

Sphere Entry and Trajectory in a Pool: Influence of Wettability

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ABSTRACT

In this study, we investigate the influence of hydrophilic, hydrophobic and superhydrophobic (SH) surfaces on spheres entering a deep pool, in particular cavity formation and secondary pinch-offs. Experiments were conducted using spheres of various densities and diameters, released from controlled heights to systematically vary the impact Froude number (Fr_i). High-speed imaging and image based volume reconstruction were employed to quantify the instantaneous displaced volume, as well as the sphere trajectory, velocity and acceleration. Results show that hydrophilic spheres do not entrain air cavities, but exhibit high fluctuating lift forces, leading to trajectories deviating largely from pure downward vertical motion. As the sphere becomes increasing hydrophobic, a stable air layer that sustains encapsulated cavities or thin air bubbles forms upon entry, leading to either terminally descending or ascending trajectories depending on the displaced volume. The scaling of the displaced volume is based on the first cavity collapse, whereas a regime map, constructed from the final cavity collapse, captures the transition between terminally descending and ascending trajectories.

1. Introduction

The entry of solid bodies into liquid pools has long been studied for its relevance to multiphase flows, cavity dynamics, and hydrodynamic impact. While previous studies have focused on geometry, cavity types, and unsteady forces, the role of surface wettability remains less explored. Surface wettability can significantly alter sphere entry behavior, stabilize air layers, and influence cavity growth, entrained air volumes, and post-impact motion (Truscott & Techet, 2006; Duclaux et al., 2007; Mansoor et al., 2014; Vakarelski, Yang, et al., 2019; Guleria et al., 2021). During the initial stages of impact, hydrophilic spheres typically do not form a sustained air cavity; instead, they penetrate the liquid while exhibiting random lateral deviations from the vertical impact (Billa et al., 2025). In contrast, hydrophobic and superhydrophobic (SH) spheres typically generate an air cavity that undergoes pinch-off, leaving a residual bubble in the wake of the sphere (Aristoff

& Bush, 2009; Truscott et al., 2012; Vakarelski et al., 2017). The size and persistence of this wake cavity depend on several factors, including the impact Reynolds number, density ratio, surface wettability, impact angle (Asfar & Moore, 1987), and possible rotation of the sphere (Techet & Truscott, 2011). Truscott (2009) and Truscott et al. (2014, 2012) further demonstrated that variations in contact angle, sphere density, and impact velocity significantly affect vortex formation and the resulting unsteady hydrodynamic forces during penetration, as revealed through Particle Image Velocimetry (PIV) measurements.

Understanding these interactions is crucial for a wide range of applications, including marine engineering and seaplane landings (Seddon & Moatamedi, 2006), vertical missile water entry (May, 1952), projectile-water impact analysis (Truscott et al., 2014) and underwater robotics (Yuh, 2000; Neira et al., 2021). A recent review by Jung (2025) highlighted the shared strategies governing water-entry and exit phenomena in both biological and engineering systems, demonstrating how insights into impact forces, body morphology, and dynamics can inspire advanced marine and aerial-aquatic designs.

The entry and penetration of a sphere into a deep liquid pool serves as a simplified yet generic representation of these practical applications. The foundational work in this area was conducted by Worthington (1883), who used high-speed photography to pioneer the study of a solid sphere impacting on the liquid. Subsequent research extended these insights, examining the influence of air pressure on splash and cavity formation (Gilbarg & Anderson, 1948), potential flow based models of cavity geometry (Richardson, 1948), the effects of surface roughness and impact angle on cavity dynamics (May & Hoover, 1965; Lavery, 2005). Later studies further explored the influence of surface grooves and impact velocity on cavity behavior (Mehri & Akbarzadeh, 2021), as well as the effects of surface heating and the formation of Leidenfrost vapor layers on sphere dynamics (Mehri & Akbarzadeh, 2020; Vakarelski et al., 2014; Mansoor et al., 2017; Vakarelski, Jetly, & Thoroddsen, 2019). Further experimental and numerical investigations have examined the unsteady motion of hydrophilic and hydrophobic spheres, including pinch-off dynamics, vortex formation, and the resulting hydrodynamic forces (Duclaux et al., 2007; Aristoff & Bush, 2009; Aristoff et al., 2008; Truscott et al., 2012; Vakarelski et al., 2011; Speirs, Mansoor, et al., 2019; Li et al., 2019). In addition, the effects of stratified liquids (Tan, 2016), as well as capillary and viscous forces (Marston et al., 2011), have been extensively investigated. The reduction of drag during underwater motion has also been the subject of recent studies (S. Zhu et al., 2022; Zhao et al., 2025). These studies have substantially advanced our understanding of cavity formation, sphere encapsulation, and deceleration processes.

Despite extensive prior research, the role of post-pinch-off air cavities and their influence on the subsequent motion of impacting spheres remains inadequately understood, particularly for spheres of lighter density ($\rho^* = \rho_s / \rho_l < 2.2$, ρ_s - density of sphere, ρ_l - density of liquid.). This knowledge gap is particularly critical for hydrophobic and superhydrophobic spheres of relatively low

density, where entrained air volumes can substantially modify drag and buoyancy forces, often leading to unexpected distinct terminal motions.

The present study addresses this gap through a systematic investigation covering a wide range of sphere diameters and impact velocities as well as spheres with static apparent contact angles (θ_a) in the $65^\circ < \theta_a < 155^\circ$. Using high-speed imaging and image based analysis, this study quantifies the instantaneous trajectories, velocities, and accelerations of spheres through spline fitting and smoothing techniques, elucidating how the entrained air volume and cavity dynamics govern the transition between terminally descending and ascending motion, depending on the primary and secondary pinch-off events. The results indicate that abrupt changes in the entrained air volume during primary and secondary pinch-off events lead to corresponding variations in buoyancy force, ultimately determining whether the sphere undergoes terminally descending or ascending motion. Furthermore, the study quantifies and semi-analytically predicts the displacement volume scaling at first cavity collapse, and motion regimes from the final collapse.

2. Experimental set-up and methodology

2.1. Experimental set-up

The experimental set-up is identical to that used for a previous study devoted to the impact and penetration of hydrophilic spheres (Billa et al., 2025) and is only briefly described here. The set-up is pictured in Fig. 1, comprising a clear, translucent acrylic container, two orthogonally placed, synchronized high-speed monochrome cameras (Phantom VEO E-340-L with a Nikon microlens 28 mm, spatial resolution of $192 \mu\text{m}/\text{pixel}$ and Phantom VEO E-340-L with a Nikon micro lens of focal length 24-85 mm, spatial resolution of $199 \mu\text{m}/\text{pixel}$), two back illumination panels (30W) and a vacuum system used to release the spheres using a pipette. The cameras operate at 1000 fps with a resolution of 1152×1100 pixels and an exposure time of $400 \mu\text{s}$. The field of view in both the cameras is $100 \text{ mm} \times 160 \text{ mm}$.

2.2. Preparation of superhydrophobic spheres

Spheres were made superhydrophobic (SH) using a commercially available coating (NeverWet spray). The static contact angle on the spheres, measured using the ellipse fit method in ImageJ and accounting for sphere curvature (Extrand, 2002; Extrand & Moon, 2012; Magos & Balan, 2021), was $155 \pm 5^\circ$. The surface generated with the coating, resulted in R_a/R_q values, measured using a Nanovea Portable Profilometer Jr25 (a non-contact 3D optical instrument) and expressed according to ISO 21920 of 5/7.7 to 6/8 microns, compared to values of 0.13/0.18 to 0.14/0.20 microns for uncoated substrates.

