

Tomographic Particle Tracking Velocimetry of Flow Around Buoyancy-Driven Transparent Solids

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ABSTRACT

Tomographic particle tracking velocimetry is used to reconstruct the coupled motion, wake, pressure field, and hydrodynamic loading of freely rising acrylic spheres in a refractive-index-matched liquid. The experiment targets a canonical buoyancy-driven fluid-structure interaction problem in which the body motion and wake evolution are strongly coupled, but direct measurement of the forces on the freely moving body remains challenging. Refractive-index matching (RIM) enables volumetric measurements around the sphere with minimal optical distortion; however, it also removes the image contrast needed to identify the transparent solid boundary. To overcome this limitation, we present an integrated RIM tomo-PTV framework in which the sphere trajectory is inferred directly from the reconstructed flow. The detection method combines three complementary signatures: the tracer-density void created by the excluded solid volume, the near-body vertical-velocity pattern induced by the no-slip surface, and the vortical wake structure identified using the Q -criterion. The recovered trajectory is then used for pressure integration and force estimation from the measured Lagrangian acceleration field. Experiments on spheres of diameter $d = 7.93, 9.53$, and 11.11 mm rising in a sodium iodide RIM solution at $Re_T = 1760\text{--}2880$ resolve wake evolution, surface pressure, and drag and lift histories over a half-cycle of the sphere motion. The measurements distinguish the nearly rectilinear two-ring wake regime of the smaller spheres from the asymmetric four-ring wake of the largest sphere. We also introduce Lagrangian-to-Eulerian reconstruction with Dynamic Masking (LE-DM), a moving-boundary reconstruction procedure that embeds the detected sphere surface into the Eulerian solve and enforces no-slip at the moving boundary. Compared with standard masked interpolation, LE-DM improves near-surface velocity consistency and yields a smoother surface-pressure reconstruction. The combined approach extends RIM tomo-PTV from a flow-only diagnostic to a coupled body-wake measurement framework for transparent moving solids.

1. Introduction

The freely rising sphere is a canonical fluid-structure interaction problem in which a geometrically simple body generates a highly coupled, three-dimensional wake. Its trajectory is governed not

only by buoyancy and inertia, but also by the unsteady vortical structures that form in its wake and feed back into the instantaneous drag and lift. Depending on the Reynolds number, Galileo number, density ratio, and moment of inertia, freely rising or falling spheres may follow rectilinear, oblique, oscillatory, helical, or chaotic paths (Jenny et al., 2004; Ern et al., 2012). These path regimes are closely linked to the wake topology. At lower Reynolds numbers, the wake is dominated by periodically shed, symmetric counter-rotating vortex rings, commonly described as the two-ring (2R) mode. At higher Reynolds numbers, the wake becomes asymmetric, the sphere undergoes lateral excursions, and the shedding involves multiple interacting ring structures associated with the four-ring (4R) mode (Horowitz and Williamson, 2010). Direct experimental access to the forces responsible for these regimes remains limited because it requires time-resolved measurements of the body position, the surrounding three-dimensional velocity field, and the pressure distribution on a freely moving surface.

Tomographic PIV and particle tracking velocimetry can, in principle, provide the volumetric flow information needed for such measurements. In practice, however, measurements near a moving curved body are often compromised by refraction, reflection, and scattering at the solid-fluid interface. Refractive-index matching (RIM) alleviates these limitations by matching the refractive index of the working fluid to that of the solid, thereby allowing light to pass through the interface with minimal distortion. Combined with fluorescent tracer particles and optical filtering, RIM enables high-quality volumetric measurements in the vicinity of transparent bodies (Bai and Katz, 2014; Poelma, 2020).

The same optical arrangement that improves the flow measurement also creates the a major obstacle: the solid body becomes nearly invisible in the raw images. Under refractive-index-matched conditions, especially with fluorescent particles and long-pass filters, the sphere has no detectable edge, silhouette, or surface signature. Standard image-based tracking, visual-hull reconstruction (Adhikari and Longmire, 2012), and surface-marker approaches (Klein et al., 2012) are therefore not applicable without altering the body or the optical configuration. The measurement contains the desired flow information, but the moving boundary that generates the flow is not directly observed.

This paper addresses that limitation through an integrated experimental and reconstruction framework. Section 3 describes a physics-informed sphere detection method that estimates the body location from the reconstructed particle and velocity fields rather than from the images. The method combines three independent flow signatures in a single optimization problem and recovers the sphere trajectory with sub-diameter accuracy (Jose et al., 2026a). Section 4 uses the detected trajectory to reconstruct wake topology, surface pressure, and time-resolved hydrodynamic loading for three sphere diameters spanning two wake regimes. Section 5 introduces Lagrangian-to-Eulerian reconstruction with Dynamic Masking (LE-DM), a moving-boundary extension of constrained Eulerian reconstruction that imposes the no-slip condition at the detected sphere surface (Jose et al.,

2026b). Together, these components provide a route to coupled body-wake measurements in RIM tomo-PTV without markers, tethers, or direct optical detection of the solid boundary.

