

Effect of angle of attack on the wake dynamics of a delta wing

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ABSTRACT

The aerodynamic performance of delta wings is well-documented; however, the spatial mechanisms and energy exchange processes governing vortex shedding at high angles of attack (α) remain poorly understood because of the limitations inherent in traditional single-point measurement techniques. This study addresses this knowledge gap by performing a volumetric experimental investigation into the near-wake dynamics of a non-slender delta wing with a 50° sweep angle. Utilizing three-dimensional (3D) Lagrangian particle tracking velocimetry, the three-dimensional, three-component (3D-3C) flow field was measured at three different angles of attack ($\alpha = 45^\circ, 60^\circ$, and 75°) at a chord-based Reynolds number of 90000. The time-averaged and instantaneous results reveal a fundamental shift in flow topology: capturing a clear transition from stable, attached leading-edge vortices at 45° to complex periodic vortex shedding at higher α . Spectral proper orthogonal decomposition (SPOD) was employed to isolate the most energetic flow structures, demonstrating that the dominant shedding frequency decreases as the angle of attack increases. By reconstructing the flow field at the dominant frequency and conducting phase-coherence testing, this research provides volumetric confirmation that vortices are shed simultaneously from the wing edges at 60° but transition to an alternate shedding pattern at 75° . Using the bispectral analysis, several triadic interactions between the frequencies are identified, revealing distinct energy transfer mechanisms. It is discovered at 60° , the dominant shedding frequency acts as an energy sink, drawing energy from lower frequencies to sustain coherent structures. On the contrary, at 75° , the dominant frequency acts as an energy source, redistributing energy to lower frequencies and resulting in a more chaotic wake state. These findings demonstrate that the wake of the considered delta wing is a low-rank system governed by specific non-linear interactions. This spatio-temporal characterization provides essential physical insights for the development of future theoretical and predictive models in high angle of attack aerodynamics for delta wings.

1. Introduction

The aerodynamic performance of delta wing planforms has remained a cornerstone of aerospace research for decades, primarily due to their unique ability to maintain high lift at high angles of attack. Consequently, several highly maneuverable supersonic fighters, such as the Eurofighter

Typhoon and Saab Gripen, as well as commercial aircraft like the Concorde, employ delta wings. Unlike conventional high-aspect-ratio wings that rely on attached flow to generate lift, the delta wing utilizes the formation of vortices, a phenomenon known as vortex lift. As the flow encounters the swept leading edge of a delta wing, it separates and rolls up into two counter-rotating structures known as leading-edge vortices (LEVs) (Rediniotis et al., 1993; Gursul, 2004; Wang et al., 2016). These vortices induce a low-pressure region on the upper surface of the wing, which significantly enhances lift beyond the predictions of classical theories. Past studies have observed that the strength and stability of these LEVs depend on several factors, including the sweep angle (Ol & Gharib, 2003; Ekaterinaris & Schiff, 1994; Gursul et al., 2005), angle of attack (α) (Tu et al., 2022; Yaniktepe & Rockwell, 2004; Furman & Breitsamter, 2013; Rediniotis et al., 1993), and Reynolds number (Gordnier et al., 2009; Gursul, 1994), among others.