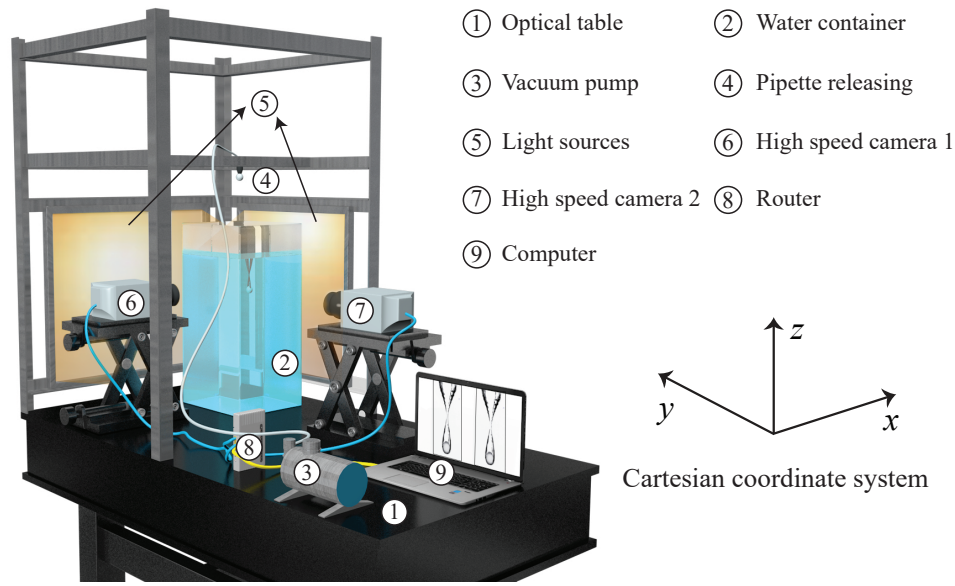


Figure 1. Schematic of the experimental set-up. The container has dimensions $200(L) \times 200(B) \times 400(H)\text{mm}^3$, whereby the largest sphere used in the study was 10 mm. The impact velocity of the spheres is varied by varying the height of sphere release.

2.3. Range of varied parameters

Table 1 provides the parameters and dimensionless quantities investigated in this study. The hydrophilic and hydrophobic spheres employed in this study exhibited different wettability, as summarized in Table 1. Consequently, the measured static apparent contact angles of the uncoated spheres varied between 64° and 100° , depending on the substrate material. These values represent the inherent wettability of the sphere surfaces prior to the application of the superhydrophobic coating. Following the coating procedure described in Section 2.2, all spheres exhibited static apparent contact angles greater than 150° , irrespective of the underlying substrate material. The impact velocity ($v_i \approx \sqrt{2gh}$) is derived from the initial release height (h) of the sphere, measured from its center to the air-water interface. In these definitions ν is the kinematic viscosity ($0.89 \text{ mm}^2/\text{s}$)

Table 1. Properties of spheres with associated parameters.

Parameter	Symbol	Definition	Range of values
Sphere diameter	D	-	4, 6, 10 mm
Static apparent contact angle	θ_a	$^\circ$	64, 75, 85, 94, 97, 100, 155 (SH)
Density ratio	ρ^*	ρ_s / ρ_l	1.12, 1.37, 2.16, 3.26, 6.08, 7.92
Impact velocity	v_i	$\sqrt{2gh}$	0.89, 3.52 m/s
Impact Reynolds number	Re_i	$v_i D / \nu$	4,000 - 39,600
Impact Froude number	Fr_i	$v_i / \sqrt{gD}$	3.07 - 17.32

of the fluid and g is gravitational acceleration (9.81 m/s^2). Lengths are rendered dimensionless using the sphere diameter D , velocities using v_i , and time using D/v_i . Dimensionless quantities are designated with a * superscript.

Anticipating the need to compute the displaced volume exactly, image processing was performed as illustrated in Fig. 2. Images in Fig. 2(a) and (b) show the air cavity prior to primary pinch-off for a hydrophobic and superhydrophobic sphere respectively. While the hydrophobic sphere exhibits a three-phase contact line near the equator, the superhydrophobic sphere is completely encapsulated in an air pocket. The computation of the displaced volume is illustrated in Fig. 2(c) and (d) just before and after primary pinch-off. The displaced volume was computed at each time step following the procedure of Billa et al. (2026), using edge detection combined with trapezoidal integration of horizontal disks in the vertical direction. The total displaced volume was then made dimensionless using the volume of the sphere, V_S , yielding the dimensionless displaced volume ($V_{\text{Disp}}^* = V_{\text{Disp}}/V_S$). In this closed cavity state, lighter spheres with an attached air cavity decelerate to rest during descent and subsequently reverse direction to ascend, as the upward buoyancy force (F_B), resulting from the displaced liquid volume exceeds the net downward gravitational force (F_G). Thus, this ascending trajectory behavior is expected under the condition

$$\frac{F_B}{F_G} = \frac{\rho_l g V_{\text{Disp}}}{m_s g} = \frac{V_{\text{Disp}}^*}{\rho^*} > 1 \quad \text{or} \quad V_{\text{Disp}}^* > \rho^*, \quad (1)$$

The dimensionless terminal velocity (v_t^*) is obtained by equating the net body forces with the hydrodynamic force, yielding

$$v_t^* = \frac{1}{v_i} \sqrt{\frac{4(\rho^* - 1)gD}{3C_D}}. \quad (2)$$

This quantity can be estimated prior to the experiment from the prescribed release height, provided that the drag coefficient (C_D) in the liquid is known.

The pinch-off time, t_p , is defined as the elapsed time between the initial impact of the sphere and the instant at which the air cavity contracts and separates ('pinches off') into a detached bubble and an attached wake cavity. In some cases, multiple pinch-off events occur: the primary pinch-off corresponds to the detachment of the cavity, while subsequent secondary pinch-offs produce smaller entrained bubbles near the wake. Hydrophilic spheres exhibit no similar air cavity up to some limiting impact Reynolds number, since the contact line upon entry moves as rapidly as the sphere penetration, completely wetting the sphere up to its apex (Duclaux et al. (2007). Under which conditions an air cavity forms has been the subject of numerous investigations and modeling, starting with May (1951) and further pursued by Duez et al. (2007); Duclaux et al. (2007) and Aristoff & Bush (2009). Truscott et al. (2012) examined the entry of hydrophobic spheres and modeled the cavity growth and collapse using a potential flow model, as in Duclaux et al. (2007). Speirs, Belden, et al. (2019) also characterized the influence of contact angle on the water entry of a sphere. Nevertheless, few studies exist using superhydrophobic spheres (maximum advancing

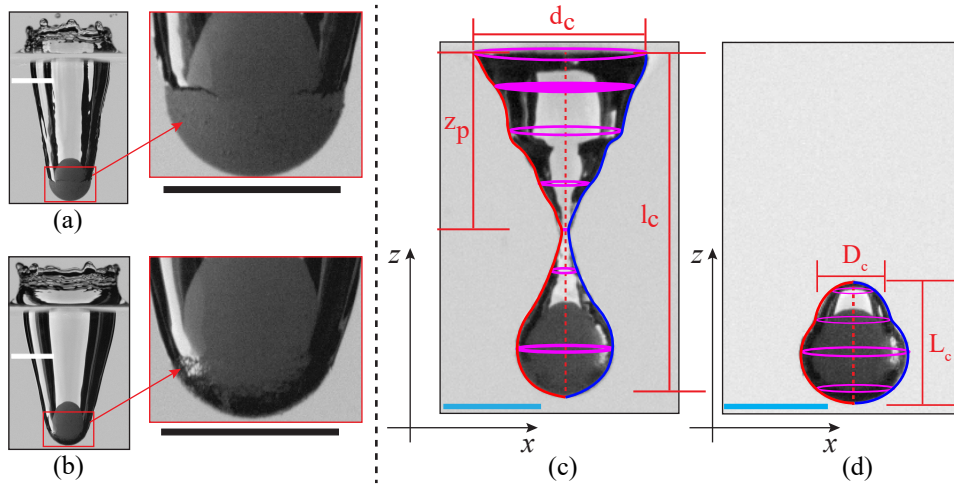


Figure 2. Images of hydrophobic and superhydrophobic spheres during water entry. (a) Hydrophobic sphere showing unstable cavity formation (indicated by red the box) and complete wetting up to the equator (inset). (b)

