomizers, effervescent atomizers can vary the amount of atomizing gas, particularly the gas-to-liquid ratio (GLR), to govern the atomization process. Effervescent atomizers were first introduced by Lefebvre (1988) and have since been continuously developed by several research groups (Roesler and Lefebvre, 1989; Sovani et al., 2001; Jedelsky et al., 2009; Jedelský et al., 2014). Today, this atomization strategy is a well-established method for spray systems. However, the indeterministic nature of drop formation has not yet been considered in their design.

The atomization process in effervescent sprays is governed by an internal two-phase flow. The structure of internal two-phase flows is sensitive to the inlet pressure (p_{in}), GLR, and atomizer geometry (Hammad et al., 2021). It may appear as a flow of bubbles (low GLR), slugs (intermediate GLR), or an annular flow regime (high GLR) (Sun et al., 2019). However, in all cases, the cause of liquid atomization was the rapid expansion of the gaseous phase, which tears the liquid into ligaments and droplets, as described by Zaremba et al. (2018). However, when slug flow is present, the discharge alternates chaotically between gas-dominant and liquid-dominant jets (Malý et al., 2025). With a further increase in the GLR, an annular internal flow is established, which discharges a thin annular liquid film with a high-speed gas core (Zaremba et al., 2018). Therefore, different atomization processes lead to diverse droplet sizes, velocities, and corresponding distributions, which are usually measured further downstream of the atomizer. An open question remains: what hydrodynamic mechanisms shape the final Drop Size Distribution (DSDn)?

Rapid momentum exchange between the liquid, atomization gas, and surrounding environment occurs after discharge. In the vicinity of the atomizer, a large relative velocity between the gas and liquid creates strong shear-disruptive forces. As the atomizing gas decelerates downstream, the strong initial gas-liquid shear transfers momentum to the dispersed phase, accelerating the droplets, particularly those with low Stokes numbers, and entraining them toward the spray core (Cejpek et al., 2026). As the liquid and atomization gas velocities become equal at some point downstream from the atomizer, the interaction with the surrounding environment dominates, causing agglomeration or clustering of droplets at the spray boundary induced by outer vortices (Heinlein and Fritsching, 2006). Further downstream from the atomizer, the droplets reach their terminal falling velocity (Beard, 1976). Understanding liquid velocity development is important because it plays a crucial role in determining the spray distance from the coated surface to achieve the desired deposition regime and the contact time during spray scrubbing to maximize scrubbing performance. Primary liquid atomization and liquid-gas interactions play crucial roles in droplet formation and spatial distribution. The atomization efficiency is generally low because only a few percent of the input energy is transferred to the surface energy of the droplets, as shown by Jedelsky and Jicha (2013), whereas most of the input energy is transferred to the droplet velocity. Therefore, a better understanding of atomization physics and injector design requires an accurate characterization of spray dynamics, which can be achieved using laser and imaging techniques.

Phase Doppler Interferometry (PDI) or high-resolution high-speed visualization is often used to characterize droplets and their spatial distributions (Zaremba et al., 2018; Milkvik et al., 2020; Cejpek et al., 2023a). However, the choice of measurement technique can limit the accuracy of droplet size data, as each technique is feasible for measuring droplets created by different mechanisms. PDI has the advantage of characterizing small droplets inside dense sprays, whereas high-resolution high-speed visualization better captures large, nonspherical liquid structures (Sijs et al., 2021; Cejpek et al., 2023b). Moreover, the selection of a representative measurement position is crucial. There are cases where a single-point measurement, preferably in the spray center, is sufficient (Hammad et al., 2021). Data from this single point refer to local spray characterization, but conclusions about the overall spray were derived. The application of line-of-sight methods integrates droplets over the entire measurement volume along the beam path, effectively averaging out spatial variations in droplet size but losing information about the droplet-size distribution across the spray (García et al., 2017; Wittner et al., 2019). In most industrial processes, droplet size (represented in terms of surface energy) is the main parameter, despite being a marginal part of the system energy, and is the parameter most commonly used to describe a spray.

If we focus on liquid atomization, although measurement techniques can capture the spray of multiple sizes, a single parameter often replaces the measured droplet size distribution (DSDn). The most common analysis is based on the parametric effect on the mean diameter values. Each mean diameter has a significance in terms of spray characterization. For example, the arithmetic mean diameter (D_{10}) is equivalent to the size that all counted droplets would have if they were equal. The Sauter Mean Diameter (D_{32}), often used in combustion applications, is the mean diameter of an area-weighted DSDn and is considered to represent the spray area. The volume-weighted mean diameter (D_{43}) represents the DSDn volume and mass, which is relevant in spray cooling applications. If j represents the weight with which we build a DSDn ($j = 0 \equiv \text{number-}$, $j = 2 \equiv \text{area-}$, $j = 3 \equiv \text{volume-weighted}$), the relative frequencies would be

$$\omega_{j,i} = \frac{n_i d_i^j}{\sum_{i=1}^k n_i d_i^j} \quad (1)$$

where n_i is the number of droplets inside class i centered at d_i , k is the number of classes dividing the size range spectrum, and $\sum_i \omega_{j,i} = 1$. A moment of the distribution is given by

$$m_{j,l} = \sum_{i=1}^k \omega_{j,i} d_i^l \quad (2)$$

and the mean diameter of a j -weighted DSDn is given by

$$D_{\alpha j} = \frac{m_{j,\alpha-j}}{m_{j,0}} \quad (3)$$

For example, the Sauter Mean Diameter corresponds to

$$D_{32} = \frac{m_{2,1}}{m_{2,0}} = \frac{\sum_{i=1}^k \omega_{2,i} d_i^1}{\underbrace{\sum_{i=1}^k \omega_{2,i} d_i^0}_{=1}} = \sum_{i=1}^k \omega_{2,i} d_i \quad (4)$$

which is why D_{32} is the mean of an area-weighted DSDn. Mean diameters are useful for performing parametric analyses of the effects of changing the operating conditions in liquid atomization for injector design. However, these quantities are limited and do not convey any information regarding the diversity of drop sizes.

To obtain a glimpse of the Drop Size Diversity (DSDy), the second moment or standard deviation ($m_{j,2}$) is a possibility, but is blind to multimodal DSDn shapes. The most common parameter is likely to be the Relative Span Factor ($RSF = (D_{j,90} - D_{j,10}) / D_{j,50}$) based on a cumulative droplet size distribution ($F_{j,i} = \sum_{s=1}^i \omega_{j,s}$) and three representative diameters ($D_{j,10}$, $D_{j,50}$, $D_{j,90}$, the size representing 10%, 50%, and 90% of the spray droplets organized in the j -weighted DSDn), although Panão et al. (2020) showed the limitations of the RSF . The q shape factor as the fractal dimension assuming the DSDn fitting a Weibull Distribution is also a common measure, as is the Geometric Standard Deviation (GSD) in a Log-normal fitting, but both assume that specific mathematical probability density functions (PDFs) describe reality without adequate verification. García et al. (2017) proposed the volume-weighted Standard Deviation SD_v to assess DSDy, without assuming a certain PDF, but like the previous proposals, it disregard the actual shape and scale of the measured DSDn [see also Roesler and Lefebvre (1989); Hammad et al. (2021); Schröder et al. (2011); Rosell-Llompart and Gañán-Calvo (2008)]. Similar to the mean quantities, these approaches do not fully capture the actual amount of information contained in the droplet size distributions of different shapes, including multimodal distributions. This study argues that capturing the diversity hidden by the mean quantities and moments of the droplet size distribution implies adding a new layer to spray characterization based on the amount of information.

2. A New Spray Language and Characteristic

Spray dynamics are typically characterized by measuring drop sizes, velocities, and fluxes. The analysis typically focuses on mean quantities, which are useful for providing data that can validate numerical models. However, although the mean quantities are sensitive to the non-deterministic nature of the spray dynamic characteristics, they do not fully express them accurately. In this study, we recognized the importance of adding an essential layer to spray characterization that fully captures its non-deterministic nature. Panão (2025a); Ferrão et al. (2025) have previously

demonstrated that infodynamics provides quantified measures of a spray's non-deterministic nature, using parameters such as *informature* that quantify the amount of information. Therefore, in this section, we show that infodynamic parameters add a new independent layer of spray characteristics that allows us to introduce spray indeterminacy in the validation of numerical models to improve them and provide deeper insights from the perspective of spray diversity. The first step involved revising the spray characterization language to highlight its physical character.

A spray made of N discrete particles, each essentially described by size and velocity, $x_i \equiv (d_i, \mathbf{u}_i)$, is analogous to a gas in that it is an ensemble of many entities where we rarely care about the identity of each entity, only statistical regularities. Therefore, the spray *microstate* is the complete list of droplet-level states in our sample, $\Omega \equiv \{(d_1, \mathbf{u}_1), (d_2, \mathbf{u}_2), \dots, (d_N, \mathbf{u}_N)\}$. To characterize the spray *macrostate*, we first need to develop a *measurement alphabet* as a set of classes or bins used to organize the spray data into histograms.