2. Experimental configuration and volumetric tracking

Experiments were performed in an octagonal acrylic tank with side length 110 mm and height 700 mm, filled with approximately 62 wt% aqueous sodium iodide (NaI). The NaI concentration was adjusted to match the refractive index of acrylic at $n = 1.489$, so that clear acrylic spheres became effectively invisible when immersed. Three sphere diameters were considered, $d = 7.93, 9.53$, and 11.11 mm, all with density ratio $m^* = \rho_s/\rho_f = 0.65$. The spheres rose through initially quiescent fluid after release from a nozzle located 50 mm below the measurement volume, allowing them to enter the imaged region at or near terminal velocity. The corresponding terminal Reynolds numbers were $Re_T \approx 1760, 2170$, and 2880 , with Galileo numbers $Ga \approx 848, 1346$, and 2397 . These conditions span the transition from nearly axisymmetric to non-axisymmetric wake dynamics.

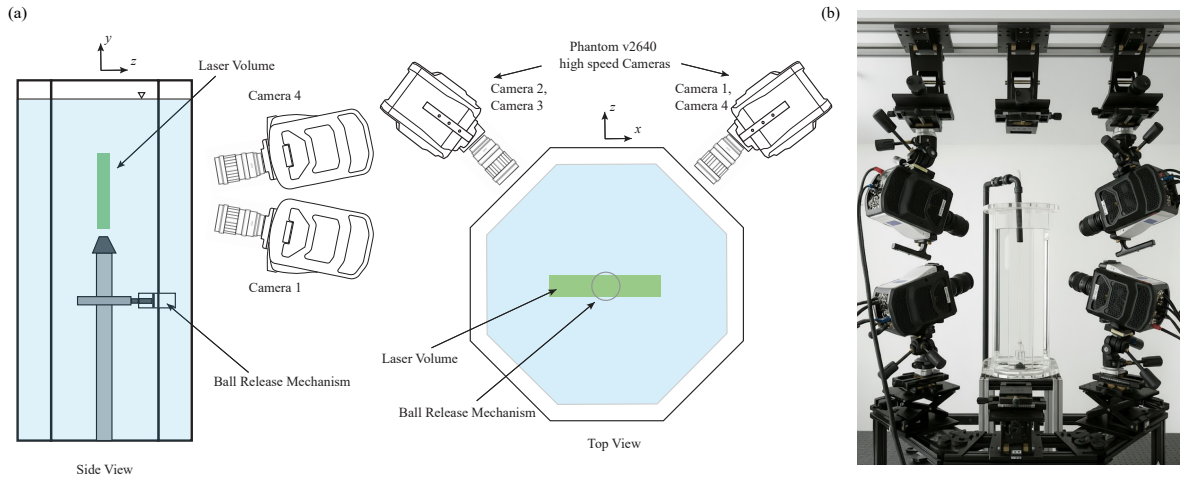


Figure 1. Refractive-index-matched tomographic PTV setup. Schematic (left) and photograph (right) of the octagonal tank, sphere release mechanism, dual-head Nd:YAG laser volume, and four Phantom v2640 cameras.

The tank was seeded with Rhodamine-B-coated PMMA tracer particles with diameters of $1\text{--}20\ \mu\text{m}$ at a concentration of approximately $0.4\ \text{particles}/\text{mm}^3$, corresponding to roughly 80,000-100,000 particles in the $110 \times 75 \times 25\ \text{mm}$ measurement volume. Four Phantom v2640 high-speed cameras, each equipped with a 550 nm long-pass filter, imaged the fluorescent tracer emission. Volumetric illumination was provided by a Photonics DM-100 dual-head Nd:YAG laser at 532 nm, and image sequences were acquired at 4800 Hz. A multi-plane dot target traversed through the illuminated depth was used for camera calibration, followed by volumetric self-calibration that reduced the mapping error to below 0.08 pixels. Lagrangian particle trajectories were reconstructed using Shake-the-Box (STB) (Schanz et al., 2016); only tracks persisting for at least four consecutive frames were retained. The resulting particle tracks were interpolated onto a Cartesian grid with

689 μm spacing using VIC# (Schneiders and Scarano, 2016), which provides a regularized Eulerian velocity field together with vorticity and Q -criterion fields at each time step.