Superhydrophobic sphere exhibiting smooth cavity formation with complete air encapsulation (indicated by the red box) of the sphere during penetration (inset). (c,d) Schematic illustration of how the displaced volume was computed following Billa et al. (2026), where the cavity profile is divided into horizontal disks and integrated along the vertical direction using the trapezoidal method. The pinch-off depth (z_p), cavity diameter near the free surface (d_c), and total cavity length (l_c) are also indicated. Panel (d) shows a sustained bubble-tail cavity with the associated geometrical parameters: bubble length (L_c) and bubble diameter (D_c). Scale bars in all images and insets represent 10 mm (equal to the sphere diameter).

contact angle of Speirs, Mansoor, et al. (2019) was 141°). Due to the large contact angle ($> 150^\circ$), all spheres in the present study exhibited an air cavity upon impact and a pinch-off, resulting in a residual air cavity in their wake, regardless of their impact velocity.

Image processing

The accumulated dimensionless path traversed by the sphere is represented as $s^* = \sqrt{x^2 + y^2 + z^2}/D$. Although the trajectory of the hydrophobic and superhydrophobic spheres exhibited very little deviation from a pure vertical motion, a dimensionless lateral displacement was computed for each time step using $r^* = \sqrt{x^2 + y^2}/D$. The velocity magnitude $|\vec{v}_s|$ is computed by differentiating the displacement data after first fitting that data with a quintic spline function. The acceleration (a_s^*) is then computed from the time derivative of the velocity. To reduce noise in the acceleration (a_s^*), the velocity is first smoothed using a piecewise spline fit applied over fixed time intervals. The resulting acceleration (a_s^*) is further smoothed using a smoothing spline, with a smoothing parameter close to one. This approach follows the methodology introduced by Epps et al. (2010), which provides a robust framework for extracting reliable derivatives from experimental data with inherent noise.

2.4. Equation of motion

In contrast to previous measurements performed using hydrophilic spheres (Billa et al., 2025), superhydrophobic spheres exhibited only very low values of r^* , indicating that only weak lift/lateral forces were acting during its descent. The dimensionless pinch-off time, defined as $t_p^* = t_p v_i / D$, remains approximately constant across all experiments, suggesting that the primary pinch-off dynamics are largely insensitive to variations in sphere density. This fact is illustrated with comparative trajectory results shown in Fig. 3(a), (b) and (c). In this figure, the trajectory traces of a 10 mm, $\rho^* = 2.16$ superhydrophobic sphere are compared with those of a 10 mm hydrophilic sphere, both impacting with a Reynolds number of 15,700. A comparison of the lateral displacements for spheres of different density ratios is shown in Fig. 3(d). Given this apparent lack of lateral lift forces for hydrophobic and superhydrophobic spheres, the assumption can be made that the flow around the sphere, including the wake air cavity, is comparably more axisymmetric. The unsteady and non-symmetric wake as a source of lift forces acting on hydrophilic spheres has been postulated by numerous authors in the past (Scoggins, 1967; Taneda, 1978; Kuwabara et al., 1983; Veldhuis et al., 2005; Veldhuis & Biesheuvel, 2007) and in some cases confirmed by PIV measurements in the wake region (Horowitz & Williamson, 2008, 2010; Truscott et al., 2012; Vakarelski et al., 2017; Jetly et al., 2018). Moreover, the symmetric wake behavior with superhydrophobic spheres is in agreement with many other studies of superhydrophobic spheres penetrating a deep pool (Aristoff et al., 2010, 2008; Truscott et al., 2012; Speirs, Mansoor, et al., 2019) and allows the implicit conclusion that the axial symmetry of the three-phase contact line arising with the superhydrophobic sphere is steadier than the boundary-layer separation around a hydrophilic sphere.

The lack of lift forces justifies the use of force terms in the equation of motion acting only in the z direction, implicitly assuming that the z direction is equivalent to the pathline (in-line). This scalar momentum equation (Boussinesq-Basset-Oseen, BBO) for the completely submerged sphere/air-cavity then reads (C. Zhu & Fan, 1998; Crowe et al., 2011):

$$\underbrace{\frac{1}{6}\rho_s\pi D^3 \frac{dv_s}{dt}}_{\text{Inertial force}} = \underbrace{\frac{1}{6}\pi D^3 \rho_s g - F_N}_{\text{Gravity}} - \underbrace{\frac{1}{6}C_A \rho_l \pi D^3 \frac{dv_s}{dt}}_{\text{Added mass}} - \underbrace{\frac{3}{2}D^2 \sqrt{\pi\mu\rho_l} \int_0^t \frac{1}{\sqrt{t-\zeta}} \frac{dv_s}{d\zeta} d\zeta}_{\text{Boussinesq-Basset term}}, \quad (3)$$

where the gravity term will be designated F_G , and F_N , outlined below, will be a net force comprising buoyancy (F_B), hydrodynamic (F_H) and, before pinch-off, surface tension (F_γ) forces. C_A is the coefficient of the added mass force. In this equation, the mass of the wake air cavity has been neglected in the inertial and gravity terms. Furthermore, no rotational forces have been considered, since observations with marked spheres confirmed that the spheres were not rotating.

An expression for the net force acting on the sphere before pinch-off and for $z_i > D$ is given by