There are two essential methods for building the measurement alphabet's "letters". The first method builds the alphabet from regular/uniform classes with equal size, and the second method uses variable/non-uniform size classes. Methods for building the measurement alphabets are explored later in this study (Sec. 3). Given the measurement alphabet, the *macrostate* of the spray is defined by the distribution of the droplet characteristics in terms of relative frequency, as shown in Eq. (1), or in terms of probability density if $\omega_{j,i}$ is divided by the class size (Δd_i) as $p_{j,i} = \omega_{j,i}/\Delta d_i$. The probability density is crucial for comparing spray macrostates because two researchers can measure the same spray microstate and report different macrostates if their alphabets are different. This clarification is essential for understanding the non-deterministic nature of a spray from an infodynamic viewpoint.

Shannon (1948) introduced a way to measure the amount of information we need to know about the system. However, he was misled to name his formulation as *entropy* due to its similarity with the formulation developed by J.W. Gibbs. Panão (2025b) recognized the value of restricting this term to the discipline of Thermodynamics, and suggested two terms as part of the different discipline of Infodynamics: *informature* and *infotropy*. This study focuses on *informature* as a new spray characteristic, showing that it is an independent parameter, even if it is based on the size or velocity of the spray droplets.

Informature is an extensive property of the system given in the natural logarithm base by

$$H_{I,j} = - \sum_{i=1}^k \omega_{j,i} \ln(\omega_{j,i}) \quad (5)$$

This implies that it depends on the measurement alphabet and its k "letters". Its conversion to an intensive property of the system is derived in Panão (2025a) and it is the *differential* informature,

including a grid term, as expressed by

$$H_{\Delta,j} = - \sum_{i=1}^k \omega_{j,i} \ln(p_{j,i}) \quad (6)$$

Once we measure the macrostate of the spray for the droplet size, for example, from our data sample, what is the maximum number of microstates for the most probable (and measured) macrostate? Considering that the maximum informature occurs when all classes have the same relative frequency, and is given by $H_{I,max} = \ln(\Omega_{max})$, equalizing $H_{I,j} = H_{I,max,j}$, then, for a certain j -weighted DSDn, the maximum number of microstates results in

$$\Omega_{max,j} = \exp(H_{I,j}) \quad (7)$$

Since the DSDn in a spray, is not usually uniform, it means that $\Omega_{max,j} \equiv k_{eff,j}$, with $k_{eff,j}$ as the number of effective classes that carry the perspective of the spray given by the j -weight of the DSDn (number-, area-, or volume). What is the value of using informature or differential informature as a new spray characteristic? As noted by Panão (2025a), informature as a quantifiable value containing complete information on droplet size distribution can improve our understanding of the complicated interplay between atomization and liquid–gas interactions. Furthermore, we suggest that this information can be used for a more precise validation of numerical simulations or analytical solutions (Park et al., 2024).

3. Implications of Measurement Alphabet on Spray Characterization

The measurement alphabet defines the number of classes in the model. One of the proposed strategies for building variable-size classes is based on interlaced Fibonacci series. Following Chang et al. (1991) and Zhang et al. (1992), the number of interlaced Fibonacci series is J , and we obtain the number of classes by effectively rounding down to the nearest whole number with MATLAB's function `floor` as

$$k = \text{floor} \left[\frac{\ln(SR)}{\ln(R_e)} \right] \quad (8)$$

with $SR = D_{max}/D_{min}$ as the size range, with D_{max}, D_{min} as the maximum and minimum droplet sizes measured, $R_e = \varphi^{1/(3 \cdot J)}$ is the average ratio between adjacent class limits, and $\varphi = (1 + \sqrt{5})/2$ is the golden ratio. We retrieved the maximum and minimum droplet sizes, considering all PDI data samples and conditions, to determine the size range of the measurement alphabet applied to characterize all measurement points. Eq. (8) is used to determine the number of classes k built with regular and variable sizes, considering $J = 3$ interlaced Fibonacci series.

To understand the implications of the measurement alphabet on spray characterization, we simulated a Log-normal DSDn with a median of $D_{50} = 50 \mu\text{m}$ and standard deviation of $s = 1/\sqrt{6}$.

MATLAB randomly generated $N = 10^5$ droplets, with a size range of $SR = 59.73$ and a $D_{min} = 5.335 \mu\text{m} > 1 \mu\text{m}$. Considering $J = 3$ interlaced Fibonacci series, the number of “letters” (classes) is $k = 76$. Fig. 1 exemplifies the results obtained for the number ($j = 0$)-weighted DSDn.

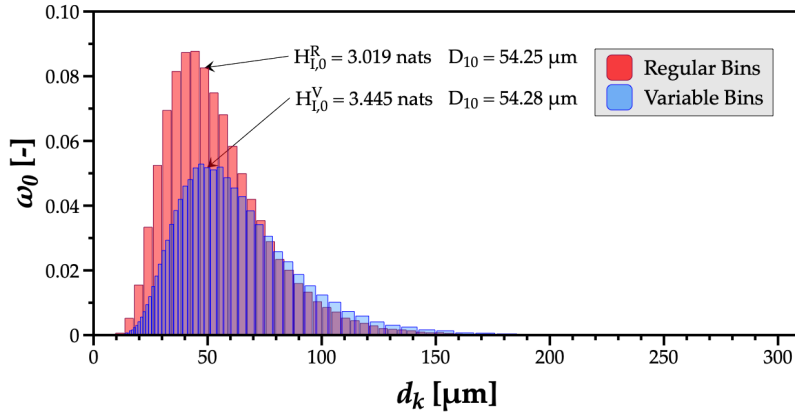


Figure 1. Example of a number($j = 0$)-weighted Droplet Size Distribution, including information of the mean diameter and the informature values considering regular ($H_{I,0}^R$), and variable ($H_{I,0}^V$) bin sizes. It shows how similar mean diameters can correspond to different amounts of information.

The mean diameters are practically equal to $D_{10} \simeq 54 \mu\text{m}$, but the shape and scale of the same data with a different measurement alphabet produced two distinct DSDn. This difference is captured by the informature, which is larger for DSDn using variable bins.

However, considering the three types of weights in DSDn ($j = 0, 2, 3$) and the different measurement alphabets (regular and variable bins), Fig. 2 shows that the variable bins maintain the shape and scale of the distribution while shifting each D_{mj} determined according to Eq. (3) to larger values. Regular bins lead to different droplet size distributions with different amounts of information, although the mean diameters are practically the same as those obtained with the variable bin measurement alphabet. These examples show how the mean diameters and informature values are independent parameters, with the latter depending on the macrostate of the spray because it is sensitive to the DSDn shape and scale.

If we express the droplet size distributions in terms of probability density, and consider the intensive form for the amount of information, the differential informature, Fig. 3 on the left shows that the number-, area-, and volume-weighted DSDn no longer depend on the measurement alphabet, and the differential informature, which also depends on the grid term (Panão, 2025a), is also independent of the measurement alphabet. The insight is that analyzing the differential informature in sprays is a more appropriate parameter to include in the new spray characterization knowledge layer.

In the next simulated experiment, we considered the mixing of two Log-normal distributions, $w = 92\%$ of one with $D_{50,1} = 30 \mu\text{m}$ and $s_1 = 0.35$, and the remaining 8% of another with $D_{50,1} = 180 \mu\text{m}$ and $s_1 = 0.25$. Fig. 3 on the right shows that the differential informature values are still

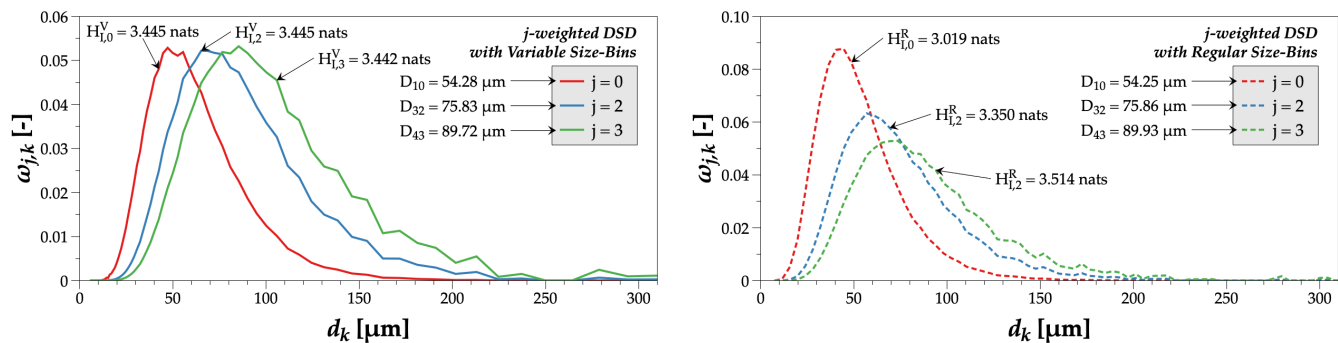


Figure 2. Example of a number($j = 0$)-, area($j = 2$)-, and volume($j = 3$)-weighted Droplet Size Distributions, using all variable (left) and regular (right) bins. The plots include information on both mean diameter and informative values ($H_{I,j}^V$).

