

The impact of swirl on a starting vortex from an impulse jet identified in PIV data

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ABSTRACT

In diffusion-limited laminar jet flows, momentum and heat transfer can be enhanced through the addition of swirl or toroidal vortices. This study utilizes Stereoscopic Particle Image Velocimetry (Stereo-PIV) to investigate the properties of a toroidal starting vortex generated by a pulsatile jet. A key focus is the modification of vortex topology via swirl, induced by 3D-printed nozzles with internal vanes. Vortex cores are identified and characterized using a Combinatorial Vortex Detection (CVD) algorithm. To ensure methodological robustness, the study analyzes how PIV data characteristics and processing schemes influence the accuracy of the CVD algorithm. The overall goal is to compare experimental results against theoretical predictions to identify the vortex models that best characterize the vortical structures in these flows.

1. Introduction

The generation of vortex rings via pulsed jets has been a subject of extensive research, where advanced diagnostic techniques such as Particle Image Velocimetry (PIV) and Stereo-PIV have provided deep insights into their structural characteristics and evolution. However, a critical question remains regarding the potential to modulate these structures by superimposing an azimuthal component upon the pulsative flow. Motivated by this gap, the present work investigates the introduction of swirl to a pulsed jet, serving as a fundamental study into the mechanics of mixing induced by large-scale motion within laminar regimes.

A vortex ring (or toroidal vortex) is a persistent, donut-shaped structure where fluid rotates around an imaginary circular axis (poloidal rotation) that forms a closed loop, a phenomenon comprehensively described in classical vortex dynamics (Saffman, 1992). These structures are typically generated by the impulsive ejection of a fluid slug through a sharp-edged orifice; as the fluid exits, the boundary layer separates at the lip, creating a cylindrical shear sheet that rolls up into a

coherent spiral due to the velocity mismatch between the jet and the ambient fluid (Maxworthy, 1977; Gharib *et al.*, 1998).

A highly effective method for generating a continuous train of these structures is the use of a pulsatile jet, defined as a fluid jet where the exit velocity varies periodically with time, often sinusoidally or as a square wave, rather than remaining steady (Glezer, 1988). The dynamics of such unsteady flows are governed by the Womersley number (α), a dimensionless parameter representing the ratio of pulsatile inertial forces to viscous forces (Womersley, 1955). It is mathematically defined as:

$$\alpha = L \sqrt{\frac{\omega}{\nu}}$$

where, L is the characteristic length scale (typically the radius of the pipe or orifice) [m], ω is the angular frequency of the pulsations, calculated as $2\pi f$ where f is the frequency [rad/s] and ν is the kinematic viscosity of the fluid [m²/s]. A high Womersley number ($\alpha > 10$) indicates that the flow frequency is sufficiently high that the velocity profile remains flat or "plug-like" in the center with thin boundary layers, ideal for the sharp separation required for distinct vortex ring formation. Conversely, a low Womersley number ($\alpha < 1$) implies that viscous forces dominate, resulting in a quasi-steady parabolic velocity profile where the fluid has ample time to develop fully during each cycle.

Vortex flows are modeled theoretically by seeking solutions to the Navier-Stokes equations, utilizing methods that range from exact viscous solutions to kinematic approximations and inviscid potential flow theory (Saffman, 1992). Common formulations, including the Rankine (Rankine, 1858), Lamb-Oseen (Oseen, 1911; Lamb, 1932), Burgers (Burgers, 1948), Sullivan (Sullivan, 1959), and Batchelor (Batchelor, 1964) vortices, are not universally applicable but are instead tailored to specific physical regimes. For example, the Rankine model approximates a solid-body core in inviscid flow, while the Burgers vortex specifically accounts for the stretching effects of external strain in viscous fluids. These models are invariably derived from a series of simplifying assumptions, such as axisymmetry, incompressibility, or steady-state behavior, and their utility is fundamentally limited by the fact that there is no rigorous, generally accepted definition of a vortex in fluid mechanics (Lugt, 1979; Jeong & Hussain, 1995). Consequently, a comprehensive understanding of these models and

their specific limitations is required to determine their relevance for complex applications, particularly when modeling the addition of swirl components to a flow field.

Swirl is intentionally introduced into a turbulent jet flow in, for example, combustion systems to significantly enhance the mixing rates of fuel and oxidant and, most critically, to induce a central recirculation zone (CRZ) that acts as an aerodynamic flame holder by transporting hot, chemically active combustion products upstream to ignite fresh reactants (Beer & Chigier, 1972; Candel *et al.*, 2014). This impartation of tangential momentum can be achieved through various passive and active methods detailed by Gupta *et al.* (1984), including the use of tangential entry ports where fluid is injected tangentially into a cylindrical chamber, twisted tape inserts placed within the supply pipe, or rotating mechanical devices such as spinning pipes. However, for practical industrial applications such as gas turbines, the most common method involves the use of stationary swirl vanes located upstream of the nozzle exit to deflect the incoming axial flow (Lefebvre & Ballal, 2010), the intensity of the induced rotation is quantified by a non-dimensional parameter known as the swirl number (S). While fundamentally representing the ratio of angular momentum flux to axial momentum flux, for the specific case of a vane-based generator, S is estimated geometrically based on the nozzle design parameters as defined by (Gupta *et al.*, 1984). The geometric swirl number is calculated as:

$$S = \frac{2}{3} \left[\frac{1 - (R_{hub} - R_{tip})^3}{1 - (R_{hub} - R_{tip})^2} \right] \tan(\theta)$$

where θ denotes the angle of the internal vanes relative to the flow axis, and the nozzle geometry is defined by the central hub radius (R_{hub}) and the outer tip radius (R_{tip}).

The flow characteristics downstream of the nozzle are heavily dependent on the magnitude of the swirl number, which is generally categorized into weak and strong swirl regimes. In the weak swirl regime ($S < 0.6$), the axial momentum dominates; while the swirling motion increases the spreading angle of the jet and improves entrainment compared to a non-swirling jet, the radial pressure gradient is insufficient to cause flow reversal along the centerline (Beer & Chigier, 1972). Conversely, in the strong swirl regime ($S > 0.6$), the strong centrifugal forces create a radial pressure gradient that results in a sub-atmospheric low-pressure region along the axis. When this adverse axial pressure gradient overcomes the axial momentum, the flow undergoes vortex breakdown, triggering

the formation of a large central toroidal recirculation zone (CTRZ) (Gupta et al., 1984). It is this strong recirculation zone that is essential for high-intensity combustion, as it anchors the flame and prevents blow-off (Lefebvre & Ballal, 2010).