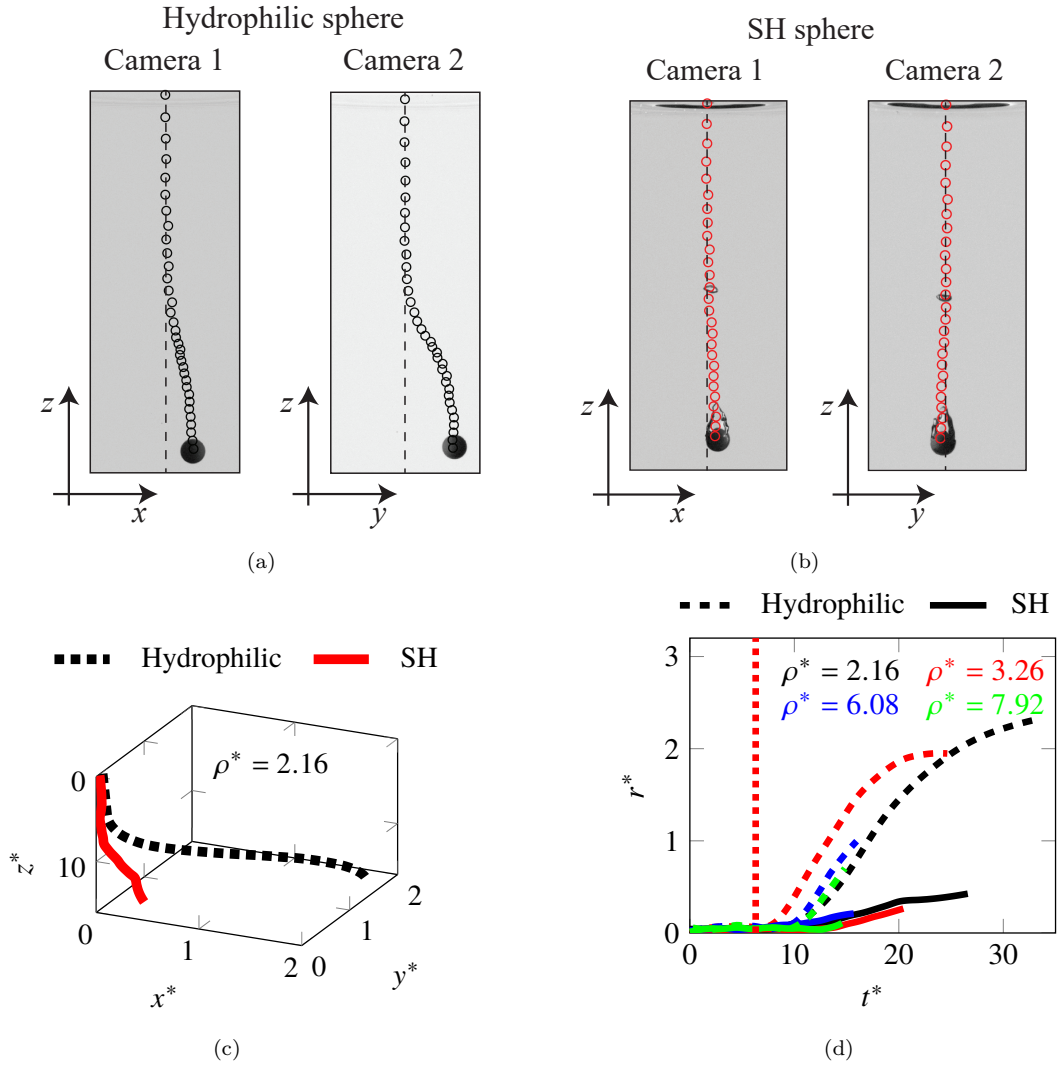


Figure 3. The trajectories of a 10 mm diameter hydrophilic and SH sphere with a density ratio $\rho^* = 2.16$ at $Re_i = 15,700$. The trajectories viewed from the orthogonally placed high-speed cameras for the (a) hydrophilic sphere and (b) SH sphere. (c) Three-dimensional rendition of the trajectories. (d) Comparison of the dimensionless lateral displacement (r^*) for 10 mm spheres of varying density at $Re_i = 15,700$. The red dashed vertical line in (d) represents the dimensionless time of primary pinch-off (t_p^*).

Billa et al. (2026) as:

$$\begin{aligned}
 F_N &= \rho_l g \left(\frac{\pi D^3}{12} - \frac{\pi D^2}{4} (z_i + D/2) \right) + p_0 \frac{\pi D^2}{4} - \frac{\pi D^2}{4} \left(p_0 - \frac{1}{2} \rho_a v_s^2 \right) + \pi D \gamma \\
 &= \rho_l g \left(\frac{\pi D^3}{12} - \frac{\pi D^2}{4} (z_i + D/2) \right) + \frac{\pi D^2}{8} \rho_a v_s^2 + \pi D \gamma.
 \end{aligned} \tag{4}$$

Given that $\rho_a \ll \rho_l$, the penultimate term in the above equation is at least one order of magnitude smaller than the last term and can be neglected. The final term in the above equation is of the order 10^{-3} N, which is approx. one order of magnitude less than the first term, but comparable to F_G for

the small light spheres; hence, this term cannot always be neglected. Note, that before pinch-off, F_N increases proportional to the depth of the sphere, z_i .

The hydrodynamic force acting on the sphere (F_H) can then be computed as:

$$F_H = \begin{cases} F_G - F_N & : \text{ before pinch-off} \\ F_G - F_B & : \text{ after pinch-off} \end{cases} \quad (5)$$

whereby before pinch-off, F_H will also include any contributions from the added mass or history term in Eq. (3), whereas after pinch-off, these terms will become negligible due to the low value of measured deceleration.

3. Results

3.1. Primary pinch-off

From Fig. 4(b) it is apparent that the pinch-off occurs at a constant dimensionless time, regardless of the impact conditions or sphere size and density. The experimental data lead to the linear relation $t_p^* = 4Fr_i/3$ yielding for the absolute pinch-off time

$$t_p = \frac{4}{3} \sqrt{\frac{D}{g}}. \quad (6)$$

This relation is confirmed by the measured t_p values and indicates that the absolute pinch-off time does not depend on the impact velocity or density of the sphere, but only on the sphere diameter. This is intuitively plausible, since the pinch-off is a phenomenon dominated primarily by capillary forces; the impact velocity only serves to determine the size of the cavity before pinch-off. A first-order estimate of the cavity height immediately following pinch-off (see Fig. 4) is given by

$$H = l_c - z_p - \frac{D}{2}.$$

For lighter spheres ($\rho^* = 1.12$ and 1.37), the volume displaced by the cavity after pinch-off can be approximated as a conical volume. In this case, the total displaced volume is considered as the sum of the conical gas cavity and a hemispherical cap at the base. For heavier spheres ($\rho^* \geq 2.16$) the maximum cavity diameter exceeds the diameter of the impacting sphere (Aristoff & Bush (2009); Vakarelski et al. (2017)). Although the cavity shape immediately after the pinch does not strictly resemble a perfect cylinder, a modified height term, denoted here as the *effective height*, is introduced in the scaling expression. This adjustment allows the simplified geometry (combined cylindrical gas cavity volume and hemispherical cap at the base) to better represent the displaced volume.