3. Flow-informed detection of the transparent sphere

The principal measurement challenge is that the sphere, while dynamically active, produces no direct optical signature in the recorded images. We therefore infer its position from the flow field that it generates. For a sphere of known diameter, three physically distinct signatures can be extracted from the reconstructed particle distribution and velocity field and used as spatial constraints on the sphere center.

The first signature is the tracer-density void associated with the excluded solid volume. Since tracer particles cannot occupy the interior of the sphere, the STB reconstruction contains a local depletion of particles at the sphere location. This void is not perfectly sharp, because ghost particles, reconstruction noise, and wake-induced particle depletion can partially obscure the boundary. Nevertheless, it provides the most direct geometric constraint. For a candidate center $\mathbf{x}$ and known radius R , the normalized void density is

$$D(\mathbf{x}) = \frac{1}{V(R)} \sum_{i=1}^N I(\|\mathbf{P}_i - \mathbf{x}\| < R), \quad V(R) = \frac{4}{3}\pi R^3, \quad (1)$$

where $\mathbf{P}_i$ is the position of the i th reconstructed particle and I is the indicator function. Lower values of $D(\mathbf{x})$ indicate that fewer particles lie inside the candidate sphere volume, as expected for the true body location.

The second signature is the near-body vertical-velocity pattern. The no-slip condition entrains fluid upward near the sphere surface, while mass conservation produces a compensating annular downwash farther from the body. This upwash-downwash structure is visible in sectional maps of V_y and provides a constraint on the sphere center and mid-plane. It is particularly useful when the particle-density void is degraded by sparse seeding or by wake-induced particle depletion.

The third signature is the vortical structure of the wake. The Q -criterion is elevated in the shear layer and vortex rings generated behind the sphere, whereas it remains low inside the solid body. A correct sphere location should therefore correspond to low Q within the candidate solid volume and enhanced Q in the surrounding wake. This constraint becomes especially effective once coherent wake structures have developed.

The three contributions are combined into a single objective function,

$$J_s(\mathbf{x}) = D(\mathbf{x}) + w_1 [-V_{\text{inner}}(\mathbf{x}) + V_{\text{outer}}(\mathbf{x})] + w_2 [Q_{\text{inner}}(\mathbf{x}) - Q_{\text{outer}}(\mathbf{x})], \quad (2)$$

where D is the normalized void density of Eq. 1, V_{inner} and V_{outer} are the mean vertical velocities in inner and outer hemispherical regions centered at $\mathbf{x}$, and Q_{inner} and Q_{outer} are the average

Q -criterion values in concentric spherical regions around $\mathbf{x}$. The weights w_1 and w_2 balance the velocity and vortex contributions against the void density term. Because the three indicators arise from different physical mechanisms, their combination reduces the likelihood of false minima that can occur when any single indicator is used in isolation. The objective function is minimized using a conjugate-gradient descent procedure. The first frame is initialized manually, while subsequent frames are initialized from the previously detected center with an allowance for the maximum expected rise distance. This temporal propagation restricts the search region, improves convergence, and suppresses spurious jumps. The calibrated weights $w_1 = 3.077$ and $w_2 = 4.415$ gave stable convergence for all sphere sizes, with negligible sensitivity to moderate perturbations. The mean positional uncertainty, estimated from the local Hessian of the cost function at convergence, is below $1.2\%d$, with a maximum below $8.1\%d$ across all datasets (Jose et al., 2026a).