Among the parameters mentioned above, the angle of attack plays a crucial role in dictating the vortex dynamics evolving over a delta wing. Ayoub & McLachlan (1987) conducted flow visualization experiments over α ranging from 25° - 90° and identified four distinct regimes of vortex evolution. Rediniotis et al. (1990) further extended this study, quantifying the unsteady characteristics of the vortex shedding from the delta wing. Based on velocity spectra derived from hot-wire measurements, the authors noted that at 35° , there was no organized shedding of vortices. However, a slight increase in α to 37° resulted in the sudden appearance of periodic vortex shedding. Furthermore, at an α of 80° , the velocity signals showed that the vortices shed alternately from the leading edges. A follow-up study by Rediniotis et al. (1993) further elaborated on these different vortex shedding behaviors. In this study, the authors noted that for α between 35° - 70° , simultaneous shedding of vortices from the leading edges occurs. However, for α greater than 70° , two shedding modes were noted: a lower-frequency simultaneous shedding and a higher-frequency alternate shedding of vortices from the leading edges. Furthermore, it was also observed that the shedding frequency decreased with an increase in α . Gursul (1994) also observed simultaneous and alternate vortex shedding patterns in their study. However, the difference in shedding behavior was attributed to the change in the aspect ratio of the delta wing. In that study, it was further noted that the influence of vortex shedding on unsteady pressure fluctuations was negligible. As such, the source of the unsteady loading on the delta wing was attributed to helical mode instability as opposed to vortex shedding. Further studies by Gursul et al. (2005) on non-slender delta wings showed that, at very low α , vortical flow formed close to the wing surface interacts with the boundary layer vorticity and might result in the formation of a dual vortex structure. However, with an increase in α , the primary vortex was observed to become stronger and move away from the wing surface, resembling the vortex structure of a slender wing. Tu et al. (2022) noted that an increase in α results in the LEVs approaching each other while departing from the wing. It was further noted that the flow at sufficiently high α was dominated by separation, similar to fully separated flow around a 2-D panel wing. However, the spanwise gradients were observed to remain relatively large near the leading edge, and it was suggested they should not be ignored.

Based on these past studies, it can be concluded that a change in α results in a shift in the vortex evolution pattern - transitioning from an attached vortex to simultaneous vortex shedding, and eventually to alternate vortex shedding. However, most studies rely on hot-wire or pressure measurements to understand vortex dynamics. Since these techniques are single-point measurements, spatio-temporal information regarding the evolution of vortical structures cannot be determined. Other studies that have conducted flow field measurements have not identified the underlying reasons for the different mechanisms of vortex shedding associated with changes in α . Furthermore, knowledge regarding the specific vortical structures shedding from the leading edge and their association with various shedding frequencies remains sparse.

As such, the aim of this study is to perform an experimental investigation to understand the effect of the angle of attack on the evolution of vortex structures in the near wake of a non-slender delta wing. These objectives are achieved by measuring the three-dimensional flow field, as detailed in section 2, along with the experimental setup and operating conditions. section 3 discusses the time-averaged and instantaneous flow fields, the spatial modes associated with different frequencies, and the interaction between frequencies present in the flow field. Finally, conclusions are presented in section 4.

2. Experimental methodology

Experiments were conducted in the University of Southampton's Boundary Layer Wind Tunnel, which has dimensions of $1.2 \text{ m} \times 1.0 \text{ m} \times 12 \text{ m}$ ($W \times H \times L$) and is equipped with a heat exchanger to maintain constant operating conditions (schematic shown in the top of Fig. 1). A triangular-shaped non-slender delta wing with a span of 230 mm and a sweep angle of 50° with the wing being tapered at the apex was used in the current study. As shown in the bottom right of Fig. 1, the delta wing was sting-mounted from the roof of the wind tunnel. In this study, three angles of attack ($\alpha = 45^\circ, 60^\circ, \text{ and } 75^\circ$) were considered, and the experiments were performed at a chord-based Reynolds number of 90,000.

In this study, the three-dimensional, three-component (3D-3C) flow field in the wake of the delta wing was measured using three-dimensional particle tracking velocimetry (3D-PTV). As shown in the bottom left of Fig. 1, four Phantom V641 high-speed cameras were used, with two cameras mounted on either side of the wind tunnel. The V641 features a maximum resolution of 2560×1600 pixels (4 megapixels) at a maximum frame rate of 1450 frames per second. Nikon 50 mm lenses with an f-number of 5.6 were utilized. Volumetric field illumination for the 3D-3C velocity measurements was achieved using a LaVision Flashlight-300 LED system, which emits light with an intensity peak in the blue spectrum (445 nm). To visualize the flow field, neutrally buoyant helium-filled soap bubbles with diameters ranging from 100 to 300 microns were used as tracer particles. These were generated using a LaVision Helium-Filled Soap Bubble (HFSB)