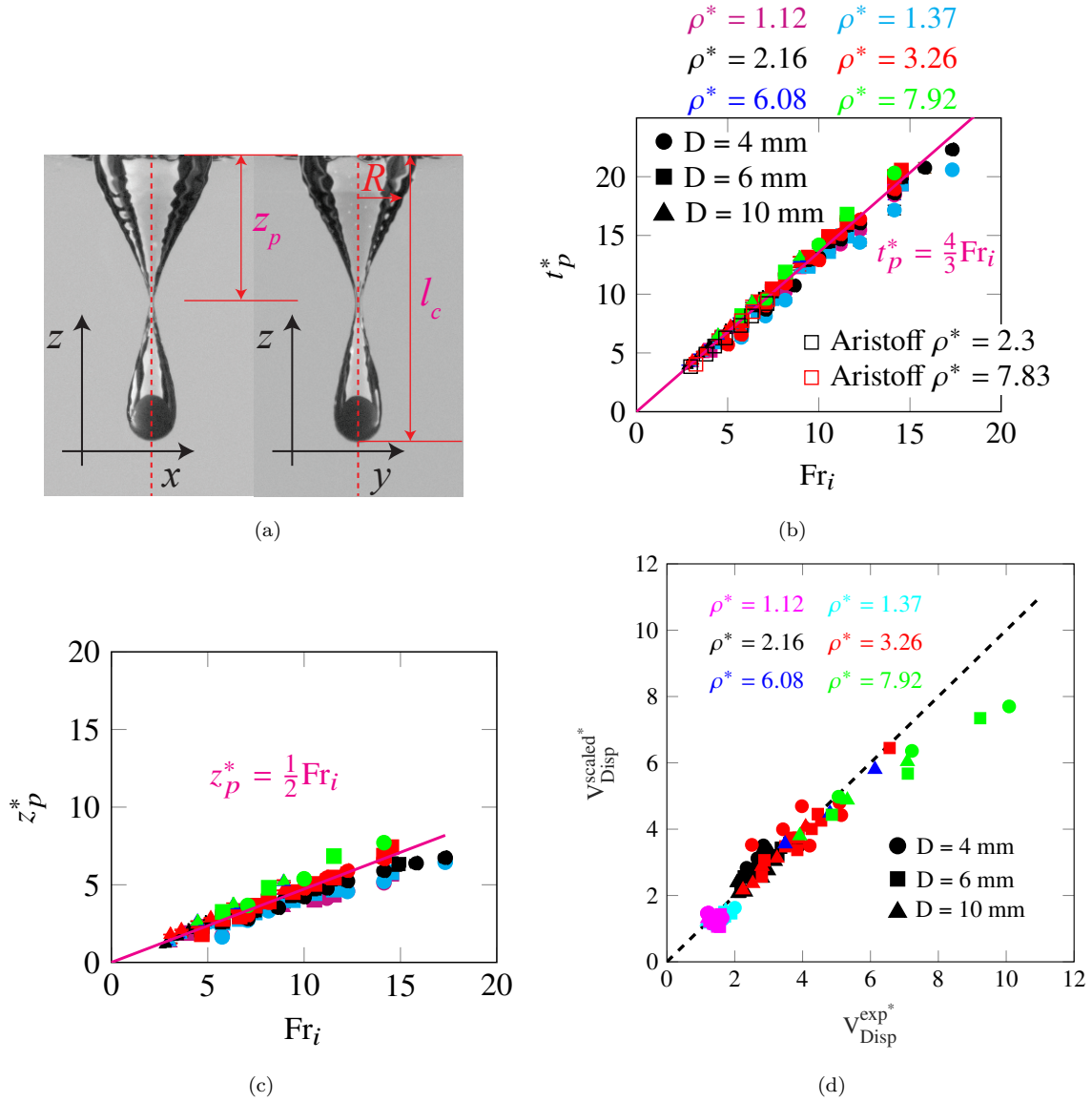


Figure 4. Pinch-off characteristics of a superhydrophobic sphere entering a deep pool: (a) Dimensions of an axisymmetric cavity observed using two high-speed cameras at the pinch-off. The influence of Fr_i on the dimensionless pinch-off time (t_p^*), and dimensionless pinch-off depth (z_p^*) across various impacting conditions for four of the investigated densities is provided in (b) and (c), respectively. (d) Comparison between scaled model predicted dimensionless displaced volumes ($V_{Disp}^{scaled*}$) and experimentally measured dimensionless displaced volumes (V_{Disp}^{exp*}) following the primary pinch-off. The measurement uncertainties in (b)–(d) are on the order of the symbol size. The open square symbols in (b) correspond to the t_p^* reported by Aristoff et al. (2010), plotted against the Fr_i . The legend in (b) applies to (c) and (d).

To represent the cavity shape consistently across all density ratios, a model for the scaled displaced

volume is proposed by Billa et al. (2026) as follows:

$$V_{\text{Disp}}^{\text{scaled}*} = \begin{cases} \frac{\pi D^2}{12} \left(H + \frac{D}{2} \right), & \rho^* \leq 1.37 \quad (\text{conical}) \\ \frac{\pi D^2}{12} \left(2H - \frac{D}{2} \right), & \rho^* \geq 2.16 \quad (\text{cylindrical}) \end{cases} \quad (7)$$

As illustrated in Fig. 4(d), the modeled values of V_{Disp} agree well with the experimentally measured values.

The influence of the primary pinch-off on the trajectory and velocity is illustrated in Fig. 5. In this figure, graph (b) shows the dimensionless velocity of the sphere over dimensionless time and noteworthy is a distinct kink and decreased deceleration rate at the time of primary pinch-off. This is clearly associated with the hydrostatic force jumping from F_N to the much lower F_B at the moment of pinch-off, as shown in graph (c). In graph (d) similar velocity curves for other dimensionless sphere densities are shown. It is evident from this last graph that the influence of hydrostatic forces is much stronger for the lighter spheres due to their lower inertial force; for heavier spheres there is no distinguishable kink.

In graph (c) a horizontal dashed line at F_G has been added, allowing condition (1) to be directly evaluated. For this particular case ($D = 6$ mm, $\text{Re}_i = 9,400$) F_B lies slightly above F_G immediately after the primary pinch-off, indicating that the sphere and attached cavity would eventually begin to rise. However, after a second pinch-off at approx. $t^* = 45$, $F_B < F_G$, indicating that the sphere and remaining cavity will continue descending. This change in buoyancy after the secondary pinch-off is also seen in graph (b), where the slope of v_s^* changes from negative (decelerating) to positive (accelerating) at $t^* = 45$, subsequently approaching the terminal velocity. It should be noted that the terminal velocity may no longer be strictly valid, as the assumed $C_D = 0.5$ to compute the terminal velocity is for a sphere, not a sphere with an attached air cavity. The heavier spheres do not approach their terminal velocity within the observational field of view.

Although the total cavity length at pinch-off is different for different density spheres, the dimensional primary pinch-off time remains virtually unchanged. The total cavity length reaches higher values for higher dimensionless densities before pinch-off; following pinch-off the buoyancy to gravity force ratio (V_{Disp}^* , Eq. 1) decreases with increasing sphere density, exhibiting values less than unity for the heaviest sphere.

3.2. Secondary pinch-off

Whether a sphere is terminally ascending or descending is strongly dependent on whether a secondary pinch-off of the wake cavity occurs, since the size of the wake cavity after the primary pinch-off following water entry generally satisfies $V_{\text{Disp}}^* > \rho^*$, i.e., the condition for terminally ascending behavior. However, after a second (or further) pinch-off(s), the submerged volume; hence,