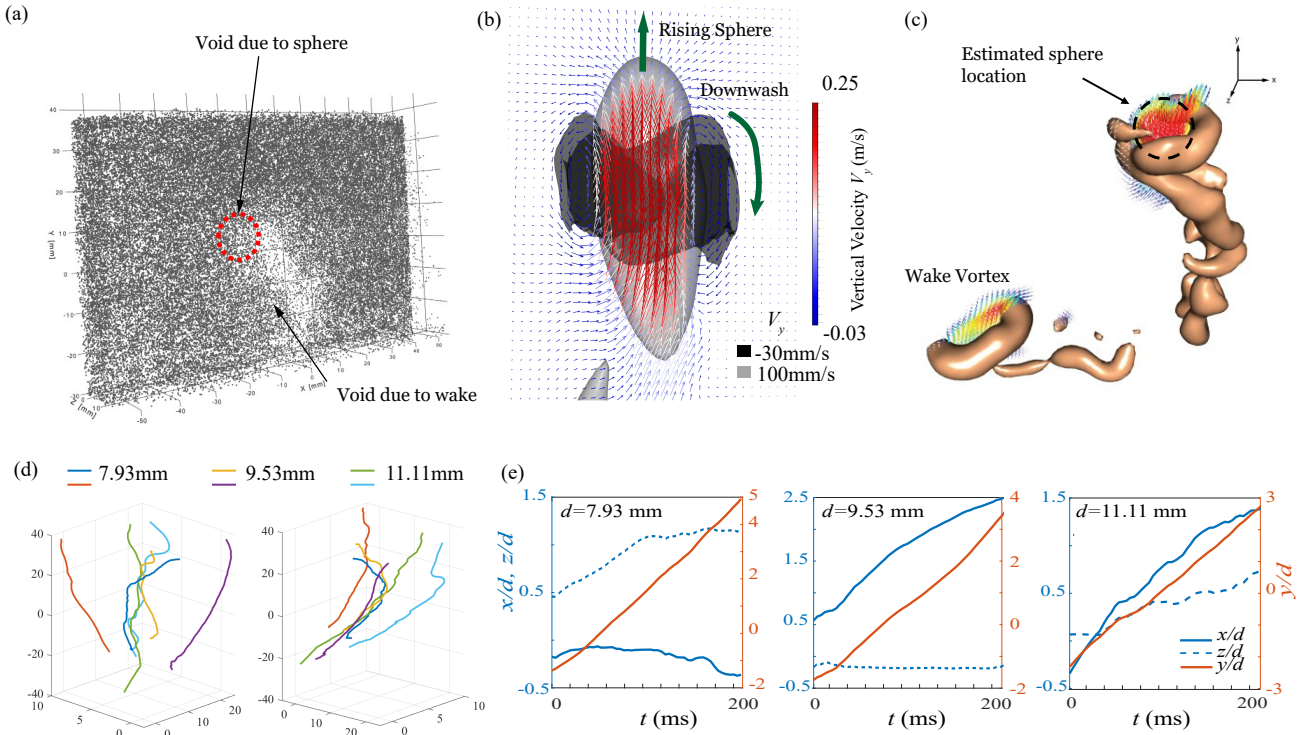


Figure 2. Detection signatures and recovered trajectories. (a) STB particle reconstruction with the detected sphere location marked (red dashed circle); the downstream void produced by the wake is also labelled. (b) Iso-surfaces of vertical velocity V_y around the sphere, showing the inner upwash region and the outer downwash shell. (c) Q -criterion iso-surface around the sphere and wake. (d) Three-dimensional trajectories for all three sphere diameters plotted from a common origin, shown from two viewing angles. (e) Normalized coordinate time histories x/d , y/d , z/d for each diameter.

Figure 2 shows the three detection signatures and the recovered trajectories. Panel (a) plots the STB particle reconstruction at a representative instant. The local depletion of tracers in the immediate vicinity of the sphere is clearly visible, together with a broader wake-induced void extending downstream. Panel (b) shows iso-surfaces of the vertical velocity V_y in concentric volumes around the sphere. The inner red region captures the upward entrainment of fluid by the rising sphere

through the no-slip condition, while the outer dark shell marks the annular downwash imposed by mass conservation. Panel (c) shows the Q -criterion iso-surface, with elevated values concentrated in the shear layer and the developing wake and near-zero values inside the solid region. The three signatures are spatially co-located at the same physical sphere position, confirming that the independent indicators reinforce each other in the cost function and reduce the likelihood of false minima. The mutual consistency of the three indicators across all snapshots is the principal check on detection robustness, since no independent ground-truth trajectory is available (Jose et al., 2026a).

The recovered trajectories in panels (d) and (e) reveal a clear diameter dependence. The 7.93 mm and 9.53 mm spheres follow nearly rectilinear paths with small lateral excursions, with motion confined predominantly to a single vertical plane. In contrast, the 11.11 mm sphere shows appreciable displacement in both lateral directions over the same time interval, characteristic of a fully three-dimensional path. This difference between rectilinear and zigzagging trajectories is consistent with the wake-regime transition examined quantitatively in the force and pressure analysis below.

4. Wake topology, pressure, and hydrodynamic loading

Once the sphere trajectory has been determined, the measured velocity field can be used to reconstruct pressure and hydrodynamic loading. Instantaneous pressure fields were obtained from the VIC# velocity data using the omni-directional integration method of Wang et al. (2019). The pressure was then integrated over the detected sphere surface to estimate the instantaneous drag and lift. This procedure provides force histories for a freely rising sphere directly from the surrounding flow field, without tethering the body or embedding force sensors.