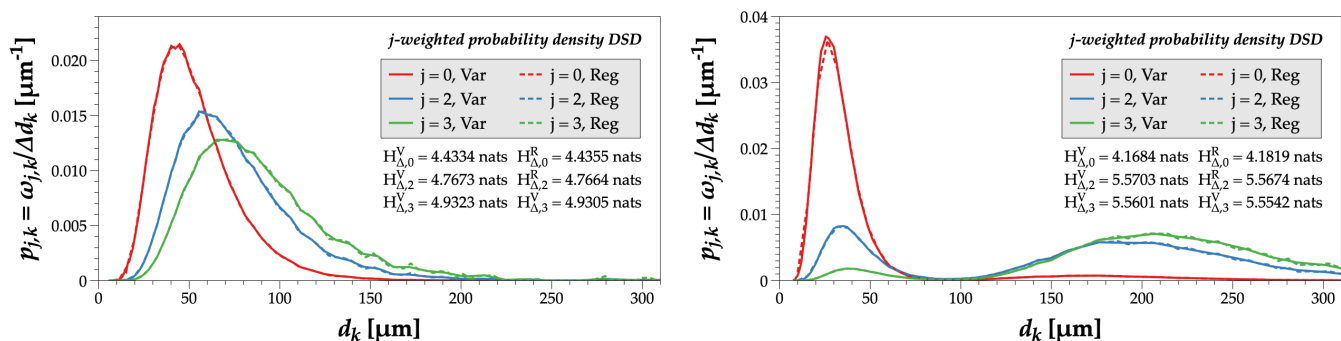


Figure 3. Number($j = 0$)-, area($j = 2$), and volume($j = 3$)-weighted probability density droplet size distributions, considering regular and variable bin sizes, and including the values for the differential informative. A unimodal example on the left, and a bimodal example on the right.

similar regardless of the measurement alphabet (regular or variable bins). Fig. 4 further shows how the mean size of each j -weighted distribution is similar and independent of the measurement alphabet, but the scale and shapes of these distributions are clearly different and fully captured by the informature value in its extensive form given by Eq. (5).

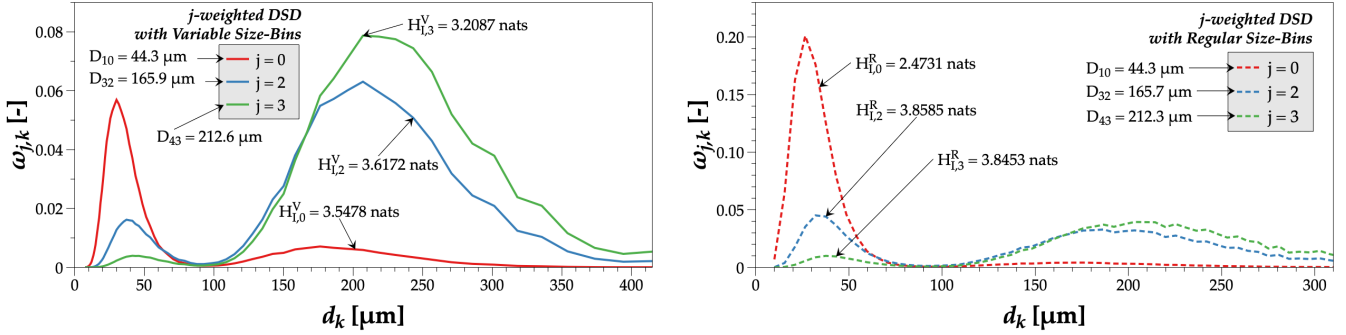


Figure 4. Example of the number($j = 0$)-, area($j = 2$)-, and volume($j = 3$)-weighted form of a bimodal Droplet Size Distribution, using all variable (left) and regular (right) bins. The plots also include information on both mean diameter and informature values ($H^V_{I,j}$).

With the variable bin measurement alphabet, the number- and volume-weighted DSDn appear to have pronounced peaks for the smallest and largest droplets, respectively, which is why their informature values are lower than that of the area-weighted distribution, where the size carrying the information of the spray area is more scattered in the relative frequency ω_j spectrum (d_k). Regarding the regular bin measurement alphabet, the evolution of the scale and shape of the part related to the distribution of smaller droplets is similar to that of the variable bins, but the differences between the representations in the larger droplet spectrum are less pronounced. However, the same pattern remained regarding the differences in the informature values, although the gap between the area- and volume-weighted droplet size distributions closed. In all the distributions shown thus far, only when the weight changes does one observe a change in the value of the mean diameter. However, the mean diameter of the number-, area-, or volume-weighted DSDn is blind to the changes induced by the measurement alphabet used. At this point, we began developing a *Representation Index*, $R_{(j)}$, to help characterize the spray from an informational perspective using the differences produced by regular and variable size classes.

The first step is to relate the number of classes effectively carrying the spray diversity from a weighted perspective using a certain measurement alphabet (MA $\equiv$ (R)egular/(V)ariable) with the total number given by Eq. (8), which we designate as an “Activation Factor”

$$\phi_j^{(MA)} = \frac{k_{eff,j}}{k} \quad (9)$$

with $\phi_j^{(R)}, \phi_j^{(V)} \in [0, 1]$. This step is followed by a second step that establishes the requirements for accurately formulating a Representation Index $R_{(j)}$:

1. Symmetry – $R_{(j)} \left(\phi_j^{(R)}, \phi_j^{(V)} \right) = R_{(j)} \left(\phi_j^{(V)}, \phi_j^{(R)} \right)$
2. Normalization – for equal activation factors: $R_{(j)}(\phi, \phi) = 1$
3. Monotonous discrepancy – as $\phi_j^{(R)}$ differentiate from $\phi_j^{(V)}$, $R_{(j)} \left(\phi_j^{(R)}, \phi_j^{(V)} \right)$ decreases
4. Scale invariance – $R_{(j)} \left(c \cdot \phi_j^{(R)}, c \cdot \phi_j^{(V)} \right) = R_{(j)} \left(\phi_j^{(R)}, \phi_j^{(V)} \right)$ for $c > 0$

We suggest that using the ratio between Activation Factors, $\rho_j = \phi_j^{(V)} / \phi_j^{(R)}$ is the simplest way to relate the measurement alphabets. However, to fulfill the requirements above, we should linearize this relation as $\eta_j = \ln(\rho_j)$. To ensure the symmetry requirement, $R_{(j)} = f(|\eta_j|)$, and if $\phi_j^{(V)} = \phi_j^{(R)}$, $|\eta_j| = 0$, and the Representation Index should result $R_{(j)} = 1$. Therefore, a simple functional relation that fulfills the requirements above is

$$R_{(j)} = \exp(-|\eta_j|) \quad (10)$$

The Representation Index quantifies how invariant the Activation Factor $\phi_j^{(MA)}$ is to changing the measurement alphabet used in the spray characterization. It is noteworthy that $R_{(j)} \in [0, 1]$. This index approaches unity when both measurement alphabets yield the same Activation Factor, for example, when the droplet population is effectively concentrated in a single size range or when both alphabets represent the active size range equivalently. This formulation is also interesting because it can be used to evaluate the additive effect of two independent Activation Factors. For example, between the number-weighted ($j = 0$) and area-weighted ($j=2$) perspectives, the outcome becomes the product of the Representative Index of each j -weighted distribution: $R(\eta_0, \eta_2) = R_{(0,2)} = R_{(0)} \cdot R_{(2)}$. Fig. 5 shows the results for the Representation Index for single j -weighted DSDn and combined cases.

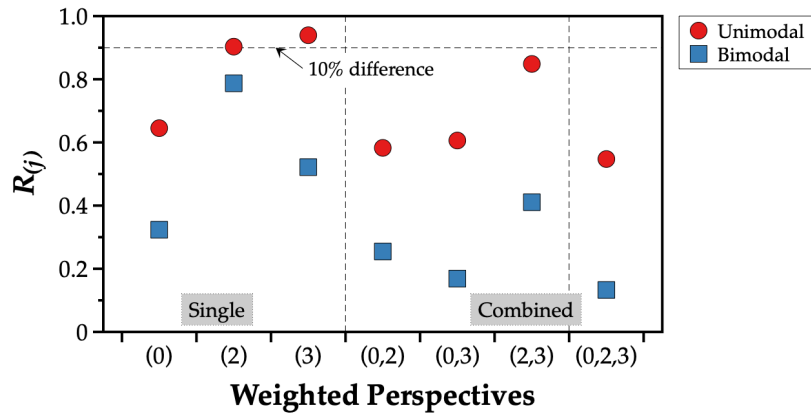


Figure 5. Example of the Representation Index $R_{(j)}$ for a unimodal and bimodal simulated cases.