While swirl is most commonly associated with turbulent combustion, it is also a critical mechanism for enhancing mixing in laminar jets, where mixing is otherwise limited to slow molecular diffusion. In the laminar regime, adding swirl promotes the onset of hydrodynamic instabilities, specifically centrifugal and shear instabilities, that would not exist in a purely axial laminar jet. Billant and Chomaz (1998) demonstrated that even at low Reynolds numbers, a sufficiently high swirl ratio triggers vortex breakdown, manifesting as distinct topological states such as the "bubble" or "spiral" breakdown. These breakdown structures create stagnation points and internal recirculation regions within the laminar flow. This process stretches and folds the fluid interfaces, dramatically increasing the interfacial area available for diffusion and transitioning the flow from simple linear mixing to chaotic mixing (Billant & Chomaz, 1998). This is particularly valuable in micro-combustion or chemical processing applications where turbulence cannot be sustained, yet rapid mixing is required.

With the potential presence of large-scale flow features in the generated flow field, a method for characterising large-scale vortices in PIV data is needed. Bussière *et al.* (2024) introduce a Combinatorial Vortex Detection (CVD) algorithm specifically designed to identify and characterize large-scale vortical structures in the wake of an oscillating wing using 2-component, 2-dimensional (2C-2D) Particle Image Velocimetry (PIV) data. Unlike traditional identification methods such as the Q-criterion (Hunt *et al.*, 1988) or λ_2 -criterion (Jeong, & Hussain, 1995), which typically require full 3D velocity gradient tensors and rely on subjective, arbitrary thresholds that can lead to significant uncertainty in experimental data, the CVD algorithm integrates multiple detection strategies, including a maximum vorticity method and a streamline winding angle method, to robustly confirm vortex cores. This combinatorial approach effectively filters out shear contamination and false positives inherent in single-parameter methods, allowing for the precise quantification of vortex properties such as circulation, core radius based on the last closed stream line, and trajectory even within the noisy and spatially limited constraints of planar PIV measurements.

The primary objective of this study is to examine the effects of swirl on the topology and evolution of a starting toroidal vortex generated by a pulsatile jet. The flow fields will be resolved using Stereo-PIV. To quantify the vortex dynamics, the Combinatorial Vortex Detection (CVD) algorithm will be applied to data sets acquired from both non-swirling and swirling nozzle configurations (utilizing a vane generator). Given that the efficacy of the CVD algorithm is dependent on the fidelity of the velocity data, a sensitivity analysis of the parameters influencing the PIV results is also included. The characterisation of these large-scale vortices will be used to assess the potential used of different vortex models in the literature for charactering the effect of swirl.

2. Theory: Modeling of Vortices

A comparison of the different models of a vortex available in the literature are outlined in Table 1. Listed are the relevant equations for the tangential velocity (v_θ) then available for the different solution. Definitions for the variables used follow those used in the original works with a full list of variables definitions available in the appendix.

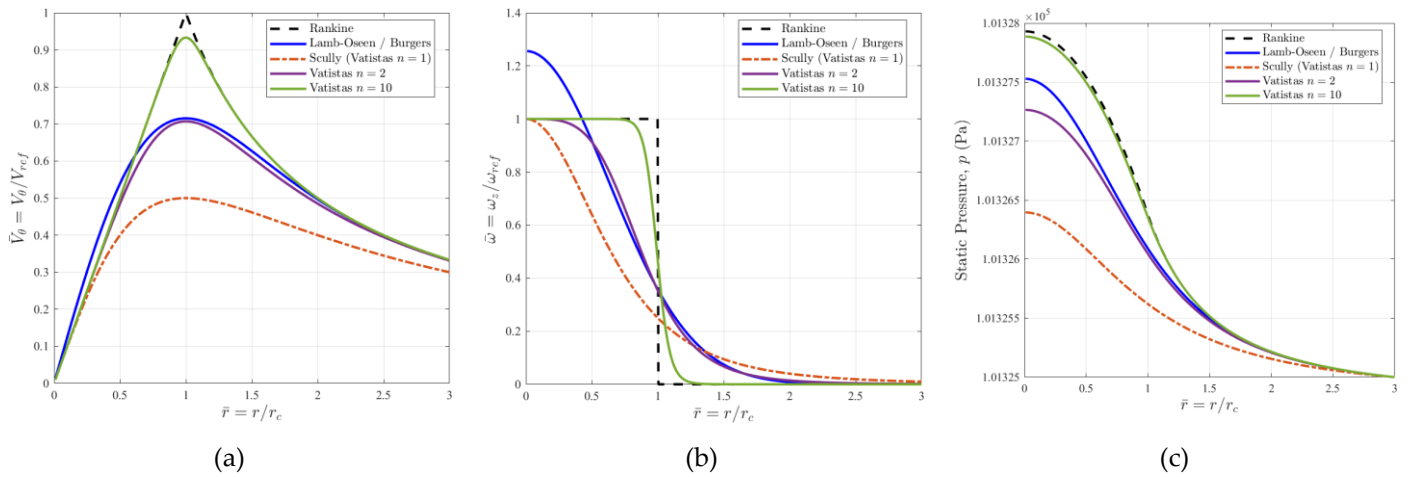


Figure 1. Plots comparing the different vortex models for the distributions of (a) tangential velocity (V_θ), (b) vorticity (ω_z) and (c) static pressure (p).

Figure 1 illustrates the radial distributions of (a) tangential velocity ($\bar{V}_\theta$), (b) axial vorticity ($\bar{\omega}$) and (c) static pressure (p) for the theoretical vortex models detailed in Table 1. Consistent with the fundamental assumptions listed in Table 1. A list of vortex models available in the literature, all presented models presume an axisymmetric, steady, and incompressible flow field where the vortices are perfectly circular with an aspect ratio of 1 (implying no elliptic distortion or stretching). The results highlight significant variations in the physical realism of the core region ($r/r_c < 1$).