generator upstream of the test section. Data acquisition was performed using LaVision's DaVis software at a frequency of 1333 Hz in time-series mode for approximately 6 seconds, resulting in 7817 images per case. For all three α cases, a consistent measurement volume was maintained with dimensions of $-180 \leq x \leq 180$ mm, $-220 \leq y \leq 180$ mm, and $-150 \leq z \leq 120$ mm. Image calibration was conducted using a custom-built 500×800 mm calibration plate featuring 10 mm dots at 25 mm intervals. Calibration was performed at three planes ($z = -120, 0$, and 120 mm) using a 3rd-order polynomial, yielding a fit error of ≤ 0.5 pixels and a scale factor of 3 pixels/mm at the focal plane. Supplementing the dot-card calibration, volume self-calibration resulted in an averaged voxel disparity of 0.14 voxels and a pixel disparity of 0.08 pixels. Based on the camera magnification and the defined volume, a particle density of 0.03 particles per pixel was achieved.

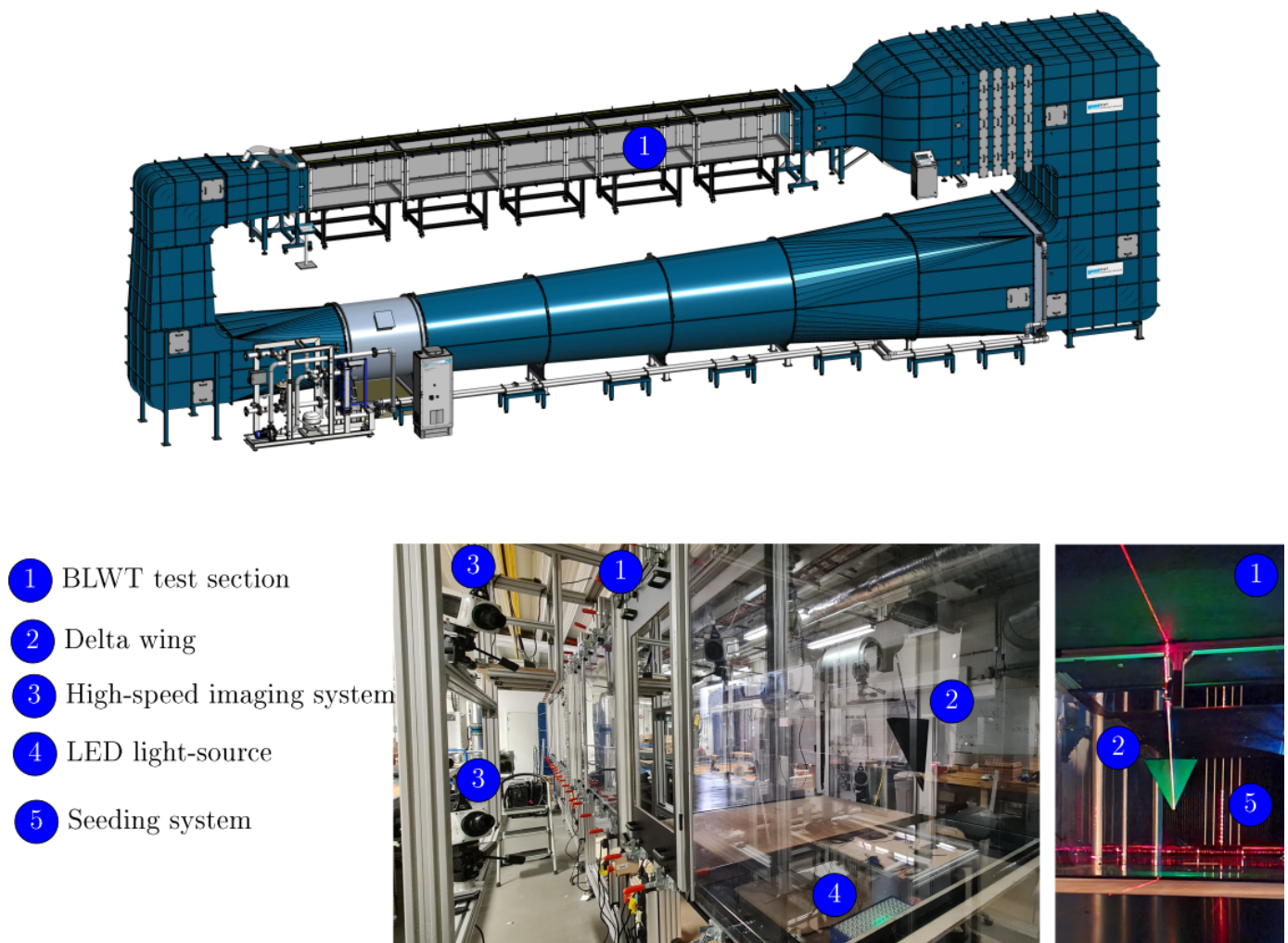


Figure 1. Top: CAD schematic of the Boundary Layer Wind Tunnel (BLWT). Bottom: Light source and camera system mounted to visualize the flow field (left) and delta wing mounted inside the wind tunnel (right).

The acquired flow field images were processed using LaVision's DaVis software, utilizing 3D Lagrangian PTV, specifically the Shake-the-Box (STB) algorithm, to process the data. Previous studies