The Representation Index $R_{(j)}$ quantifies whether the Activation Factor $\phi_j^{(MA)}$ is invariant to the measurement alphabet (regular vs. variable size classes). Differences in $R_{(j)}$ across j reveal the

physical meaning of the spray that carries its informational structure: number-carriers (fine droplets), area-carriers (interfacial), or volume-carriers (mass). In the unimodal case, the Representation Index for area- and volume-weighted distributions shows that DSDn has a coherent information carrier because the number of effective classes does not change significantly, leading to $R_{(j)} \rightarrow 1$. Therefore, in the unimodal spray, the interfacial area ($j = 2$) and mass ($j = 3$) physical meanings are carried by a comparatively coherent scale band, robustly represented under either alphabet, whereas the number ($j = 0$) of droplets is spread over finer scales and is therefore more representation-sensitive, diminishing $R_{(j)}$.

In the bimodal case, we already know that the droplet size distribution is composed of two distinct clusters: an expressive (92%) cluster of smaller droplets and a less-expressive (8%) cluster of larger droplets. In practice, multiple clusters induce multiple informational carriers, increasing the effect of incompatibility between representing drop sizes with an additive D scale in regular bins and representing them with a multiplicative $\ln(D)$ scale in variable bins. The fact that $R_{(0)}$ and $R_{(3)}$ decrease indicates that the number and mass carriers reside in distinct scale regions whose activation depends on whether the size is partitioned additively or multiplicatively.

The combined indices $R_{(0,2)}$, $R_{(0,3)}$, and $R_{(0,2,3)}$ quantify multi-carrier coherence. Their collapse in the bimodal case demonstrates that no single alphabet yields a consistent informational description across number-, area-, and volume-induced views, which is a quantitative signature of carrier switching when j changes and scale separation because the regions in the droplet spectrum that carry a particular informational aspect of the spray can have different scales. It is noteworthy that there is no right or wrong measurement alphabet to represent drop sizes, but we can benefit from their differences to identify when multiple clusters of sizes carry the spray informational characteristics. This analysis provides a basis for interpreting the experimental infodynamic patterns discussed in the following sections of this paper. The next section presents the experimental setup and diagnostic techniques that allowed the study of the infodynamic patterns in effervescent sprays.

4. Experimental Setup and Diagnostic Technique

A single-orifice effervescent atomizer was tested in this study at a constant injection overpressure of 100 kPa for several GLRs ranging from 2.5 to 10%. Water and air were mixed inside a mixing chamber with a diameter of 15 mm. Air was injected into the mixing chamber through 64 aerator orifices, each with a diameter of 0.6 mm. The resulting two-phase flow was then discharged through an exit orifice with a diameter of $d_0 = 1.2$ mm. A schematic representation of the atomizer is illustrated in Fig. 6.

Water was pumped from the storage tank to the atomizer using a gear pump. The water flow rate was measured using a Coriolis mass flow meter (Mass 2100 Di3), fitted with a Mass 6000 trans-

mitter (Siemens AG., GE), with an accuracy of $\pm 0.1\%$ of the actual flow rate. The inlet pressures of the water and atomization gas were measured using piezoresistive pressure sensors (BD Sensor DMP 331i, BD SENSORS s.r.o., CZ), with a measurement accuracy of ± 2 kPa. The atomization air flow rate was measured using an FMA-A2100 mass flow meter (Omega Engineering, USA), with an accuracy of $\pm 1\%$ of the actual flow rate.

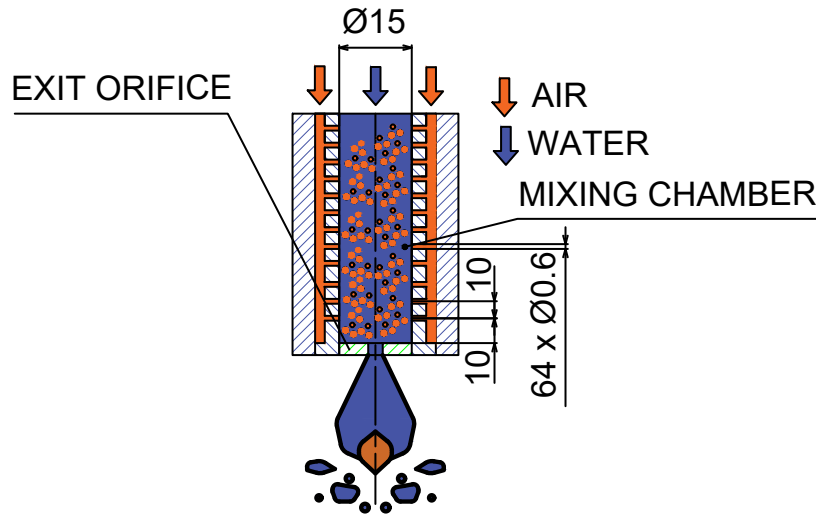


Figure 6. Schematic layout of the effervescent atomizer (adapted from Cejpek et al. (2023a)).

Droplet sizes and velocities were acquired using a two-component fiber-based Phase Doppler Interferometry (PDI) measurement system (Dantec Dynamics A/S, Skovlunde, Denmark). Non-coincident two-component velocity measurements were performed simultaneously with the droplet sizing. Several points along two perpendicular axes were probed at an axial distance of $z = 100$ mm from the atomizer tip. Either 35 000 samples were acquired, or a 30 s acquisition duration was reached, whichever occurred first. However, using the infodynamic criterion of the temporal stabilization of the differential informature, $dH_{\Delta,j}/dt \approx 0$ (Ferrão et al., 2025), all points fulfilled the criterion, including those with the lowest number of data samples. Dantec BSA software version 5.2 was used to control the measurements. The PDI settings are summarized in Table 1. The PDI configuration and acquisition protocol followed the diagnostic approach previously used by Cejpek et al. (2023a) for effervescent atomizer spray measurements.

5. Application to Study Effervescent Sprays

This section presents the application of the new infodynamic characterization layer based on the quantification of drop size diversity, including an assessment of spray symmetry and the effect of GLR on drop size and spray informational structure. First, we need to clarify how we processed

Table 1. PDI setup parameters.

Parameter	Value
Laser power output	350 mW
Scattering angle	72°
Receiver mask	C
Receiver spatial filter	0.2 mm
Focal length (transmitting/receiving optics)	500/500 mm
Wavelengths	488 nm, 514.5 nm
Velocity components	Axial, Radial
Velocity centre	25.7 m/s, 0 m/s
Velocity span	51.4 m/s, 24.4 m/s
Sensitivity	950 V, 1080 V
Signal-to-noise ratio	0 dB, 0 dB
Signal gain	8 dB, 10 dB
Level validation ratio	8

the data to build the measurement alphabet.

The PDI technique has a limitation when measuring droplet sizes below $1 \mu\text{m}$ because these approach the scale of the laser beam wavelength. Thus, if $D_{\min} < 1 \mu\text{m}$, we define the first measurement alphabet "letter"/class in the case of variable bins as the interval between $D_{\min}$ and $D_0 = 1 \mu\text{m}$, and subsequent edges as $D_l = D_0 \cdot SR^{l/k}$, $\forall l \in \{1..k\}$. If $D_{\min} \geq 1$, then $D_0 = D_{\min}$. With regular size classes, we divide the size range by the number of k classes.

The measurement grid points were in the $(x, 0)$ and $(0, y)$ -axes from -25 mm : $\Delta x = \Delta y = 2.5 \text{ mm}$: 25 mm , corresponding to 42 measurement points in the three GLR (2.5%, 5%, 10%) conditions. The data processing algorithm evaluated the maximum size range of all experiments considered and used this value to establish the regular and variable alphabets used to perform the informational characterization of the spray radial profile. These measurement alphabets affect only the infodynamic characteristics of the spray, except for the differential informature given by Eq. (6). Fig. 7 exemplifies how the differential informature ($H_{\Delta,0}$) for the number-weighted drop size distribution is independent of the measurement alphabet. In contrast, informature ($H_{I,0}$) is sensitive to whether size classes are regular or variable. Therefore, the informational structure characteristics of the spray rely on differential informature (an intensive informational parameter such as temperature), whereas the Representation Index $R_{(j)}$ retrieves further insights from the differences in the informature (an extensive informational parameter such as mass) values generated by the measurement alphabet.