Table 1. A list of vortex models available in literature

Model Name	Relevant Equations (Tangential Velocity v_θ)	Fluid Conditions & Assumptions	Notes (Nature of Solution)	Citation
Rankine Vortex	$v_\theta(r) = \begin{cases} \frac{\Gamma r}{(2\pi r_c^2)} r \\ \leq r_c \frac{\Gamma r}{(2\pi r_c)} \quad r > r_c \end{cases}$	• Inviscid (mostly) • Incompressible • Steady-state • Assumes a solid-body rotation core matched to a potential flow outer field.	Kinematic Approximation Physically unrealistic due to discontinuous vorticity at the core radius R . Useful for simple estimations.	Rankine, W. J. M. (1858).
Lamb-Oseen Vortex	$v_\theta(r, t) = \frac{\Gamma}{(2\pi r)} \left(1 - e^{-\frac{r^2}{4\nu t}}\right)$	• Viscous ($\nu > 0$) • Incompressible • Unsteady (time-dependent decay) • Initial condition is a potential line vortex (δ-function) at $t = 0$.	Exact Solution to Navier-Stokes Describes the diffusion of a line vortex over time due to viscosity. The vorticity distribution is Gaussian.	Oseen, C. W. (1911). Lamb, H. (1932).
Burgers Vortex	$v_\theta(r) = \frac{\Gamma}{(2\pi r)} \left(1 - e^{-\frac{ar^2}{2\nu}}\right)$	• Viscous • Incompressible • Steady-state • Assumes an axisymmetric strain field (a) stretches the vortex to counteract viscous diffusion.	Exact Solution to Navier-Stokes Represents a "one-cell" structure. Widely used to model fine-scale structures in turbulence (worms).	Burgers, J. M. (1948).
Sullivan Vortex	$v_\theta(r) = \frac{\Gamma}{(2\pi r)} \frac{H\left(\frac{ar^2}{2\nu}\right)}{H(\infty)}$	• Viscous • Incompressible • Steady-state • Similar strain field to Burgers but allows for more complex axial flow.	Exact Solution to Navier-Stokes Represents a "two-cell" structure. Unlike Burgers, this features a region of reverse flow (downdraft) within the core.	Sullivan, R. D. (1959).
Kaufmann (Scully) Vortex	$v_\theta(r) = \frac{\Gamma}{(2\pi r)} \frac{r^2}{r^2 + r_c^2}$	• Inviscid (generally) • Incompressible • Steady-state • Chosen to fit experimental data where velocity profiles are smooth.	Kinematic Approximation Algebraic profile (unlike the exponential Lamb-Oseen). No singularities. Standard in helicopter rotor wake analysis.	Kaufmann, L. G. (1962). Scully, M. P. (1975).
Batchelor (q -vortex)	$v_\theta(r) = \frac{V_{max} r_c}{r} \left(1 - e^{-\left(\frac{r}{r_c}\right)^2}\right)$	• Viscous • Incompressible • Steady (in moving frame) • Boundary layer (slender-flow) approximation.	Similarity Solution (Viscous) Models trailing vortices (e.g., wing tips). Includes an axial velocity defect or jet (ΔU) alongside the swirl.	Batchelor, G. K. (1964).
Vatistas Vortex	$v_\theta(r) = \frac{\Gamma}{(2\pi r_c)} \frac{\frac{r}{r_c}}{\left[1 + \left(\frac{r}{r_c}\right)^{2n}\right]^{\frac{1}{n}}}$	• Parametric / Kinematic • Incompressible • Steady-state • Variable n allows tuning between different vortex "shapes".	Kinematic Approximation A generalized family of profiles. $n = 1$ recovers the Scully vortex; $n = \infty$ recovers the Rankine vortex.	Vatistas, G. H. et al. (1991).

The Rankine model represents an idealized inviscid limit, characterized by a linear "solid-body" velocity profile and a discontinuous "top-hat" vorticity distribution. In contrast, the Lamb-Oseen and Vatistas models exhibit continuous, smooth transitions across the core radius, which better reflect the effects of viscosity and turbulence. The Vatistas family specifically demonstrates the geometric sensitivity of the vortex core; as the shape parameter increases from $n = 1$ (Scully) to $n = 10$, the velocity profile steepens and the vorticity distribution transitions from a gradual decay to a near-

vertical drop, asymptotically approaching the Rankine solution. Consequently, the core pressure deficit shown in Figure 1(c) is most severe for the "sharper" profiles (Rankine and Vatistas $n = 10$), while the more diffuse vorticity distributions of the Scully and Lamb-Oseen models result in a shallower pressure gradient.

The Lamb-Oseen vortex (Oseen, 1911; Lamb, 1932) serves as a fundamental exact solution to the Navier-Stokes equations, describing the unsteady viscous decay of a line vortex where vorticity diffuses outward in a Gaussian distribution over time. To model the steady-state "worm" structures observed in turbulent flows, Burgers (1948) modified this approach by superimposing an external radial inflow and axial strain field; this stretching mechanism counteracts viscous diffusion, effectively locking the vortex core at a constant radius. Sullivan (1959) subsequently generalized the Burgers solution to accommodate more complex axial dynamics, discovering a "two-cell" solution that permits a region of reverse flow (downdraft) along the axis, unlike the unidirectional updraft of the "one-cell" Burgers model.

Transitioning to aerodynamic applications, the Kaufmann (1962) vortex, often cited via Scully (1975), was developed specifically for helicopter rotor wakes, utilizing a simple algebraic profile to fit experimental data and eliminate the central singularity without relying on a specific viscous time-history. Similarly, the Batchelor (1964) (or q -vortex) model was derived to describe trailing wing-tip vortices, incorporating an axial velocity defect or jet alongside the swirl to account for the wake deficit in high-Reynolds-number flows. Finally, the Vatistas (1991) model synthesizes these kinematic approaches into a generalized algebraic framework; by introducing a tunable shape parameter n , it creates a versatile family of profiles that can smoothly interpolate between the broad, diffusive core of the Scully vortex ($n = 1$) and the sharp, solid-body transition of the Rankine limit ($n = \infty$), making it a robust tool for fitting a wide variety of experimental vortex structures.

3. Experimental Setup

3.1. Experimental Facility and PIV Setup

The experiments were conducted within a custom-fabricated hexagonal water tank featuring a side length of 18 cm and a height of 35 cm, designed to provide full optical access for flow visualization. Flow generation was driven by a piston-cylinder mechanism (40 mm piston diameter, 100 mm length) coupled to a stepper motor. To strictly regulate the flow cycle, two check valves were integrated into the hydraulic circuit one positioned on the suction line and the other on the injection line, Figure 2. This arrangement ensured unidirectional flow: during the suction phase, the injection line remained sealed to prevent fluid intake from the nozzle, while during the injection phase, the suction line was closed to preclude backflow, thereby forcing the fluid exclusively through the nozzle located at the top of the test section. For the data presented, the piston executed a 25 mm stroke over a duration of 1.0 s. Image acquisition was triggered at $t = 0.4$ s into the injection phase. Based on these kinematic parameters and the nozzle geometry, the flow Reynolds number was calculated to be $Re = \frac{\rho V_{outlet} D_{out}}{\mu} \approx 2000$. Three-component velocity measurements were obtained using a Stereoscopic Particle Image Velocimetry (Stereo-PIV) configuration. Two LaVision Imager Pro X 4M cameras were positioned with a separation angle of $\sim 60^\circ$ to ensure optimum 3D motion capture. The cameras featured a sensor resolution of 2048×2048 pixels and a $\Delta t_{piv} = 1000 \mu s$ between the two pulses from the 532 nm Nd:YAG laser.

Calibration was performed using a 3D calibration target (Type 11, LaVision GmbH), and a 3rd-order polynomial method was applied to map the physical coordinates. The time-averaged flow field was computed from 20 instantaneous image pairs. Data processing was conducted using a stereo cross-correlation algorithm with a multi-pass interrogation scheme of decreasing window sizes: two passes at 64×64 pixels followed by two passes at 32×32 pixels, both with a 50% overlap.