Figure 3 shows the wake evolution of the 11.11 mm sphere over the observed half-cycle, together with the trajectory coloured by surface pressure and four representative surface-pressure snapshots. Three phases of the wake life-cycle can be distinguished in the Q -criterion sequence. Early in the half-cycle, two elongated counter-rotating streamwise vortex structures form behind the sphere and lengthen as the body rises. This double-thread topology corresponds to an attached asymmetric wake. The surface pressure at $t = 41.67$ ms (first pressure panel) shows a high-pressure region on the upstream face and an extended low-pressure region on the downstream side, with a lateral asymmetry aligned with the stronger of the two streaks. The strong front-to-back pressure difference during this phase corresponds to a positive drag force on the sphere.

The attached streaks subsequently weaken and pinch off. The surface pressure at $t = 81.33$ ms shows a markedly more uniform distribution, indicating that the downstream suction has eased. Between this instant and the next, the streaks detach fully and convect downstream as discrete vortex rings, completing the two-ring formation associated with one 4R half-cycle. At later times,

localized pressure cores reappear close to the sphere surface, marking the initiation of a new near-body vortical structure that will govern the next half-cycle. The trajectory excursions in the lower-left panel track this sequence: lateral drift is largest during the attached-wake phase, where the asymmetric pressure imprint imposes the strongest transverse loading. The full sequence of attached double-thread wake, detachment into vortex rings, and renewed near-body vortex formation is consistent with the 4R wake regime reported by Horowitz and Williamson (2010) at comparable Reynolds number and density ratio.