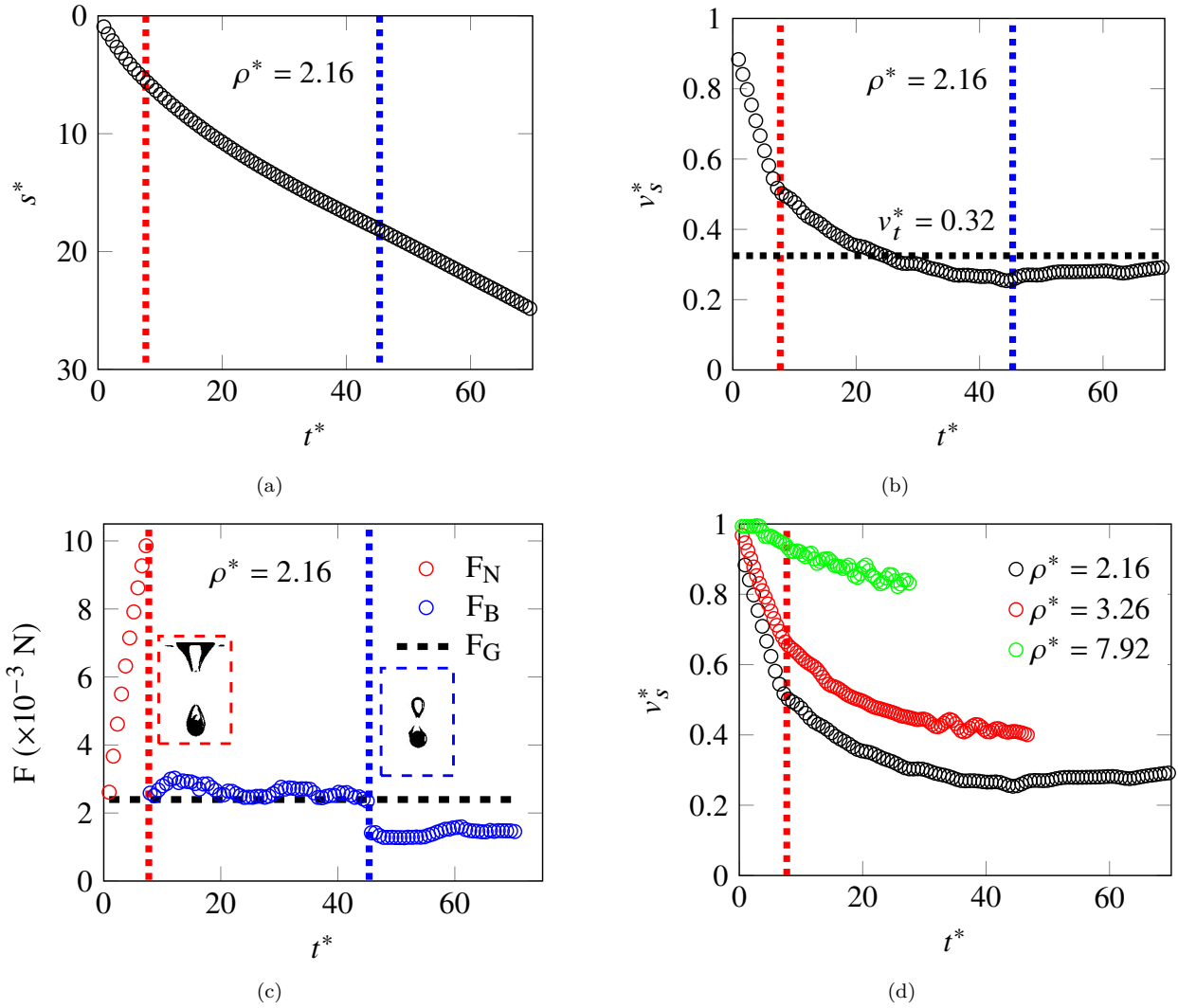


Figure 5. The influence and magnitude of hydrostatic forces on the sphere ($\rho^* = 2.16$) trajectory as a function of dimensionless time. (a) Dimensionless sphere path s^* , (b) dimensionless velocity v_s^* , (c) forces acting before (F_N) and after (F_B) the primary pinch-off, and (d) dimensionless sphere velocity for various dimensionless densities. Vertical dashed red and blue lines designate the dimensionless time of primary and secondary pinch-off, respectively. Data is for $D = 6$ mm and $Re_i = 9,400$. The horizontal dashed black line in (b) denotes the dimensionless terminal velocity (v_t^*) obtained from Eq. 2. The horizontal dashed magenta line in (c) marks F_G , allowing the condition in Eq. (1) to be directly evaluated. The inset in (c) shows captured images immediately following the primary and secondary pinch-off events, highlighted by red and blue dashed boxes, respectively.

the buoyancy, is smaller and gravity dominates. Whereas the mechanics of primary pinch-off have been well studied both experimentally and theoretically, the physics of secondary or tertiary pinch-offs has rarely been systematically investigated.

A first indication of the parametric influences is illustrated in Fig. 6. All three examples in this figure relate to a sphere of relative density $\rho^* = 1.12$, the first for a 4 mm sphere and the latter two for a 6 mm sphere, i.e., relatively light spheres. Whereas the superhydrophobic sphere in

Fig. 6(a) is terminally ascending and the heavier, less hydrophobic ($\theta_a = 100^\circ$) 6 mm sphere with higher impact velocity in Fig. 6(c) is terminally descending, the 6 mm sphere with a lower impact velocity, but the same wettability as in Fig. 6(b), exhibits varying behavior - sometimes terminally ascending, sometimes terminally descending. This figure includes also still photographs from the high speed videos, indicating the nature of the wake cavity at different times.

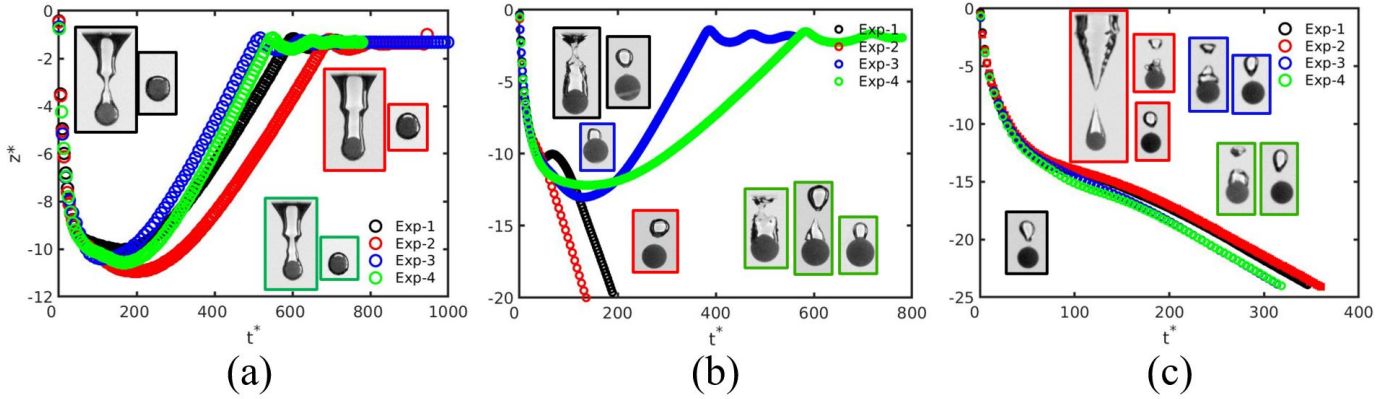


Figure 6. Dimensionless penetration depth (z^*) of a $\rho^* = 1.12$ sphere as a function of dimensionless time (t^*) with photographs of wake cavities: (a) $D = 4$ mm, $v_i = 1.4$ m/s, $\theta_a = 155^\circ$; (b) $D = 6$ mm, $v_i = 1.4$ m/s, $\theta_a = 100^\circ$; (c) $D = 6$ mm, $v_i = 2.8$ m/s, $\theta_a = 100^\circ$

The combined influence of impact velocity (v_i), apparent contact angle (θ_a), and density ratio (ρ^*) on the terminal motion of the sphere is summarized in Fig. 7. Figure 7(a) presents the three-dimensional parameter space, distinguishing terminally ascending and terminally descending motions. Figure 7(b) shows the (v_i, θ_a) plane, indicating that terminally descending motion is observed only for $\theta_a < 97^\circ$. For $\theta_a \gtrsim 97^\circ$, both terminally ascending and terminally descending motions are observed, depending on whether secondary pinch-off occurs. The combined influence of impact velocity and density ratio on the terminal motion of the sphere is illustrated in the (v_i, ρ^*) plane (Fig. 7(c)). Spheres with larger density ratios possess greater weight and therefore exhibit a stronger tendency toward terminally descending behavior. In particular, for $\rho^* \geq 2.16$, only the $D = 4$ mm spheres exhibit terminally ascending motion under limited conditions, whereas larger spheres predominantly remain descending (Billa et al., 2026). In contrast, spheres with lower density ratios ($\rho^* = 1.12$ and 1.37) frequently exhibit terminally ascending motion, although the final outcome still depends on the sphere diameter, impact velocity, and the occurrence of secondary pinch-off events.