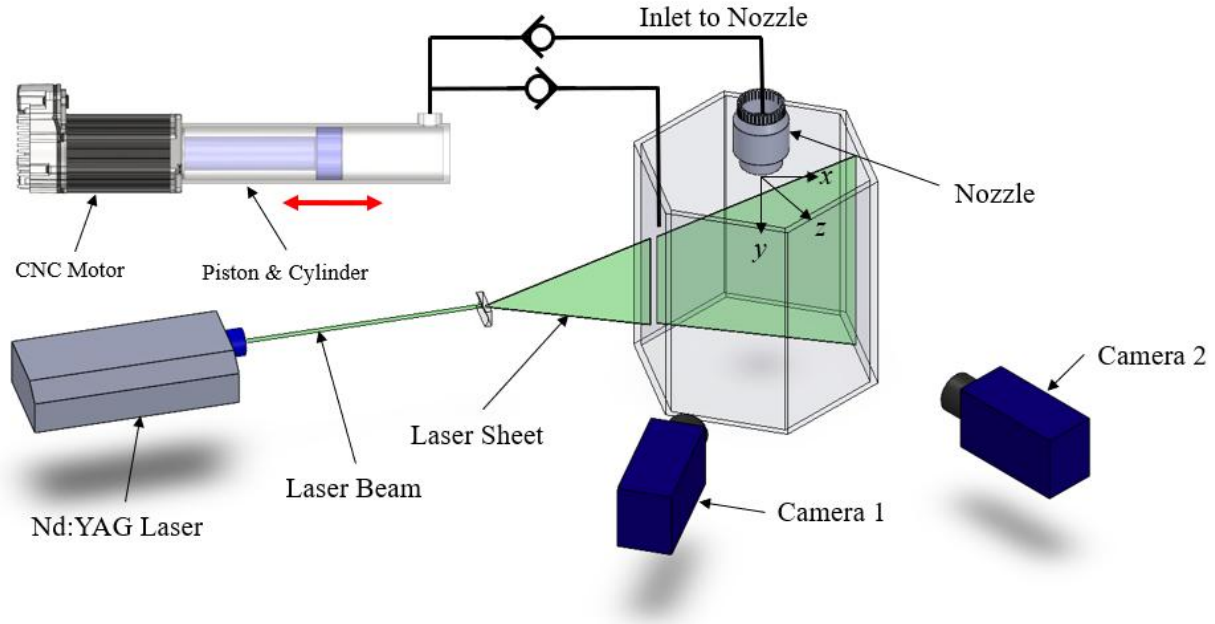


Figure 2. Layout of the experimental apparatus illustrating the piston-driven vortex generator and the optical arrangement for Stereoscopic PIV measurements.

3.2. Nozzle and Swirler Design

The nozzle used in experiments is shown in Figure 2. The assembly consisted of a main housing and an internal swirler, both of which were fabricated via 3D printing to ensure complex geometric fidelity. The 50 mm diameter swirler was rigidly fixed within the nozzle using a dual-notch locking mechanism, Figure 3(a). This component was designed with a height of 63.5 mm and vanes featuring a three-quarter revolution twist, yielding a geometric swirl number of $S \sim 1.22$, Figure 3(b). Downstream of the swirler, a 5.5 mm vertical settling gap was maintained before the flow entered the contraction section. The nozzle geometry narrowed through a 30° slope to a final outlet diameter of 20 mm, Figure 3(a); this convergence angle was selected to accelerate the flow and enhance shear generation at the exit. Finally, the experimental coordinate system was established with the origin positioned immediately downstream of the nozzle exit plane, aligned with the geometric centerline of the assembly.

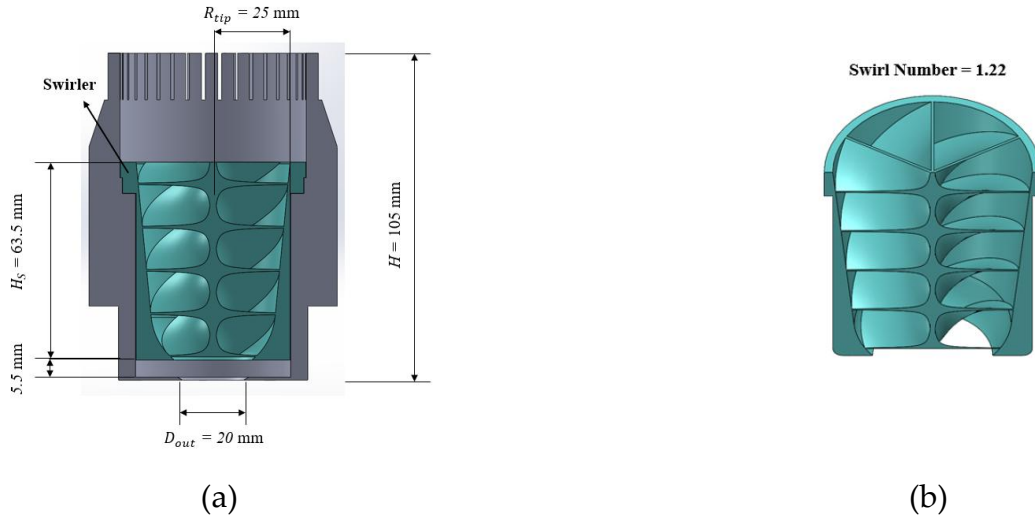


Figure 3. Design details of the experimental nozzle. (a) Dimensional drawing of the nozzle housing and internal swirler ($D_{out} = 20 \text{ mm}$). (b) Cross-section of the helical insert corresponding to a swirl number of 1.22.

4. Results and Discussion

An illustration the time-resolved evolution of the flow field and vorticity contours (ω) at three representative non-dimensional time steps ($t/t_s = 0.3, 0.5$, and 0.7) during a 1.0 s piston stroke is shown in Figure 4. The left column (panels a, c, and e) depicts the baseline flow development through a regular orifice without a swirler, while the right column (panels b, d, and f) presents the modified flow topology under the influence of a swirler. To minimize the confounding effects of cycle-to-cycle variations, localized turbulent fluctuations, and experimental unsteadiness, each presented time step is obtained via an ensemble average of 100 instantaneous frames. The image data was computed using a multi-grid processing scheme PIV consisting of two passes. For presentation purposes, the velocity vector fields shown here have been uniformly down-sampled relative to the original high-density computational grid to ensure visual clarity and prevent spatial crowding.