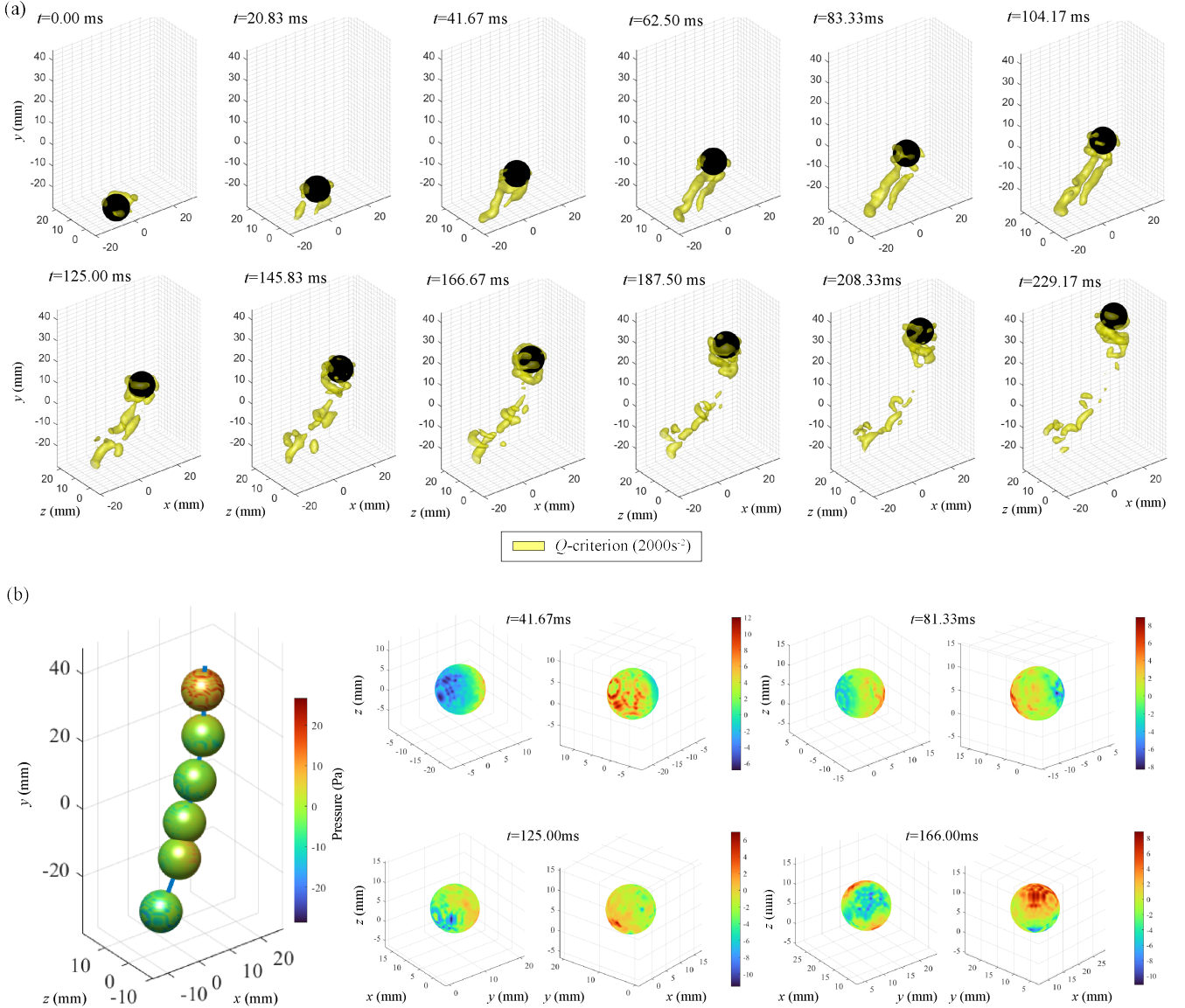


Figure 3. Wake evolution and surface pressure for the 11.11 mm rising sphere ($Re_T \approx 2880$). (top) Q -criterion iso-surfaces ($Q = 1800\text{s}^{-2}$, sphere in black) at 12 equally spaced time instants from $t = 0$ to $t = 229.17$ ms. (bottom left) Three-dimensional trajectory coloured by instantaneous surface pressure. (bottom right) Surface pressure distribution on two orthogonal views of the sphere at four representative time steps.

The corresponding force histories and near-body velocity fields are shown in Figure 4. The drag

and absolute-velocity records in panel (a) vary nearly out of phase, as expected for a buoyancy-driven body whose instantaneous motion is governed by the evolving pressure loading. The drag peaks during the attached double-thread phase, when the elongated low-pressure region behind the sphere exerts sustained streamwise suction, and drops sharply once the streaks pinch off. The sphere then accelerates over the next interval, reaching its maximum rise speed shortly before the next near-body structure begins to form. The lift history in the same panel remains positive and of comparable magnitude to the drag fluctuations throughout the recorded half-cycle, reflecting the persistent transverse pressure imbalance from the asymmetric wake. The trajectory inset in panel (b) shows the corresponding lateral excursion: the sphere drifts most strongly during the interval 40–85 ms, which coincides with the largest sustained lift in panel (a).

The instantaneous flow at $t = 62.5$ ms is shown in panels (c)–(e). The sectional V_y field in panel (c) captures both the upward entrainment immediately above the sphere and the counter-rotating vortex pair convecting downstream, with the in-plane vectors confirming the rotational sense of the wake structures. The inset Q -criterion iso-surface places these structures in their full three-dimensional context. Panels (d) and (e) show the lateral and out-of-plane velocity components at the same instant. Both reveal coherent, signed lobes around the body, indicating that the flow is fully three-dimensional rather than confined to a symmetric meridional plane. The magnitudes of V_x and V_z are several times smaller than the rise speed, so the body motion remains predominantly vertical even when the wake itself has lost axisymmetry. The combination of finite lateral velocity components, persistent lift, and lateral trajectory drift is consistent with the asymmetric 4R wake regime.