have suggested that for flow fields dominated by vortical flows with high particle densities, the STB algorithm can achieve higher reconstruction accuracy compared to conventional tomographic particle image velocimetry algorithms (Schröder et al., 2015; Schanz et al., 2016). Prior to performing STB, raw images were preprocessed using a background subtraction methodology based on the minimum intensity over time. Following image preprocessing, the STB algorithm was applied to determine particle tracks. To enhance the accuracy of particle positions, forward and backward passes in time were implemented, resulting in a total average of 30,000 particle tracks. Subsequently, the VIC# (vorticity time-marching method in fine-scale reconstruction) was utilized to obtain the velocity field. A $24 \times 24 \times 24$ grid was used during the VIC# process, resulting in a final vector grid of $45 \times 50 \times 34$ with a vector spacing of 8.131 mm ($\approx 0.033c$). Finally, a Gaussian time and space filter was applied using a $5 \times 5 \times 5$ spatial grid and 11 time steps to smooth the vector field for further analysis.

3. Results and discussions

The results section of this paper is organized into three primary subsections. In section 3.1, the discussion focuses on the evolution of both the instantaneous and time-averaged flow fields within the near wake of the delta wing. Section 3.2 then examines the various frequencies identified in the flow field, the spatial structures linked to those frequencies, and the corresponding reconstructed velocity. Finally, section 3.3 addresses the interactions between these frequencies and the energy transfer mechanisms associated with them.