Is the spray symmetric? Fig. 8 evaluates whether or not this effervescent spray is symmetric by

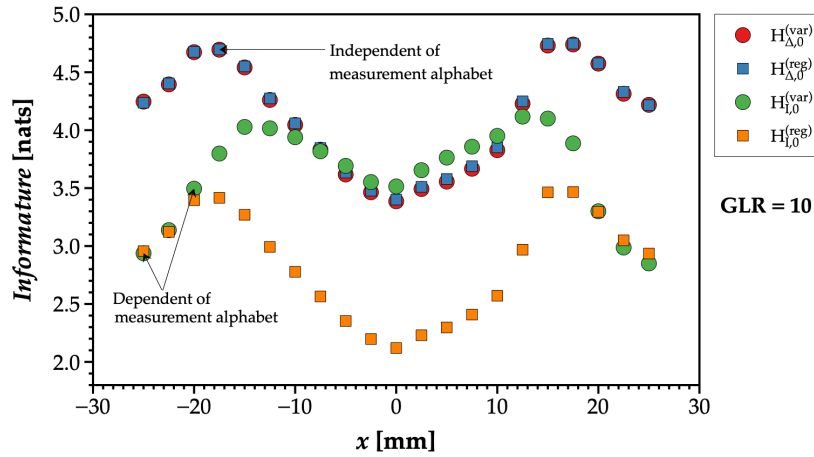


Figure 7. Comparing the role of the measurement alphabet ($\circ$ - variable; $\square$ - regular) in informature and differential informature.

comparing the $(x, 0)$ and $(0, y)$ radial profiles for the three mean diameters (number-, area-, and volume-weighted) and three GLR (2.5%, 5%, and 10%) conditions. The results show that the spray is relatively symmetric, although asymmetries are accentuated for higher j -weights. However, in the case of the spray differential informature, these asymmetries appear to be less pronounced. Asymmetry was present across all three GLR conditions and showed no systematic trend with increasing GLR, suggesting that it was related to the atomizer geometry or atomizer mounting rather than a consequence of the internal flow regime transition. The radial profiles of the differential informature in Fig. 9 show, for the first time, the spray informational structure as an *info-hollow-cone spray*. The local values of the differential informature express the amount of information required to determine the diversity of drop sizes. This diversity exhibited a local peak at the location where we expected the interface region between the spray and the environment. In Cejpek et al. (2023a), the spray cone angles (SCA) obtained from high-speed imaging were between 20° - 30° , and using the number-weighted differential informature, the SCA was approximately 20° . The SCA boundary corresponds to the radial position enclosing approximately 90% of the cumulative droplet volume, located at a radius of 17–20 mm. This agreement confirms that the differential informature peak identifies the spray/ambient interface from the informational structure of the droplet size distribution alone. The lowest values of $H_{\Delta,j}$ were consistently observed in the spray center, where a high-speed gas core was present and the droplet size distribution was relatively narrow. Two complementary mechanisms explain this phenomenon. First, the large shear at the spray axis promotes the formation of the smallest droplets, which are subsequently entrained by the high-speed gas and concentrated near the axis, thereby reducing size diversity. Second, these small droplets, which have low Stokes numbers, tend to follow the gas flow collectively, further narrowing the local droplet size distribution. The minimum informature is most pronounced at GLR = 10%, where annular internal flow produces a gas core with the largest velocity.

Another noteworthy feature is the asymmetry observed in the regions $(x > 0, 0)$ and $(0, y > 0)$ of

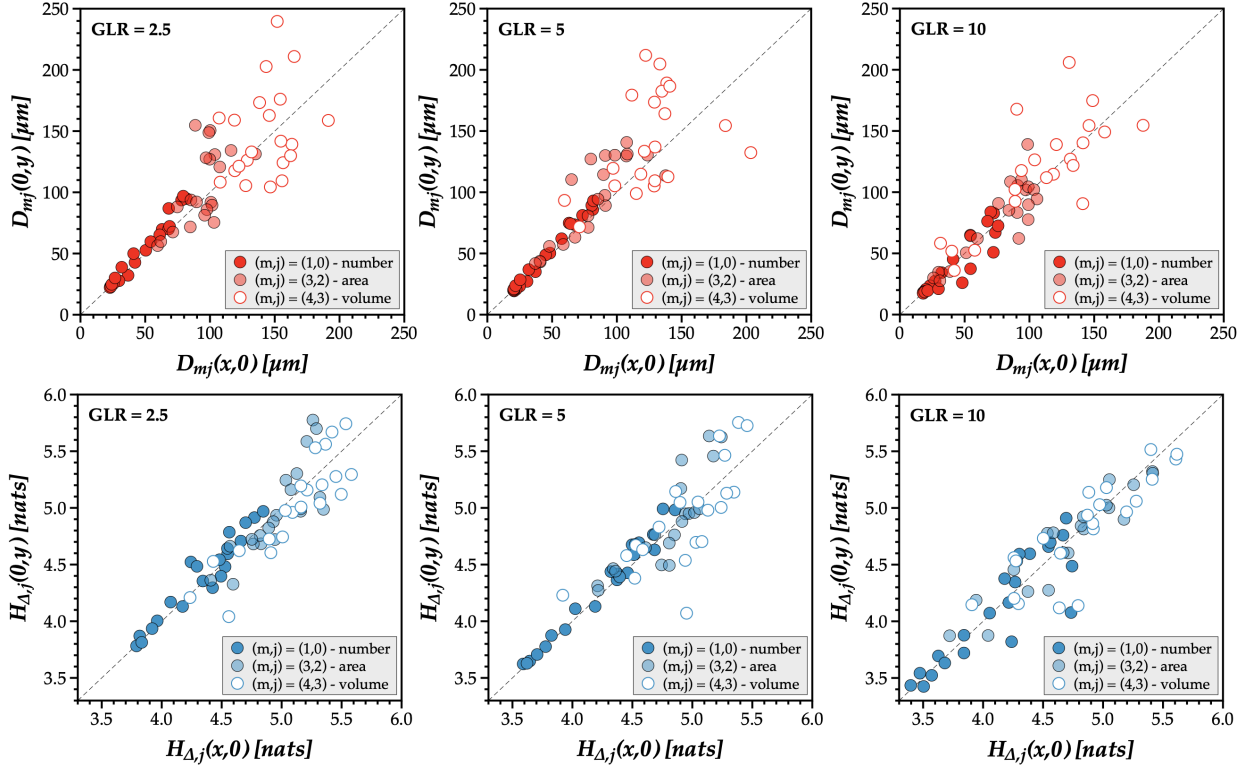


Figure 8. Symmetry of the spray from the mean diameter D_{mj} (with $m = j + 1$) on top, and below from the differential informtation $H_{\Delta,j}$ perspective for the GLR of 2.5, 5 and 10%.

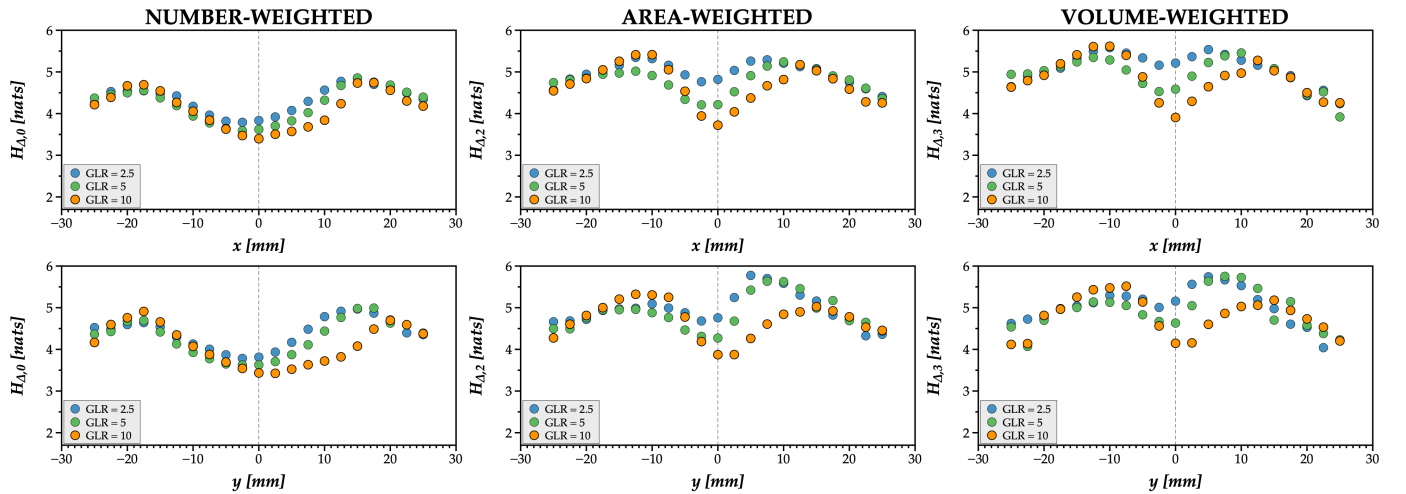


Figure 9. Spray Informational Structure and effect of GLR on the radial profiles of the differential informtation $H_{\Delta,j}$.

the radial profiles with respect to the effect of the GLR. In all j -weights considered, we observe a loss in the amount of information needed to determine the diversity inside the info-hollow cone spray. The contribution from the PDI measurement principle should also be considered for the asymmetry observed in the regions $(x > 0, 0)$ and $(0, y > 0)$. The signal from droplets located at positive x and y positions must travel larger distances to reach the receiving optics. This signal path intersects with the dense inner spray region. Consistently lower velocity validation rates were observed for these positions, whereas the spherical validation remained symmetric. The maximum difference in the overall validation rate between the opposite sides was less than 6%.