A comparative analysis of the flow topologies reveals distinct differences in vortex dynamics and structural stability between the two configurations. In the baseline case without a swirler (left column), a highly symmetric, coherent pair of counter-rotating primary vortices forms near the orifice and propagates linearly downstream in the negative y -direction. The vorticity cores remain intensely concentrated and well-defined, preserving their structural integrity even as they reach the lower domain ($y = -35 \text{ mm}$) at $t/t_s = 0.7$. Conversely, the introduction of the swirler (right column) significantly alters the jet evolution. While a primary vortex structure is initially visible at $t/t_s = 0.3$,

pronounced destabilization and vortex breakdown occur as the stroke progresses to $t/t_s = 0.5$ and 0.7 . The induced azimuthal velocity component triggers rapid core dissipation, shear-layer asymmetry, and the generation of secondary, small-scale vorticity fluctuations along the centerline. This structural disintegration highlights the swirler's effectiveness in accelerating flow breakdown and enhancing spatial mixing within the fluid domain.

To quantify the three-dimensional dynamics within the vortex ring structure, an automated core tracking and extraction routine was implemented. The algorithm first detects the primary vortex core centers and computes the corresponding core boundaries and radii (R_c). For each identified core region, the local out-of-plane velocity (V_z) components are spatially isolated and averaged. To capture the overall magnitude of the axial motion regardless of its direction, the absolute values of these individual core averages are subsequently averaged to yield a single representative metric, denoted as the area-averaged $|V_z|$. Applying this analysis to the swirled flow condition (Figure 5 b, d, f) reveals that the area-averaged absolute out-of-plane velocity peaks during the early stages of evolution at a non-dimensional time of $t^* = 0.30$, reaching a substantial value of 0.0376 m/s. As the vortex system propagates downward, this out-of-plane motion experiences a moderate decay by $t^* = 0.50$ to a value of 0.0348 m/s, before undergoing a sharp reduction down to 0.0136 m/s at $t^* = 0.70$. In contrast, the unswirled benchmark flow (Figure 5 a, c, e) consistently registers minimal area-averaged $|V_z|$ values, remaining tightly constrained between 0.006 m/s and 0.009 m/s throughout the entire observed timeline.

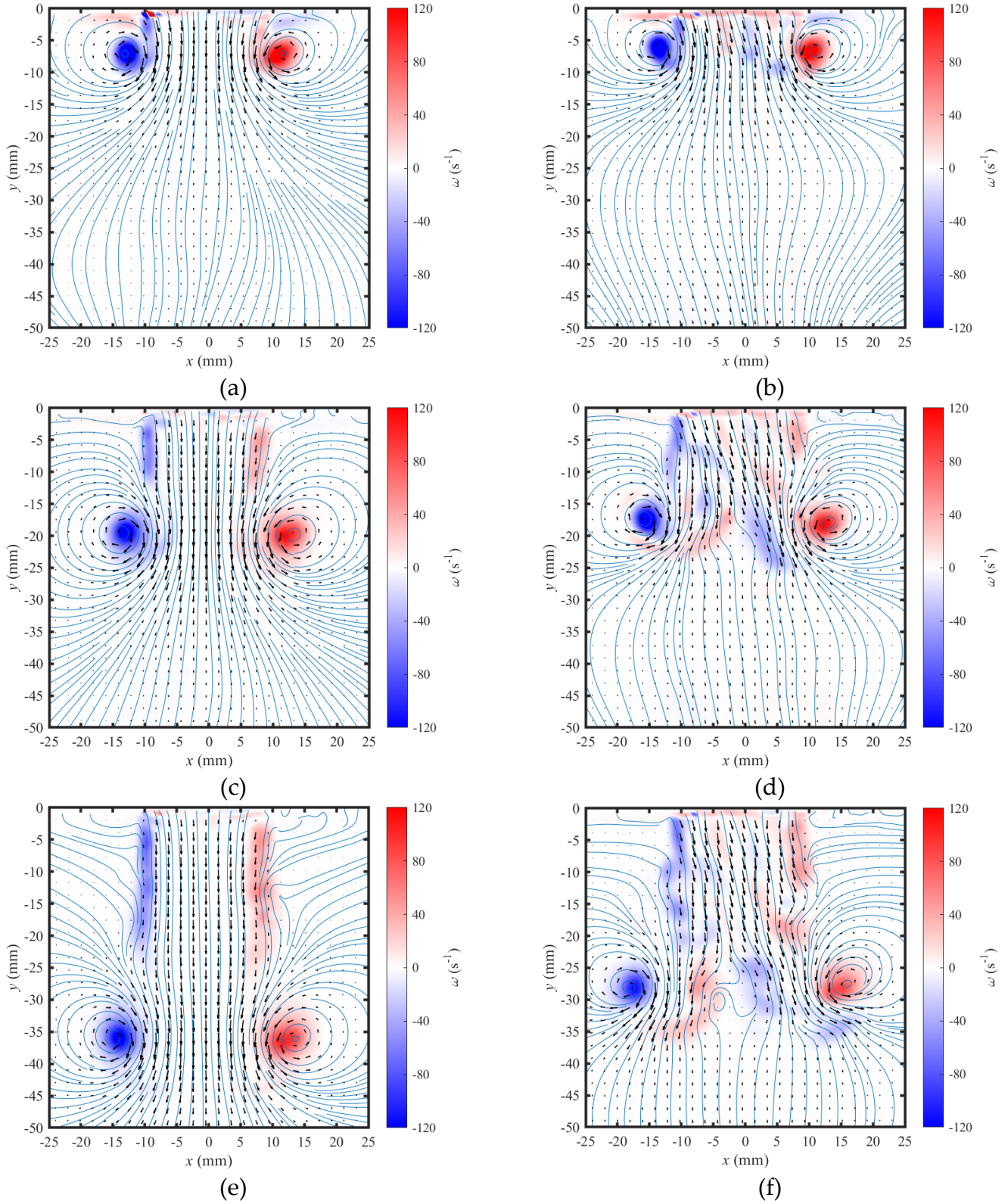


Figure 4. PIV-derived ensemble-averaged flow fields comparing jet structures without a swirler (a, c, e) and with a swirler (b, d, f) at $t/t_s = 0.3$ (top), 0.5 (middle), and 0.7 (bottom). Velocity vectors are superimposed over vorticity contours (ω) with overlaid streamlines; vector fields are down sampled for visual clarity.

The physical interpretation of these results highlights how the introduction of azimuthal swirl fundamentally alters the core topology, transforming a standard planar vortex structure into a highly complex, three-dimensional helical flow field. In the swirled case, the elevated early-stage $|V_z|$ values signify the successful injection of intense helical pitch and axial momentum directly into the core region upon formation (Figure 5b). As time progresses to $t^* = 0.50$ and 0.70 (Figure 5 d, f), the progressive attenuation of $|V_z|$ reflects the structural relaxation and viscous dissipation of the vortex ring as it expands and mixes with the ambient fluid. Conversely, for the unswirled configuration, the base flow is expected to be strictly two-dimensional with a theoretical out-of-plane velocity of zero. The marginal, non-zero baseline $|V_z|$ values ($0.006 - 0.009$ m/s) recorded in these unswirled cases do not represent a true physical axial jetting effect; instead, they are attributed to unavoidable experimental and numerical artifacts. These include minor angular misalignments of the laser sheet relative to the nominal measurement plane, structural vibrations within the experimental tank induced during the generation stroke, and inherent mathematical uncertainties coupled with the cross-correlation calculations and PIV post-processing routines.