4. Analytical modeling

Before introducing a model to explain the observed terminal trajectory behavior of the spheres, we note that a hydrophobic sphere is shown to descend following a secondary pinch-off, whereas

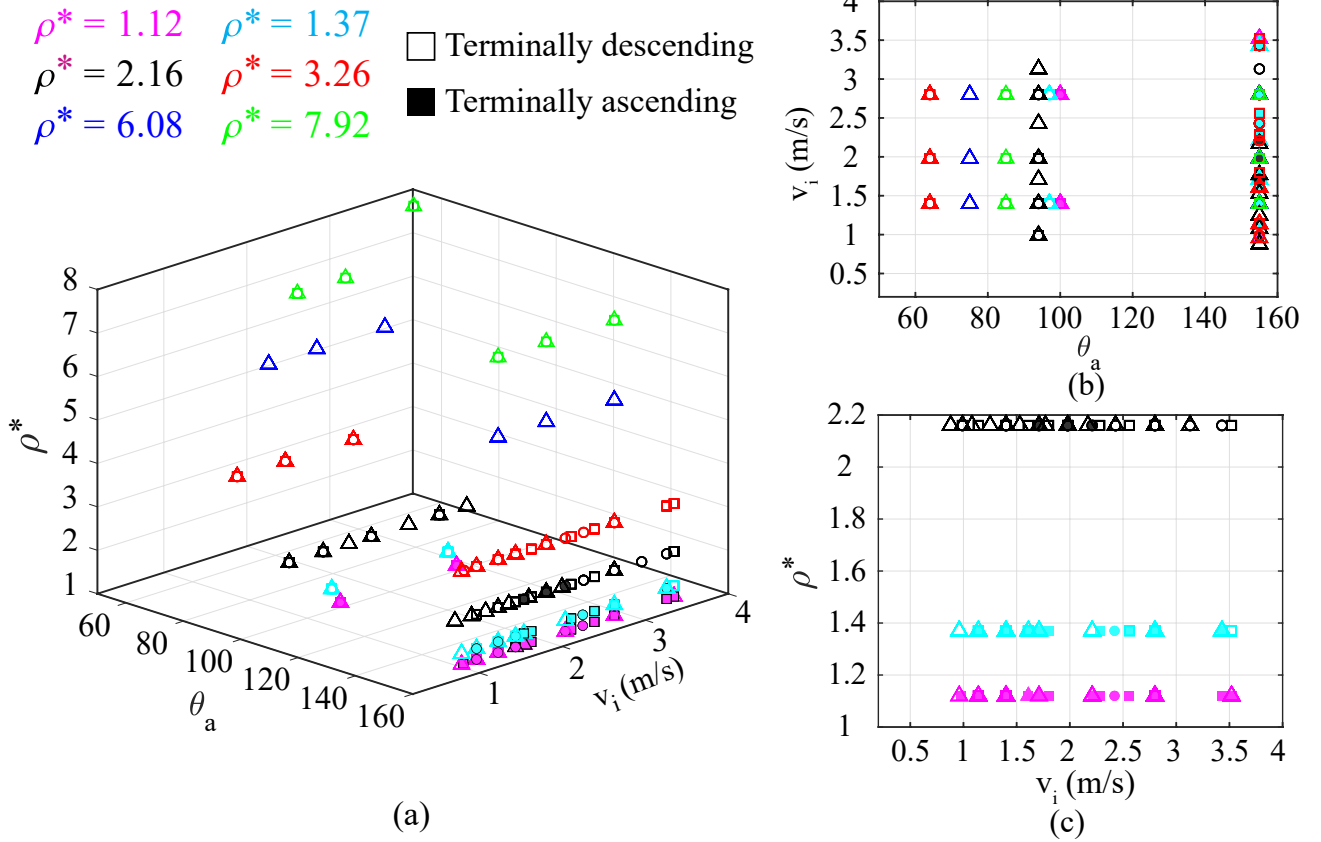


Figure 7. (a) Three-dimensional representation of the parameter space showing the variation with density ratio (ρ^*), static apparent contact angle (θ_a), and impact velocity (v_i) over the entire range of conditions. (b) Two-dimensional projection illustrating the variation of v_i with θ_a , where terminal ascending motion is observed only for $\theta_a > 97^\circ$, while all cases below this threshold exhibit terminal descending behavior. (c) Two-dimensional projection showing the variation of ρ^* with v_i . Both terminal ascending and descending motions are observed; however, for $\rho^* > 2.16$, all spheres, irrespective of θ_a , D , or v_i , consistently exhibit terminally descending motion.

a superhydrophobic sphere exhibits no pinch-off, but reverses direction and ascends. The superhydrophobic sphere is completely encapsulated within an air bubble, beginning already before primary pinch-off (Fig. 2(b)). This encapsulation and the resulting ascending of the sphere is peculiar to the lightest ($\rho^* = 1.12$) superhydrophobic spheres of all diameters and a few slightly heavier spheres ($\rho^* = 2.16$) for $D = 4$ and 6 mm and at moderate impact velocities (see also Fig. 7). No secondary pinch-off is involved and alone the relation (1) explains this behavior.

However, to explain the observations (not) involving a secondary pinch-off, a simple force model to describe the pinch-off is developed, since the secondary pinch-off is instrumental in deciding the terminal buoyancy force. The concept is schematically illustrated in Fig. 8, depicting the sphere with attached air cavity following primary pinch-off. This represents the observed cases in which the air cavity exhibits a necking immediately prior to secondary pinch-off. The model postulates that when the net body forces acting at the cavity neck exceed the surface tension force acting

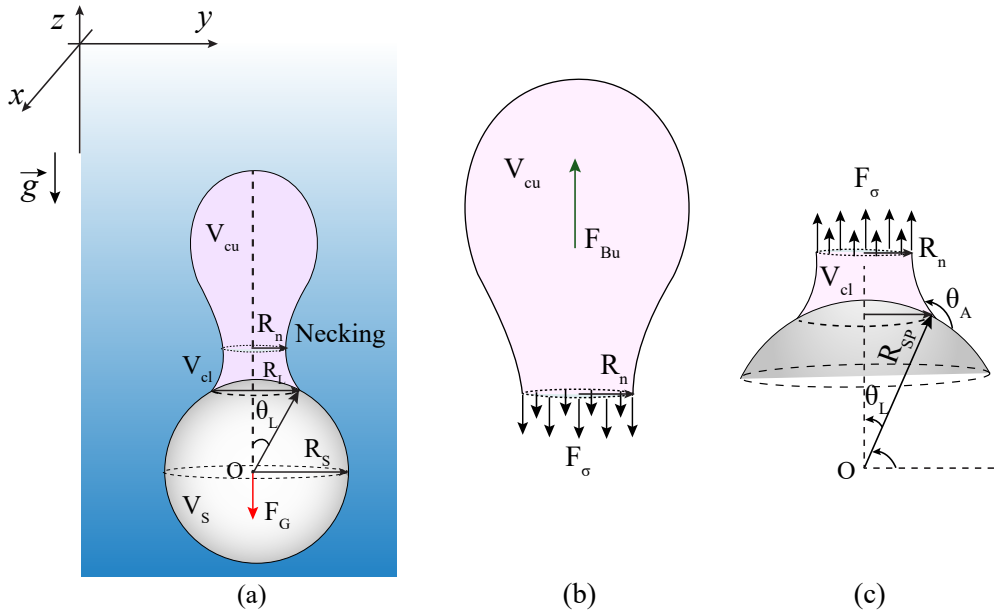


Figure 8. (a) Illustration of the geometric parameters of the sphere and air cavity following primary pinch-off, but immediately prior to secondary pinch-off. (b) and (c) Schematic of a free-body problem in which the air cavity is cut at the neck. Representation of forces acting on the upper (b) and lower (c) sections.

around the bubble neck, then the cavity will pinch-off. This is a highly simplified model in the sense that shear and pressure forces on the cavity (drag) are neglected to first order.