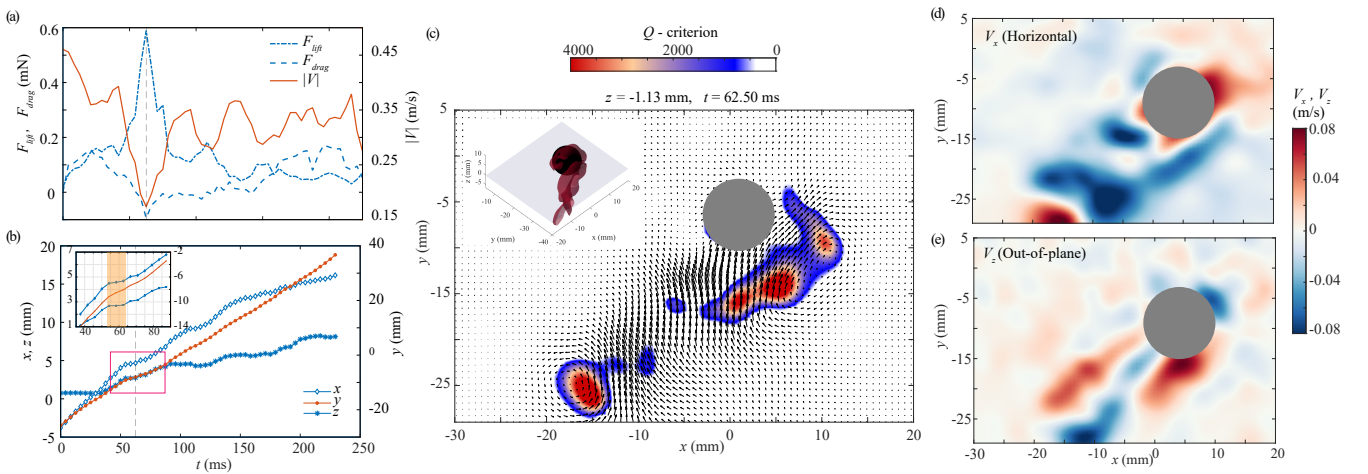


Figure 4. Hydrodynamic forces, trajectory, and near-body flow for the 11.11 mm rising sphere. (a) Temporal histories of drag F_{drag} , lift F_{lift} , and absolute velocity $|V|$. (b) Three-dimensional trajectory with an inset over the interval 40–85 ms. (c) Sectional V_y map with in-plane vectors at $z = -1.13$ mm and $t = 62.5$ ms; inset shows the corresponding Q -criterion iso-surface. (d, e) Horizontal (V_x) and out-of-plane (V_z) velocity maps at the same instant.

The two smaller spheres, $d = 9.53$ and 7.93 mm with $Re_T \approx 2170$ and 1760 , exhibit a different response. Their trajectories remain close to rectilinear, with much smaller lateral excursions than the

11.11 mm sphere. The wake is dominated by a symmetric pair of counter-rotating vortex rings that form and detach periodically, consistent with the 2R mode. In this regime, the transverse force oscillates about zero because the wake remains approximately balanced over successive half-cycles. The drag and sphere velocity are phase-linked, with deceleration and re-acceleration associated with vortex growth and detachment. For the 7.93 mm sphere, the Q -criterion structures remain more compact and closer to the surface, suggesting that the wake dynamics at the lower Reynolds number are governed more strongly by near-body shear than by the larger-scale detached structures observed at higher Re_T . The weaker forces and smaller velocity fluctuations are consistent with reduced vortex-induced loading.

These measurements identify a transition between the wake behavior of the 9.53 and 11.11 mm spheres and demonstrate that the framework resolves the coupled body-wake dynamics with sufficient fidelity to distinguish 2R and 4R shedding regimes using only experimentally reconstructed flow, pressure, and trajectory data.

5. Near-body reconstruction with LE-DM

The results in Section 4 use the VIC# reconstruction described in Section 2, with the sphere trajectory from the detection step used to define the solid region and to integrate pressure over the sphere surface. The underlying VIC# reconstruction, however, does not explicitly account for the moving solid boundary. In standard masked interpolation, grid nodes inside the detected sphere are excluded from the reconstructed fluid domain, but no condition is imposed on the velocity immediately outside the surface. Thus, the reconstructed velocity at near-wall grid nodes is not required to match the sphere velocity, even though the physical flow must satisfy no-slip. This inconsistency affects the near-body velocity gradients and material acceleration, and can therefore propagate into the pressure integration and force estimates. LE-DM, introduced below, is an alternative CCM-based reconstruction (Agarwal et al., 2021) that additionally enforces no-slip at the detected moving boundary.

LE-DM (Lagrangian-to-Eulerian reconstruction with Dynamic Masking) addresses this issue by incorporating the detected moving boundary directly into the Eulerian reconstruction (Jose et al., 2026b). The sphere is represented on the fixed Cartesian grid by the signed-distance function

$$\phi(\mathbf{x}, t_k) = \|\mathbf{x} - \mathbf{X}_s(t_k)\| - R, \quad (3)$$

where $\mathbf{X}_s(t_k)$ is the detected center and R is the sphere radius. Nodes with $\phi < 0$ are classified as solid interior and excluded from the fluid solve, nodes with $\phi > \Delta/2$ are treated as regular fluid nodes, and nodes in the shell $0 \leq \phi \leq \Delta/2$ are treated as boundary nodes adjacent to the moving sphere. At these boundary-shell nodes, LE-DM imposes the sphere translational velocity $\mathbf{U}_s(t_k)$ as a hard constraint, providing a discrete no-slip condition. The no-slip constraints are combined with

one-sided divergence constraints near the wall, so that incompressibility is enforced without finite-difference stencils crossing into the solid. The resulting reconstruction is written as the saddle-point system

$$\begin{bmatrix} A^\top W(t_k)A + \kappa S(t_k) & B(t_k) \\ B(t_k)^\top & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} A^\top W(t_k)\mathbf{y} + \kappa S(t_k)\tilde{\mathbf{u}}_{\text{prev}}(t_k) \\ \mathbf{g}(t_k) \end{bmatrix}. \quad (4)$$

Here A maps the Eulerian grid to the particle-track locations, $\mathbf{y}$ contains the measured track velocities, $W(t_k)$ is a diagonal weighting matrix, $S(t_k)$ provides temporal regularization, and $B(t_k)$ contains the divergence-free and no-slip equality constraints enforced through the Lagrange multipliers $\boldsymbol{\lambda}$. The right-hand side $\mathbf{g}(t_k)$ contains the prescribed constraint values.