To quantitatively trace the spatial and temporal evolution of the vortex structures, a Core Vortex Detection (CVD) algorithm was implemented to locate the precise coordinates of the vortex ring cores from a non-dimensional time of $t^* = 0.20$ to 0.90 with increments of 0.05 . The horizontal distance between these detected core centers defines the vortex ring diameter, as plotted in Figure 6a. Both the swirled and unswirled vortex rings initiate at an identical core diameter of 23.47 mm at $t^* = 0.20$, a value slightly larger than the physical nozzle exit diameter ($D_o = 20$ mm). However, their downstream expansion rates diverge significantly as time progresses. The unswirled vortex ring grows linearly to a final diameter of 30.32 mm at $t^* = 0.90$, exhibiting a steady radial growth rate of 9.79 mm per unit non-dimensional time ($R^2 = 0.983$). Conversely, the introduction of azimuthal swirl drastically accelerates this radial expansion, forcing the ring to a final diameter of 37.68 mm at $t^* = 0.90$. This represents a notable 24.27% increase in the final diameter compared to the unswirled counterpart, driven by a much steeper growth rate of 20.30 mm per unit non-dimensional time ($R^2 = 0.986$).

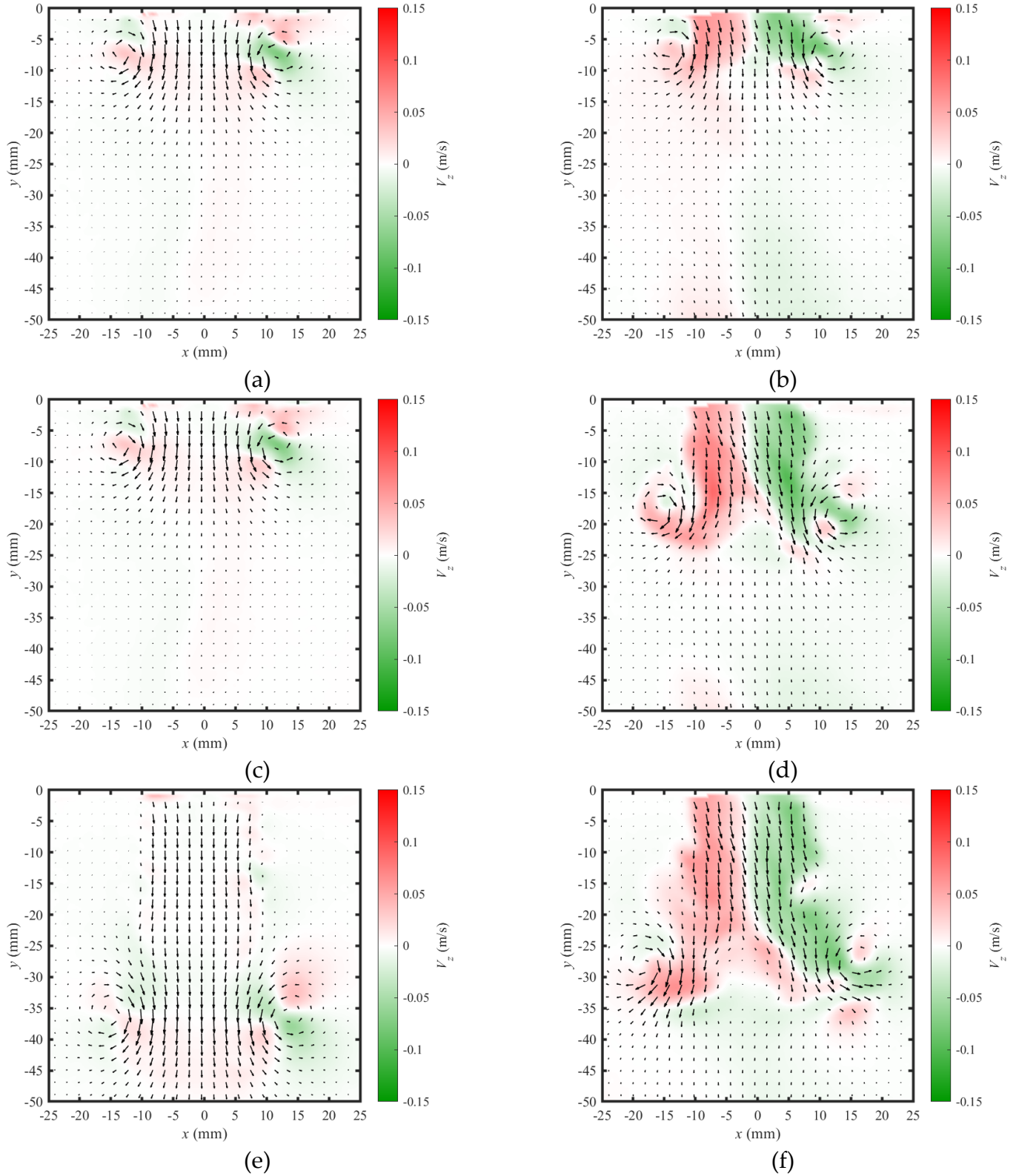


Figure 5. Contours of out-of-plane velocity (V_z) overlaid with in-plane velocity vectors at non-dimensional times $t^* = 0.30$ (top row: a, b), $t^* = 0.50$ (middle row: c, d), and $t^* = 0.70$ (bottom row: e, f). The left column (a, c, e) represents the unswirled flow condition, and the right column (b, d, f) represents the swirled flow condition. Color mapping indicates the axial velocity magnitude (V_z) in m/s, and vectors illustrate the local in-plane velocity direction and magnitude.

In addition to the radial expansion, the axial migration of the vortex cores was analyzed by tracking their average vertical position along the y -axis, where $y = 0$ mm corresponds strictly to the exit plane of the nozzle tip (Figure 6b). Given that the piston stroke time (t_{Stroke}) is exactly 1.0 s, the non-dimensional time variable ($t^* = t/t_{Stroke}$) scales identically to real physical time in seconds, rendering the temporal displacement rates directly equivalent to physical velocities in mm/s. Both cases launch their downstream propagation simultaneously from an initial average vertical core coordinate of -7.88 mm at $t^* = 0.20$. For the unswirled configuration, the vortex ring moves rapidly downward through the tank, reaching a final tracked axial position of -56.30 mm at $t^* = 0.90$, which yields a mean downward core propagation velocity of -69.17 mm/s. In contrast, the swirled vortex ring displays a conspicuously slower downward migration, terminating at a shallower depth of -42.36 mm at the same timestamp ($t^* = 0.90$). This corresponds to a reduced mean downward core propagation velocity of -49.26 mm/s. This kinematic trade-off confirms that the intense centrifugal forces generated by the swirler redirect a substantial portion of the jet's forward linear momentum outward into radial expansion, producing a wider but noticeably slower-moving vortex structure.