The last preliminary results concern the newly developed Representation Index $R_{(j)}$ for the three GLR conditions, and each plot includes all j -weights carrying information on the spray and combinations (see Fig. 10). Despite the different GLR conditions used, an infodynamic pattern emerged, allowing the distinction of three regions. An outer region, $|x| > 15$ mm where the information is mostly carried by the number of droplets. This region corresponds to the zone where small droplets with low Stokes numbers are entrained by outer vortices toward the spray boundary Heinlein and Fritsching (2006), forming a coherent population with a relatively narrow size distribution, thus a high $R_{(0)}$. An intermediate region from approximately 5 to 15 mm represents the spray region, characterized by strong liquid–gas shear (Zaremba et al., 2018), where droplets predominantly carry information on the interfacial area of the spray, resulting in elevated $R_{(2)}$. For $\text{GLR} \geq 5\%$, a central region emerged with droplets predominantly carrying information on the volume/mass of the spray. The progressive widening of the central low- $R_{(j)}$ region with an increasing GLR for both $j = 2$ and $j = 3$ reflects the expansion of the high-speed gas core associated with the transition to annular internal flow. At $\text{GLR} = 2.5\%$, slug flow produced random bursts of liquid that distributed droplets more uniformly across the spray cross-section, including the center, and this condition also produced the largest fraction of droplets exceeding $250 \mu\text{m}$. At $\text{GLR} \geq 5\%$, annular flow confines the large-droplet population to the periphery, creating a well-defined central region where volume information carriers are absent, and $R_{(3)}$ decreases. The onset of this central $R_{(3)}$ dip specifically at $\text{GLR} = 5\%$ is therefore a quantitative signature of the slug-to-annular flow regime transition in the infodynamic representation of the spray.

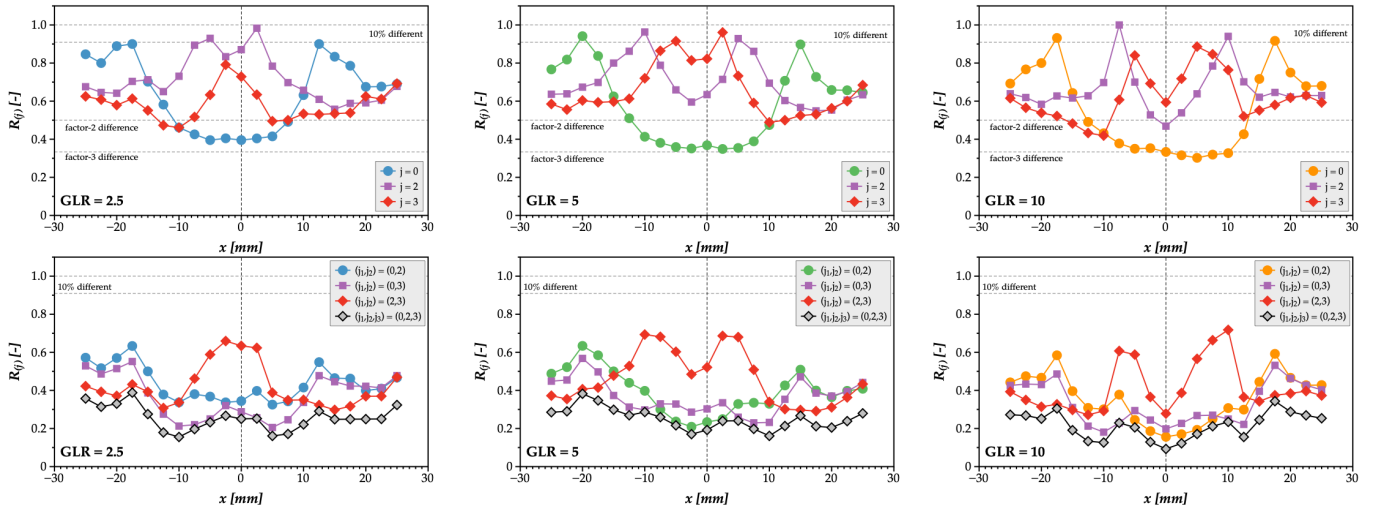


Figure 10. Representation Index $R_{(j)}$ for the three GLR conditions and combinations.

The low $R_{(j)}$ values near the spray axis at high GLR values may also be partially attributable to measurement limitations. In the dense spray center, signal losses for small droplets may reduce the number of validated events and alter the apparent size distributions. The repeatability of $R_{(j)}$ is shown in Fig. 11. We can see that $R_{(j)}$ structure is similar, slightly shifted, but the values agree well.

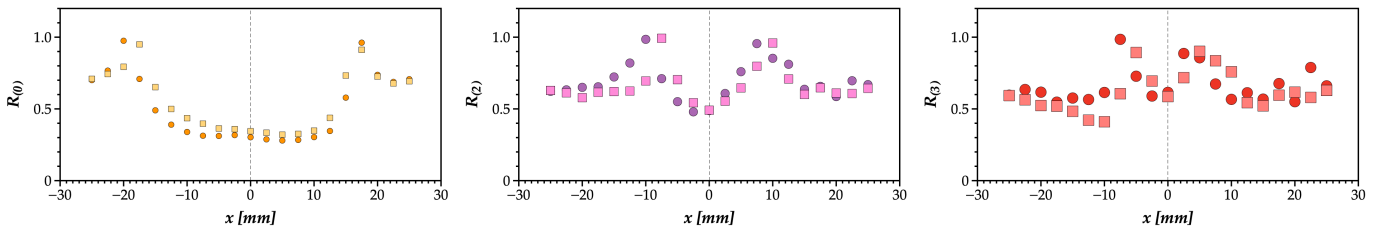


Figure 11. $R_{(j)}$ repeatability for the 10% GLR condition

To provide a spray dynamic interpretation of the observed infodynamic regions, a local Stokes number analysis was performed using raw PDI data. The local gas velocity was estimated using the approach described in the Appendix. The particle relaxation time was computed as $\tau_p = \rho_l d^2 / (18 \mu_g)$, and the characteristic flow time scale as $\tau_f = d_o / U_G$, yielding the local Stokes number $St = \tau_p / \tau_f$ for each droplet. A small number of droplets were recorded at the spray boundary, which implies that the local estimation of gas velocity and Stokes analysis may not be statistically representative. Three regimes were identified: tracer droplets ($St < 0.1$), transitional droplets ($0.1 \leq St \leq 10$), and ballistic droplets ($St > 10$). Fig. 12 shows the spatial distribution of these fractions weighted by the number across the spray cross-section.