3.1. Flow field evolution in the near wake

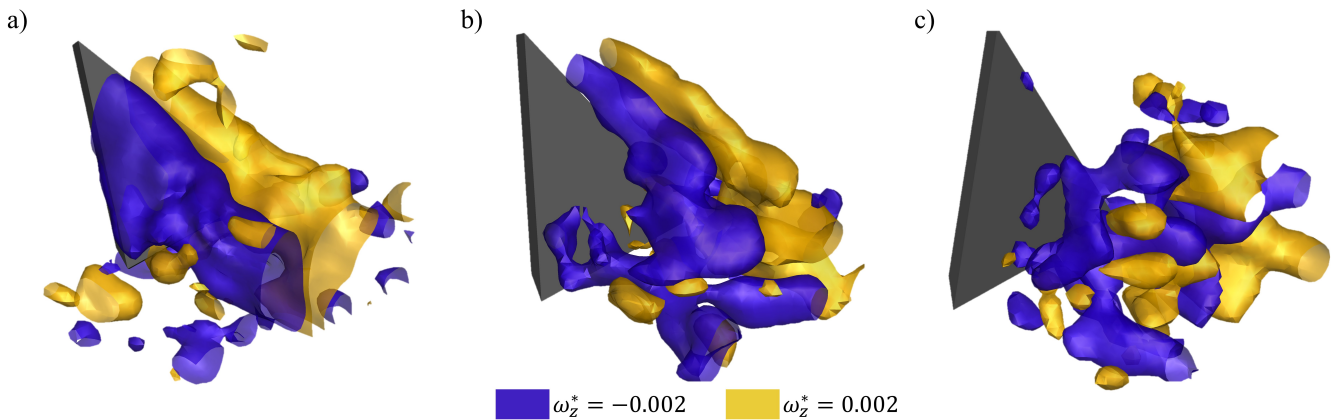


Figure 2. Instantaneous flow field represented by the iso-surface of z-vorticity ($\omega_z^* = \frac{\omega_z c}{U_\infty}$) for (a) 45° , (b) 60° , and (c) 75° . The blue and yellow iso-surface represent the negative ($\omega_z^* = -0.002$) and positive ($\omega_z^* = 0.002$) z-vorticity.

Fig. 2 illustrates the instantaneous flow field evolving over the delta wing under different α conditions. In this visualization, the flow is represented by positive (yellow) and negative (violet) iso-surfaces of streamwise vorticity, $\omega_z^* = \frac{\omega_z c}{U_\infty}$, where ω_z is the streamwise vorticity, c is the wing chord, and U_∞ is the freestream velocity. For $\alpha = 45^\circ$, two counter-rotating vortices of opposing magnitudes are clearly visible on the delta wing and extend into the wake (Fig. 2(a)). These vortices remain attached to the wing surface. Consequently, the time-averaged flow field at 45° shows two coherent vortices emanating from the leading edges, remaining attached and extending into the wake, as shown in Fig. 3(a). In Fig. 3, the flow is represented by the iso-surface of non-dimensional vorticity magnitude, $|\omega|^* = \frac{|\omega|c}{U_\infty} = 0.006$, where $|\omega|$ is the vorticity magnitude. As α increases to 60° , multiple vortex structures begin to form and shed, and the clear coherent structures observed at 45° are no longer present. Two distinct sets of vortices emerge: (i) a pair arising from the apex of the delta wing, and (ii) a pair arising near the trailing-edge region (Fig. 2(b)). Due to this continuous formation and shedding, the time-averaged flow reveals vorticity shedding primarily from the edges of the wing, as shown in Fig. 3(b). With a further increase in α to 75° , the flow field becomes increasingly chaotic, with vortices shedding from multiple locations along the wing edges, as shown in Fig. 2(c). Similar to the 60° case, the time-averaged flow at 75° displays vorticity shedding from the edges, as seen in Fig. 3(c). The primary difference between the 60° and 75° cases is the expansion of the wake at the higher α , which is attributed to the increased projected area of the delta wing.