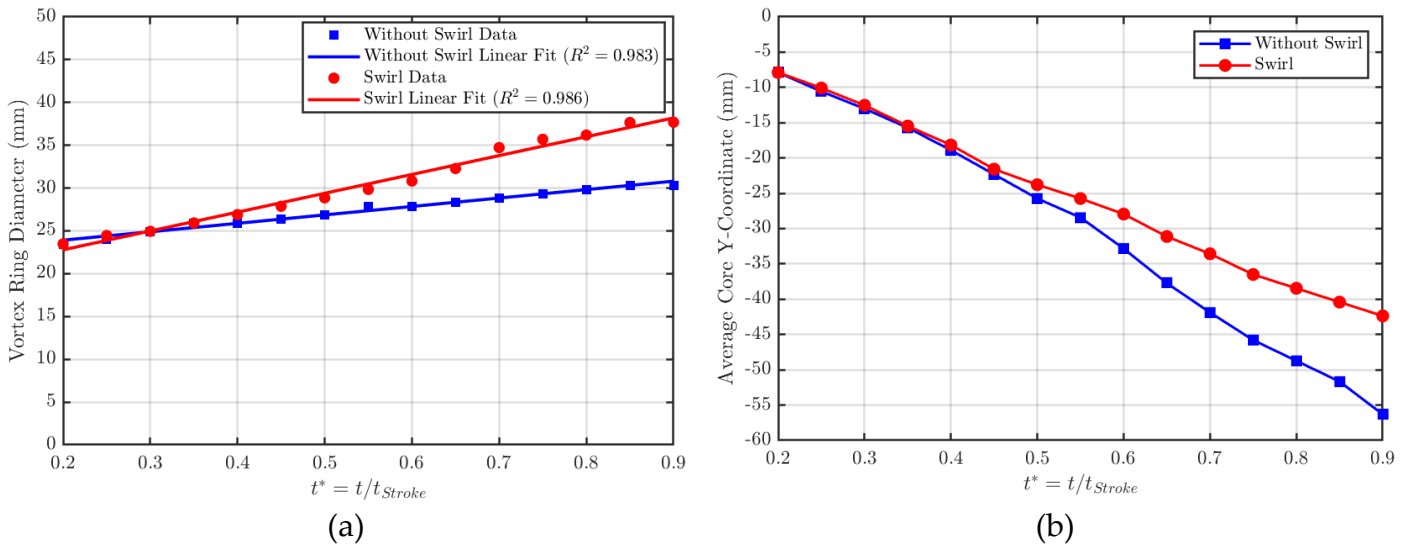


Figure 6. Vortex core kinematics from $t^* = 0.20$ to 0.90 for unswirled (blue squares) and swirled (red circles) flows: (a) ring diameter expansion with linear regression fits, and (b) mean vertical core trajectory.

A custom-developed MATLAB algorithm (R2024b) was employed to post-process the vector fields and characterize the vortex structure located on the right side of the domain. The algorithm automatically identifies the vortex center based on the peak vorticity. It then determines the vortex core radius (r_c) by calculating the radial distance from the center to the point of maximum tangential

velocity ($V_{\theta,max}$), Figure 7. Based on this characteristic length, the algorithm extracts the velocity profile over a radial domain extending from the vortex center ($r = 0$) to three times the core radius ($r = 3r_c$). A quantitative comparison reveals that the introduction of the swirl generator significantly impacts the vortex structure. Specifically, the core radius decreased from 2.4381 mm in the baseline case without swirl, Figure 7a, to 1.9810 mm with the swirl generator, Figure 7b. This corresponds to a core size reduction of approximately 18.8%.

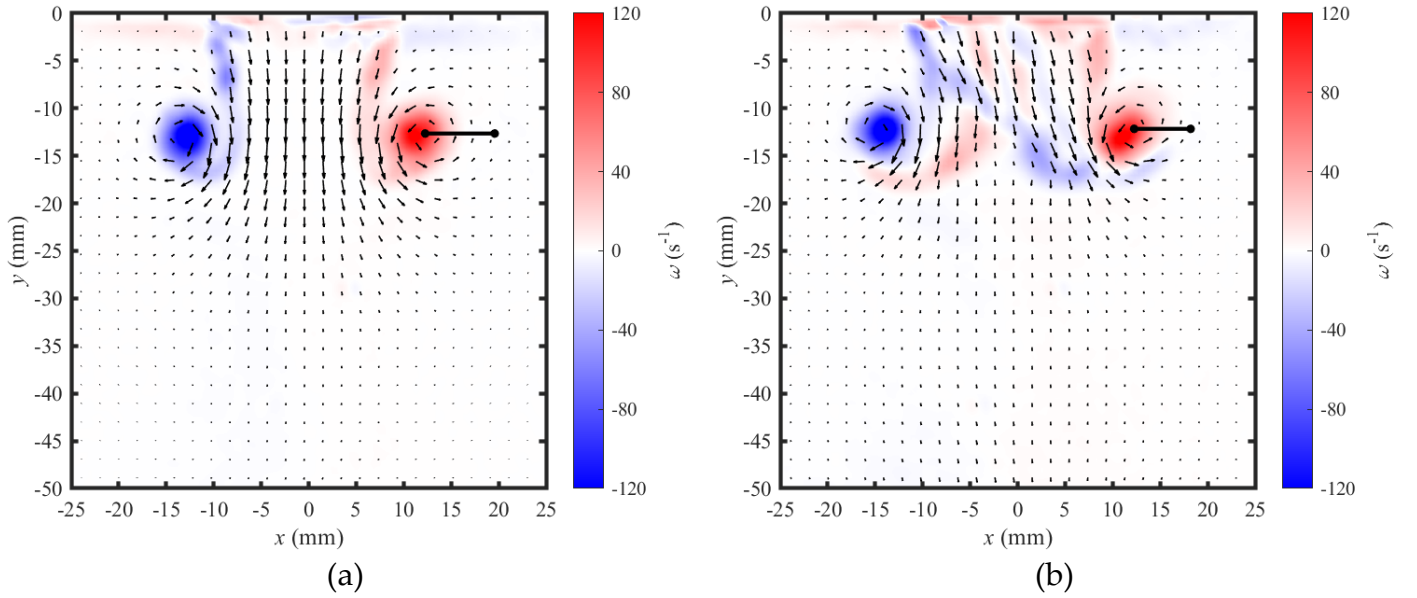


Figure 7. PIV-derived flow fields comparing the vortex structure (a) without and (b) with the swirl generator. The solid black line and indicators denote the tracked radial line used for the vortex investigation in each case.