The tracer fraction was the largest at the boundary, corresponding directly to the decrease in $R_{(0)}$ near the spray center, as described previously. The transitional regime fraction exhibited local

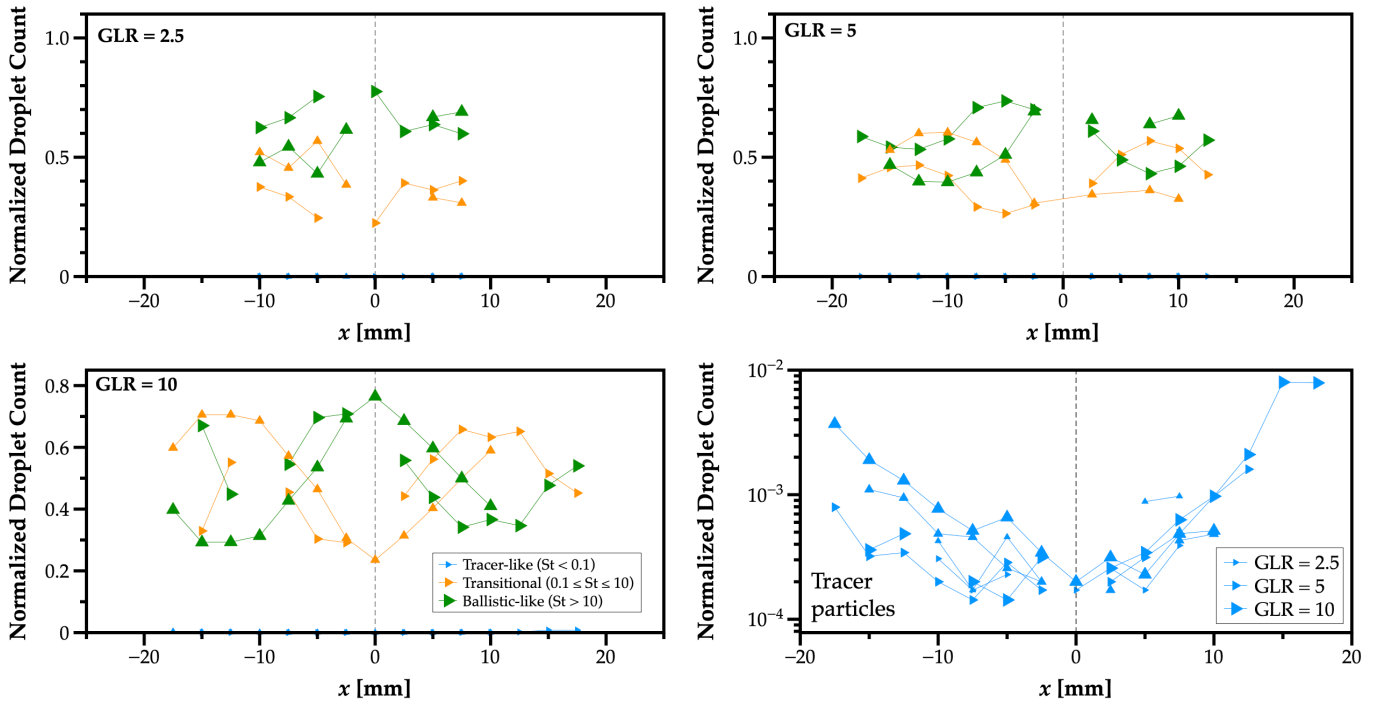


Figure 12. Normalized count profiles of tracer-like, transitional and ballistic-like droplets for $GLR = 2.5, 5, 10$ and a detailed plot of tracer particles ($St < 0.1$) because of smaller value range.

maxima followed by saturation in the intermediate spray region at approximately $|x| > 10$ mm. This is the zone of strong liquid–gas shear described by Zaremba et al. (2018), where droplets of intermediate size and Stokes number carry the largest fraction of the spray interfacial area. The spatial coincidence of this transitional area-fraction peak with the elevated $R_{(2)}$ region confirms the physical interpretation of area-carriers as a coherent population of intermediate-Stokes droplets. The peak is present in the spray center for the ballistic fraction, but does not feature a sharp dip at $x = 0$ mm as in Fig. 10. The Stokes number classifies droplets based on their mean dynamical response but does not capture the internal size diversity within each regime. Droplets originating from distinct mechanisms are present in the center, and $R_{(j)}$ correctly describes them. The ballistic fraction decreased toward the outer spray region, reflecting the ballistic trajectory of large droplets restricted to the core region of the spray.

6. Conclusions

This study investigated the infodynamic patterns emerging from the characterization of effervescent sprays, moving beyond the mean droplet diameters and conventional statistical descriptors to capture the informational structure associated with droplet size diversity. By introducing informature, differential informature, the Activation Factor, and the Representation Index $R_{(j)}$, we developed a novel characterization layer capable of expressing aspects of spray indeterminacy that

are hidden when the droplet size distribution is reduced to a single mean value. The Activation Factor represents the fraction of effective size classes that actively carry a given number-, area-, or volume-weighted view of the droplet size distribution.

The analysis showed that the measurement alphabet used to organize droplet sizes into classes affects the informature obtained from the number-, area-, and volume-weighted droplet size distributions. Regular and variable size classes can represent the same droplet sample with similar mean diameters but different amounts of information (informature) because they construct different (most probable) macrostates from the same spray microstates. In contrast, the differential informature, which includes the class-size contribution through the probability density, remained essentially independent of the measurement alphabet. This makes differential informature a more appropriate intensive parameter for describing the intrinsic informational structure of a measured spray.

The Representation Index $R_{(j)}$ was proposed to quantify the invariance of the Activation Factor with respect to the measurement alphabet. Synthetic unimodal and bimodal droplet size distributions demonstrated their physical meaning. In unimodal distributions, the area- and volume-weighted Activation Factors were comparatively invariant, indicating that the interfacial area and liquid volume were carried by coherent size ranges. In bimodal distributions, lower $R_{(0)}$ and $R_{(3)}$ values indicate that the number and volume carriers occupy distinct scale regions. The combined indices, such as $R_{(0,2,3)}$, further quantify multi-carrier coherence and show when no single measurement alphabet provides a consistent informational description across number-, area-, and volume-induced views.

The application of the framework to PDI measurements of an effervescent atomizer at different gas-to-liquid ratios (GLR) revealed a consistent radial infodynamic structure. Differential informature exposed an “info-hollow-cone” spray, with lower informature values near the center and higher values close to the spray/ambient interface. This structure indicates that the centerline region is characterized by a narrower morphological (droplet size) diversity, which is associated with the high-speed gas core and the entrainment of smaller droplets, whereas the spray boundary contains a richer diversity of droplet sizes. The location of the differential-informature peak was consistent with the expected spray cone boundary, indicating that the spray/ambient interface could be identified from the informational structure of the droplet size distribution alone.

The radial profiles of $R_{(j)}$ revealed three infodynamic regions. In the outer spray region, the number of droplets predominantly carried the spray information, which was consistent with the entrainment of small droplets toward the spray boundary. In the intermediate region, the elevated $R_{(2)}$ values indicated that the interfacial area was carried by a coherent population of intermediate droplets, which is in agreement with the local Stokes-number analysis. For GLR values equal to or above 5%, a central region emerged in which the volume/mass carriers were more represented, and $R_{(3)}$ surpassed $R_{(2)}$ and $R_{(0)}$. This central dip is consistent with the expansion of the high-

speed gas core and provides an infodynamic signature of the transition from slug-dominated to annular internal flows.

Overall, the proposed infodynamic approach provides a physically grounded methodology for characterizing the non-deterministic nature of complex two-phase spray systems. Differential infomature describes the spatial organization of droplet size diversity, whereas the Representation Index identifies whether the number, interfacial area, and liquid volume are carried by coherent or scale-separated droplet populations. This novel characterization layer does not replace conventional spray quantities such as D_{10} , D_{32} , or D_{43} , but complements them by quantifying how the measured droplet size distribution conveys information across physically meaningful, weighted views. Such information can support a more rigorous validation of numerical models by comparing not only the mean sizes and velocities but also the informational structure generated by atomization, transport, and gas–liquid interaction mechanisms.

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Nomenclature

d_i	Droplet diameter in class i [μm]
d_k	Droplet diameter [μm]
d_o	Atomizer exit-orifice diameter [m]
D_0	Reference lower edge for variable size classes [μm]
D_l	Diameter class edge indexed by l [μm]
D_{10}	Arithmetic mean diameter [μm]
D_{32}	Sauter Mean Diameter (SMD) [μm]
D_{43}	Volume-weighted mean diameter [μm]
D_{50}	Median diameter of a log-normal distribution [μm]
$D_{j,10}$	10th percentile diameter of the j -weighted DSDn [μm]
$D_{j,50}$	50th percentile diameter of the j -weighted DSDn [μm]
$D_{j,90}$	90th percentile diameter of the j -weighted DSDn [μm]
D_{max}	Maximum droplet size measured [μm]

D_{min}	Minimum droplet size measured [μm]
$D_{\alpha j}$	$(\alpha - j)$ -moment diameter of a j -weighted DSDn [μm]
D_{mj}	Mean diameter associated with a j -weighted DSDn [μm]
$F_{j,i}$	Cumulative distribution of the j -weighted DSDn [-]
$H_{I,j}$	Informature of the j -weighted distribution [nats]
$H_{\Delta,j}$	Differential informature of the j -weighted distribution [nats]
$H_{I,max,j}$	Maximum informature for the j -weighted distribution [nats]
j	Weight type index: 0 (number), 2 (area), 3 (volume) [-]
J	Interlaced Fibonacci series index [-]
k	Number of classes/bins [-]
$k_{eff,j}$	Number of effective classes for the j -weighted distribution [-]
l	Class-edge index [-]
m	Mean-diameter order index [-]
$m_{j,\ell}$	Moment of order ℓ of the j -weighted distribution [μm^ℓ]
n_i	Number of droplets in class i [-]
N	Total number of droplets [-]
p_{in}	Inlet pressure [Pa]
$p_{j,i}$	Probability density of class i in the j -weighted distribution [μm^{-1}]
q	Weibull shape factor or fractal-dimension parameter [-]
R_e	Average ratio between adjacent class limits [-]
$R_{(j)}$	Representation Index for the j -weighted distribution [-]
$R_{(j_1,j_2,\dots)}$	Combined Representation Index for multiple weighted distributions [-]
s	Standard deviation [-]
SR	Size range, D_{max}/D_{min} [-]
St	Stokes number [-]
u_i	Droplet velocity [m/s]
U_G	Continuous phase velocity [m/s]
w	Weight fraction [-]
x_i	Droplet-level state vector, $x_i \equiv (d_i, u_i)$ [-]
x, y, z	Measurement coordinates [mm]