Two additional treatments are used to improve robustness near the moving boundary. First, tracks close to the sphere surface are down-weighted in $W(t_k)$ using a proximity-based factor, because these tracks are more likely to be affected by residual reflections, sharp seeding gradients, or uncertainty in the detected surface location. The decay length is set by the detection uncertainty $\sigma_s(t_k)$. Second, nodes that were inside the solid at the previous time step but become exposed to the fluid after the sphere moves are assigned a physically meaningful temporal prior based on the local rigid-body velocity. This prevents newly uncovered fluid nodes from inheriting artificial values associated with the previous solid region.

The resulting velocity field is divergence-free over the fluid domain, satisfies a discrete no-slip condition at the moving sphere surface, and is obtained in a single linear solve at each time step without post-processing corrections. Because the constraint matrix $B(t_k)$ is rebuilt from the detected sphere state at every snapshot, the same formulation can handle arbitrary sphere trajectories. On a synthetic oscillating-sphere benchmark with closed-form ground truth, LE-DM reduces the first-cell velocity error from 14.0% to 2.9% of the oscillation amplitude relative to standard CCM, with similar improvements in the bulk and ring-profile reconstructions (Table 1). These improvements are concentrated within one grid spacing of the sphere surface, where standard masked interpolation is most sensitive to sparse near-body tracks and to the absence of an explicit boundary condition. A full derivation, synthetic validation against an oscillating-sphere solution, and experimental tests on the present rising-sphere dataset are reported in Jose et al. (2026b).

6. Conclusions

We presented an integrated RIM tomo-PTV framework for measuring the coupled motion, wake, pressure field, and hydrodynamic loading of transparent freely rising spheres. The framework combines flow-informed sphere detection, pressure reconstruction from volumetric velocity data, and a moving-boundary-aware Eulerian reconstruction. It therefore addresses a central limitation of refractive-index-matched measurements: the same optical matching that enables undistorted near-body flow measurements also removes the direct image signature of the solid boundary.

Table 1. LE-DM versus standard CCM on the synthetic oscillating-sphere benchmark at Womersley number $Wo = 2$ ($\delta_{BL}/\Delta = 1.96$). Bulk error is over fluid nodes with $\phi > \delta_{BL}$; first-cell error is over the single cell adjacent to the surface; ring RMS is the deviation from the analytical magnitude on a sampling ring at $\phi = 2\Delta$. All errors are normalised by the oscillation amplitude U_0 . Full validation including a Womersley sweep, resolution study, and experimental rising-sphere comparison is reported in Jose et al. (2026b).

Metric	Base CCM	LE-DM
Bulk error / U_0	3.0%	1.1%
First-cell error / U_0	14.0%	2.9%
Ring-profile RMS / U_0	5.4%	1.4%

The sphere detection method recovers the body trajectory from three complementary flow signatures: the particle-density void associated with the excluded solid volume, the near-body vertical-velocity structure induced by the no-slip surface, and the coherent vortical structures in the wake. Their combination yields smooth, physically consistent trajectories for acrylic spheres of diameter 7.93, 9.53, and 11.11 mm rising in a NaI solution, with mean positional uncertainty below 1.2%*d*.

Using the detected trajectory, the measurements provide simultaneous access to wake topology, surface pressure, and time-resolved drag and lift over a half-cycle of the motion. The 11.11 mm sphere at $Re_T \approx 2880$ exhibits an asymmetric 4R wake, including an attached double-thread phase, vortex-ring detachment, reduced downstream suction, and lateral loading associated with three-dimensional wake asymmetry. The smaller spheres at $Re_T \approx 2170$ and 1760 remain closer to rectilinear motion and show dynamics consistent with a symmetric 2R shedding mode.

LE-DM extends the framework by incorporating the detected moving boundary into the Eulerian reconstruction. Unlike standard masking, which removes the solid interior but leaves the interface unconstrained, LE-DM imposes the sphere velocity on a boundary shell, uses one-sided incompressibility constraints near the wall, and updates the constraint structure at each time step. This produces a divergence-free, no-slip-consistent velocity field and improves near-surface pressure continuity relative to masked interpolation.

Together, the detection and LE-DM reconstruction methods extend RIM tomo-PTV from a flow-only diagnostic to a coupled body-wake measurement platform. Although demonstrated here for buoyancy-driven spheres, the approach can be generalized to other moving bodies provided that the body geometry, a signed-distance representation, and a rigid-body velocity estimate are available. This makes the framework relevant to settling/rising particles, non-spherical bodies, and multi-body configurations in refractive-index-matched experiments.

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