To evaluate the mathematical characteristics of the generated vortices and validate their structural behavior against classic aerodynamic theory, the experimental PIV datasets were fitted using the generalized Vatisas formulation. The tangential velocity profiles for both the baseline configuration without swirl, Figure 8a and the swirled jet, Figure 8b case were compared against theoretical curves corresponding to shape factors of $n = 1, 2, 3$, and 5. For both configurations, a non-linear least-squares curve fit reveals a convergence to an optimal shape factor of $n = 2.00$, illustrating that the normalized data points trace the $n = 2$ theoretical curve from the vortex center through the peak velocity core boundary ($r/r_c = 1.0$), and into the outer potential flow decay region ($r/r_c > 1.0$). As the Vatisas $n = 2$ model serves as an algebraic approximation of the classic Lamb-Oseen vortex, this correlation demonstrates that the physical vortices generated in this experiment behave as classic decaying viscous vortices. Furthermore, the scaling alignment under dimensionless coordinates

($\bar{V}_\theta$ Vs $\bar{r}$) proves that while the introduction of the swirl generator alters physical properties like the absolute core size and velocity limits, the underlying structural topology of the vortex family remains self-similar and governed by standard viscous core mechanics.

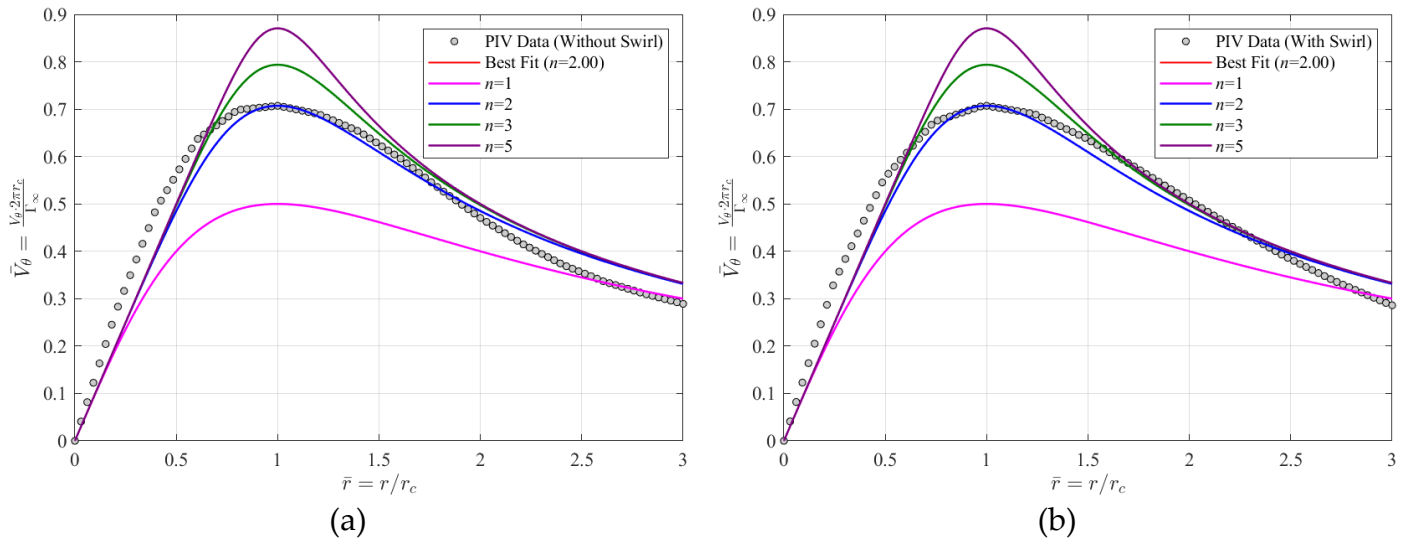


Figure 8. Profiles of the normalized tangential velocity ($\bar{V}_\theta$), non-dimensionalized by the theoretical infinite circulation. (a) without swirl, and (b) with swirl.

5 Conclusion

This study investigated the structural and kinematic evolution of a pulsatile starting vortex ring when modulated by an azimuthal swirl component. The introduction of a 3D-printed swirler fundamentally transformed the baseline planar vortex into a highly complex, three-dimensional helical flow field. Quantitatively, the addition of swirl accelerated the radial expansion of the vortex ring by 24.27% and noticeably reduced its downward propagation velocity, demonstrating a redirection of linear momentum into radial expansion. Furthermore, the imparted azimuthal velocity triggered rapid core dissipation, accelerated vortex breakdown, and resulted in an 18.8% reduction in the vortex core radius. Despite these pronounced topological changes, the swirled vortices maintained self-similar velocity profiles that closely matched the Vatistas $n = 2$ theoretical model, confirming they still behave as classic decaying viscous vortices. However, future work will require improvements to the experimental setup. Minor out-of-plane velocities recorded during the unswirled baseline testing revealed the presence of experimental artifacts, which must be addressed in subsequent studies by mitigating laser sheet misalignments and dampening structural vibrations induced by the flow generation mechanism.

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7. Appendix

6.1. List of Variables

- $v_{\theta}(r)$: Tangential velocity at radius r [m/s].
- r : Radial distance from the vortex center [m or mm].
- r_c : Vortex core radius; the radial location where the tangential velocity reaches its maximum (V_{max}) [m or mm].
- V_{max} (or $V_{\theta,max}$): The maximum tangential velocity magnitude occurring at the core radius [m/s].
- Γ : Circulation strength of the vortex [m²/s].
- Γ_{∞} : Total circulation in the far field (for Vatistas model).
- n : Shape parameter (specifically for the Vatistas model) that controls the "sharpness" of the transition from the core to the outer flow [dimensionless].
- t : Time (appears in the decaying Oseen-Lamb model) [s].
- S : Geometric Swirl Number (theoretical estimation based on vane geometry) [dimensionless].
- R_{hub} : Radius of the central hub of the swirl generator [m or mm].
- R_{tip} : Radius of the tip (outer edge) of the swirl generator vanes [m or mm].
- θ : Vane angle (pitch) of the swirl generator [degrees or radians].
- D_{out} : Outlet diameter (characteristic length for Reynolds number calculation) [m].
- ρ : Fluid density [kg/m³].
- μ : Dynamic viscosity [Pa·s or N·s/m²].
- ν : Kinematic viscosity ($\nu = \mu / \rho$) [m²/s].
- V_{outlet} : Mean flow velocity at the outlet [m/s].
- α : Womersley number (dimensionless parameter describing the ratio of pulsatile flow frequency to viscous effects).
- L : Characteristic length scale (typically radius of pipe/orifice for Womersley model) [m].
- ω : Angular frequency of pulsations ($\omega = 2\pi f$) [rad/s].
- f : Frequency of the pulsation/flow oscillation [Hz].
- Δt_{piv} : Time interval between laser pulses for Particle Image Velocimetry [s].