Greek Letters

α	Moment-diameter order index [-]
Δd_i	Class size [μm]
$\Delta x, \Delta y$	Measurement spacing in the x - and y -directions [mm]
η_j	Linearized ratio of Activation Factors, $\eta_j = \ln(\rho_j)$ [-]
μ_g	Dynamic viscosity of the gas phase [Pa s]
ρ_j	Ratio of Activation Factors, $\rho_j = \phi_j^{(V)}/\phi_j^{(R)}$ [-]
ρ_l	Liquid density [kg m^{-3}]

τ_f	Characteristic flow time scale [s]
τ_p	Particle relaxation time [s]
Ω	Spray microstate [-]
$\Omega_{max,j}$	Maximum number of microstates for the j -weighted distribution [-]
$\phi_j^{(MA)}$	Activation Factor for measurement alphabet MA [-]
φ	Golden ratio, $(1 + \sqrt{5})/2$ [-]
$\omega_{j,i}$	Relative frequency of class i in the j -weighted distribution [-]

Acronyms

DSDn	Droplet Size Distribution
DSDy	Droplet Size Diversity
GLR	Gas-to-Liquid Ratio
GSD	Geometric Standard Deviation
MA	Measurement Alphabet
PDI	Phase Doppler Interferometry
PDF	Probability Density Function
RSF	Relative Span Factor
SCA	Spray Cone Angle
SDv	Volume-weighted Standard Deviation
SMD	Sauter Mean Diameter

Appendix

Algorithm to obtain the continuous phase velocity U_G

The continuous phase velocity U_G was estimated locally from the raw PDI samples using a response-coherent tracer-selection method without imposing a fixed droplet-size threshold. The method is based on the premise that droplets representative of the continuous phase should satisfy two conditions: they must be dynamically capable of responding to the gas flow within the selected flow time scale and belong to a locally coherent velocity population. The first condition is associated with the classical response-time definition of the Stokes number, whereas the second condition avoids estimating U_G from isolated droplets or droplets belonging to distinct hydrodynamic histories.

For each valid PDI droplet event (d_k, u_i) , the particle relaxation time was computed, assuming Stokes drag, as

$$\tau_{p,k} = \frac{\rho_l d_k^2}{18\mu_g}, \quad (11)$$

where d_k is the droplet diameter, ρ_l is the liquid density, and μ_g is the dynamic viscosity of the gas phase, respectively. The characteristic flow length was taken as the nozzle exit orifice diameter, d_o , which provides a physically consistent length scale for the gas–liquid discharge in the effervescent atomizer. For a local estimate of the continuous phase velocity U_G , the corresponding flow time scale is

$$\tau_f = \frac{d_o}{U_G}. \quad (12)$$

Thus, the classical response-time Stokes number is

$$\text{St}_{\text{flow},k} = \frac{\tau_{p,k}}{\tau_f} = \frac{\tau_{p,k} U_G}{d_o}. \quad (13)$$

Because U_G is not known a priori, the method was implemented iteratively. At iteration m , using the current estimate $U_G^{(m)}$, droplets were first tested for dynamic response capability according to

$$\text{St}_{\text{flow},k}^{(m)} = \frac{\tau_{p,k} U_G^{(m)}}{d_o} < \text{St}_{\text{tr}}, \quad \text{St}_{\text{tr}} = 0.1. \quad (14)$$

Equivalently, for each droplet size, this condition defines a critical continuous-phase velocity

$$U_{\text{crit},k} = \frac{\text{St}_{\text{tr}} d_o}{\tau_{p,k}}, \quad (15)$$

so that a droplet of diameter d_k is dynamically capable of tracer-like response if

$$U_G^{(m)} < U_{\text{crit},k}. \quad (16)$$

This step identifies droplets with a sufficiently short response time relative to the flow timescale. This should be interpreted as a response capability condition, not yet as proof that the droplet is an adequate tracer of the local gas velocity.

A velocity coherence condition was then applied to the response-capable droplets. The median velocity of the response-capable subset was computed as

$$\tilde{U}^{(m)} = \text{median}(u_i), \quad (17)$$

and a robust velocity scale was estimated from the median absolute deviation,

$$\sigma_{\text{rob}}^{(m)} = 1.4826 \text{ median} \left(|u_i - \tilde{U}^{(m)}| \right). \quad (18)$$

The factor 1.4826 (inverse of the 75th percentile of the standard normal distribution) makes the median absolute deviation consistent with the standard deviation for a Gaussian velocity population, although it is used only as a robust scale parameter. A droplet was accepted as part of the tracer-like velocity population when

$$|u_i - \tilde{U}^{(m)}| \leq C_{\text{MAD}} \sigma_{\text{rob}}^{(m)}, \quad (19)$$

where C_{MAD} is the prescribed robustness factor. This second condition rejects response-capable droplets whose velocities are inconsistent with the dominant local tracer-like velocity population.

The continuous phase velocity was then updated from the median velocity of the droplets satisfying both the response capability and velocity coherence conditions:

$$U_G^{(m+1)} = \text{median} \left(u_i \mid \text{St}_{\text{flow},k}^{(m)} < 0.1 \text{ and } |u_i - \tilde{U}^{(m)}| \leq C_{\text{MAD}} \sigma_{\text{rob}}^{(m)} \right). \quad (20)$$

The iteration was stopped when the accepted droplet set became unchanged, the variation in U_G between successive iterations fell below a prescribed tolerance, or the maximum number of iterations was reached. If fewer than five droplets satisfied the final combined criterion, U_G was not evaluated at that point.

Because some spray regions contained a limited number of accepted tracer-like droplets, the reliability of each local estimate was also assessed. Points with at least ten accepted droplets were classified as reliable when they satisfied the convergence and velocity coherence checks. Points with five to nine accepted droplets were retained only as tentative estimates if a leave-one-out test showed that U_G was stable and the final tracer/transitional/ballistic fractions were insensitive to the resulting uncertainty interval in U_G . Points failing these tests were rejected from the Stokes number interpretation.

After obtaining U_G , all valid droplets at the same measurement point were classified using the classical response-time Stokes number,

$$\text{St}_{\text{flow},k} = \frac{\tau_{p,k} U_G}{d_o}. \quad (21)$$

Droplets were counted as tracer-like for

$$\text{St}_{\text{flow},k} < 0.1, \quad (22)$$

transitional for

$$0.1 \leq \text{St}_{\text{flow},k} \leq 10, \quad (23)$$

and ballistic-like for

$$\text{St}_{\text{flow},k} > 10. \quad (24)$$

The corresponding counts were normalized by the total number of valid droplets at the same measurement point and GLR condition.

A slip-consistency parameter was also computed as a diagnostic quantity,

$$\text{St}_{\text{slip},k} = \frac{\tau_{p,k} |u_i - U_G|}{d_o}, \quad (25)$$

However, it was not used as the primary criterion for classifying droplets. Instead, $St_{\text{slip},k}$ was used to evaluate whether droplets dynamically capable of a tracer-like response were also locally consistent with the estimated continuous phase velocity. In this sense, this method separates the response capability, velocity coherence, and final droplet classification.

This approach differs from the procedure of Rácz et al. (2022), who estimated the gas velocity using a relative-velocity Stokes number to determine a critical droplet diameter D_{crit} , and then recalculated the gas velocity from droplets smaller than the critical diameter. Their method is a practical and valuable procedure for estimating the gas-phase velocity from PDA data, especially when no independent gas-phase measurement is available. However, it ultimately converts the tracer selection problem into a local diameter cut-off. In the present effervescent spray, droplets of the same size may have different hydrodynamic histories, and small droplets may originate from breakup, clustering, or intermittent structures, rather than from a locally equilibrated tracer population. Therefore, the present method does not assume that all droplets below a critical diameter are equally representative of the continuous phase. Instead, it accepts droplets only when they are both dynamically response-capable and part of a coherent local velocity population, which is more appropriate for an effervescent spray in which tracer-like, transitional, and ballistic-like (inertial) droplets may coexist at the same measurement location.

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