A free-body problem is envisioned, in which the air cavity is conceptually cut at the neck of the air cavity into an upper volume (V_{cu}) and a lower volume (V_{cl}). The surface tension force acting at the cavity neck is simply $F_\sigma = 2\pi R_n \sigma$. The buoyancy force of the upper cavity portion and acting upwards at the neck is $F_{Bu} = \rho_l g V_{cu}$. The body force acting downward at the cavity neck is the weight of the sphere ($F_G = \frac{4}{3}\rho_s g \pi R_s^3$), diminished by the buoyancy of the sphere ($F_{Bs} = \rho_l g \frac{4}{3}\pi R_s^3$) and the lower cavity volume (V_{cl}). Some further simplifications are made, namely that the upper cavity volume V_{cu} is approximately equal to V_{Disp} minus the sphere volume V_s , i.e., $V_{cu} = V_s(V_{disp}^* - 1)$. Moreover, we assume that the lower cavity volume contributes negligible weight or buoyancy to the force acting on the lower portion of the body. Thus, the postulated condition for a secondary pinch-off becomes

$$F_{Bu} + (F_G - F_{Bs}) > F_\sigma \quad (8)$$

which can also be written in several variations as

$$\begin{aligned}
 V_{\text{Disp}}^* &> \frac{3}{2} \frac{R_n \sigma}{g \rho_l R_s^3} + 2 - \rho^* \\
 &> \frac{3}{2} \frac{R_n \lambda_c^2}{R_s^3} + 2 - \rho^* \\
 &> \frac{3}{2} \frac{R_n}{R_s} \text{Bo} + 2 - \rho^*
 \end{aligned} \tag{9}$$

where the capillary length is defined as $\lambda_c = \sqrt{\sigma / \rho_l g}$ and the Bond number is $\text{Bo} = \lambda_c^2 / R_s^2$. These inequalities illustrate the fundamental factors favoring secondary pinch-off (terminally descending), in particular a heavier sphere (higher ρ_s^*), a larger relative air cavity (V_{Disp}^*), a larger sphere radius (R_s) or a smaller (relative) neck radius (R_n / R_s). Note that all quantities in inequality (9) except R_n are fixed in magnitude following primary pinch-off.

Fig. 9 compares the dynamics of hydrophobic spheres ($\rho^* = 1.12$ and $\theta_a = 100^\circ$) under identical impact conditions, highlighting two distinct outcomes: terminally ascending and descending motion. Together with their dimensionless depth trajectories, the forces in Eq. (8) are plotted as a function of time for the case of: (a)-(c) spheres with $D = 4$ mm, $v_i = 2.8$ m/s, and; (d)-(f) spheres with $D = 6$ mm, $v_i = 1.4$ m/s. It is evident from these curves that the terminally descending cases exhibit a decrease of R_n with time; hence, favoring the secondary pinch-off, while the terminally ascending cases do not exhibit a time dependence of R_n (F_σ). Despite identical impact conditions, the sphere-cavity system can evolve toward two distinct terminal states (see Fig. 9). The occurrence of secondary pinch-off emerges as the key mechanism governing this bifurcation, ultimately determining whether the sphere undergoes terminally ascending or terminally descending.

5. Conclusions

This study has characterized the trajectory of spheres following entry into a deep pool. In particular, the influence of sphere wettability on the entrainment of an air cavity and its sustainability throughout the trajectory has been systematically investigated. A parameter range has been identified over which some spheres end up terminally descending, while others ascend back to the pool surface. This depends strongly on the wettability and whether a secondary pinch-off occurs or not. Models derived from force balances have been introduced to predict the volume of the entrained air and the occurrence of a secondary pinch off.

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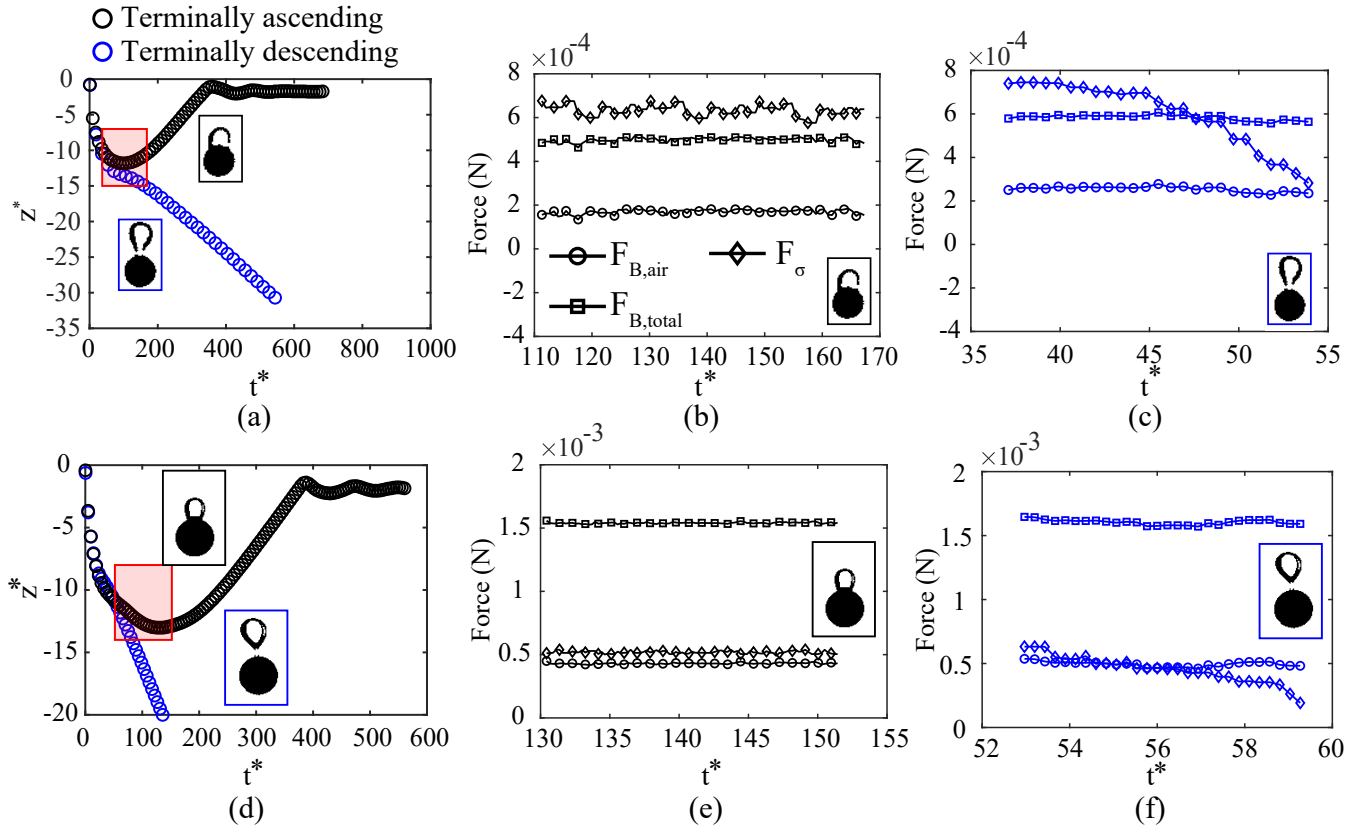


Figure 9. Distinct terminal motion of hydrophobic spheres under identical impact conditions ($\rho^* = 1.12$ and $\theta_a = 100^\circ$). Panels (a–c) correspond to $D = 4$ mm and $v_i = 2.8$ m/s, while panels (d–f) correspond to $D = 6$ mm and $v_i = 1.4$ m/s. Black and blue symbols denote terminally ascending and terminally descending motions, respectively. The shaded regions in (a) and (d) indicate the time intervals examined in (b,c) and (e,f), respectively. Panels (b) and (e) show the force evolution for the terminally ascending cases, whereas panels (c) and (f) correspond to the terminally descending cases. The legend shown in (b) also applies to (c), (e), and (f). The inset images show the final cavity configuration. For terminally ascending motion (black box), the air cavity remains intact without pinch-off, whereas terminally descending motion (blue box) results in cavity pinch-off and complete rupture of the entrapped air cavity.

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Declaration of interests

The authors report no conflicts of interest.

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