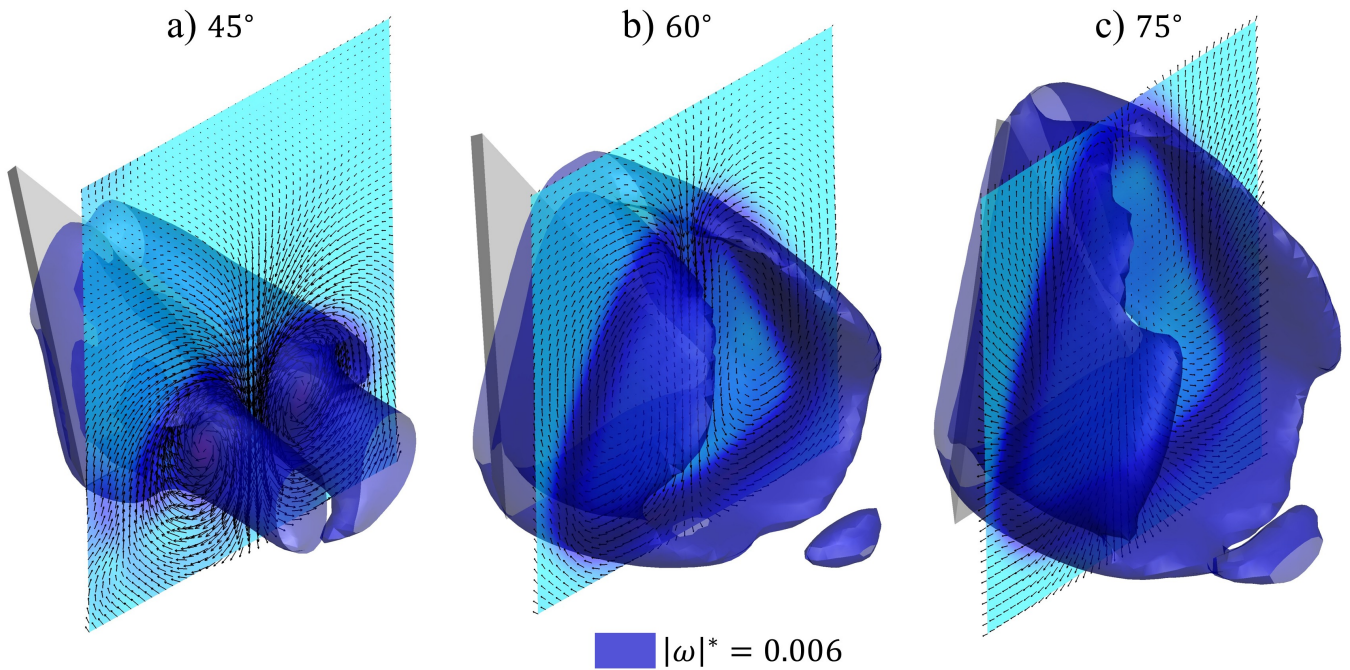


Figure 3. Time-averaged flow field represented by the iso-surface of vorticity magnitude ($|\omega|^* = \frac{|\omega|c}{U_\infty} = 0.006$) for (a) 45° , (b) 60° , and (c) 75° . The 2D slice is shown in the mid-volume and shows the contour of vorticity magnitude with the velocity vectors overlaid.

Because multiple vortex structures are formed and shed into the wake for $\alpha = 60^\circ$ and 75° , a Power Spectral Density (PSD) analysis was performed to determine the dominant frequencies associated with vortex shedding. Fig. 4 illustrates the spatial variation of the PSD performed at $\frac{x}{s/2} = -0.5, \frac{y}{c} = 1.5$ (a,c) and $\frac{x}{s/2} = 0.5, \frac{y}{c} = 1.5$ (b,d) for $\alpha = 60^\circ$ (a,b) and 75° (c,d). In this figure, the contour represents the power spectral density (PSD), determined using the *pwelch* function in *MATLAB*; the x -axis denotes frequency, and the y -axis represents the streamwise location behind the trailing edge of the delta wing. From the figure, it can be noted that the PSD is very weak for frequencies above 20 Hz, suggesting that higher frequencies do not contain significant energy. Furthermore, for both cases, the PSD is significantly weak across all frequencies for streamwise locations below $z/c < 0.4$. Above $z/c = 0.4$, at frequencies below 10 Hz, the PSD becomes stronger, and a dominant frequency can be clearly observed, revealing a major difference between the two cases. For $\alpha = 60^\circ$, the dominant frequency is $f = 6.5$ Hz, while for 75° , it is 4.5 Hz. In other words, the dominant frequency, which likely corresponds to the vortex shedding frequency, decreases as α increases. A similar pattern of vortex shedding was reported by Rediniotis et al. (1993), where the vortex shedding frequency decreased with an increase in α . In that same study, Rediniotis et al. (1993) also observed simultaneous and alternate shedding of vortices for a delta wing mounted at $\alpha = 60^\circ$ and 75° , respectively. However, a simple PSD cannot determine if a similar phenomenon occurs in the present scenario. Additionally, the PSD shows that multiple other frequencies are present alongside the dominant ones, which may be related to other vortical structures shedding from the wing edges. Therefore, it is necessary to extract the flow structures associated with the dominant frequency and study the associated flow physics, which is discussed in the following section.

3.2. Vortex dynamics

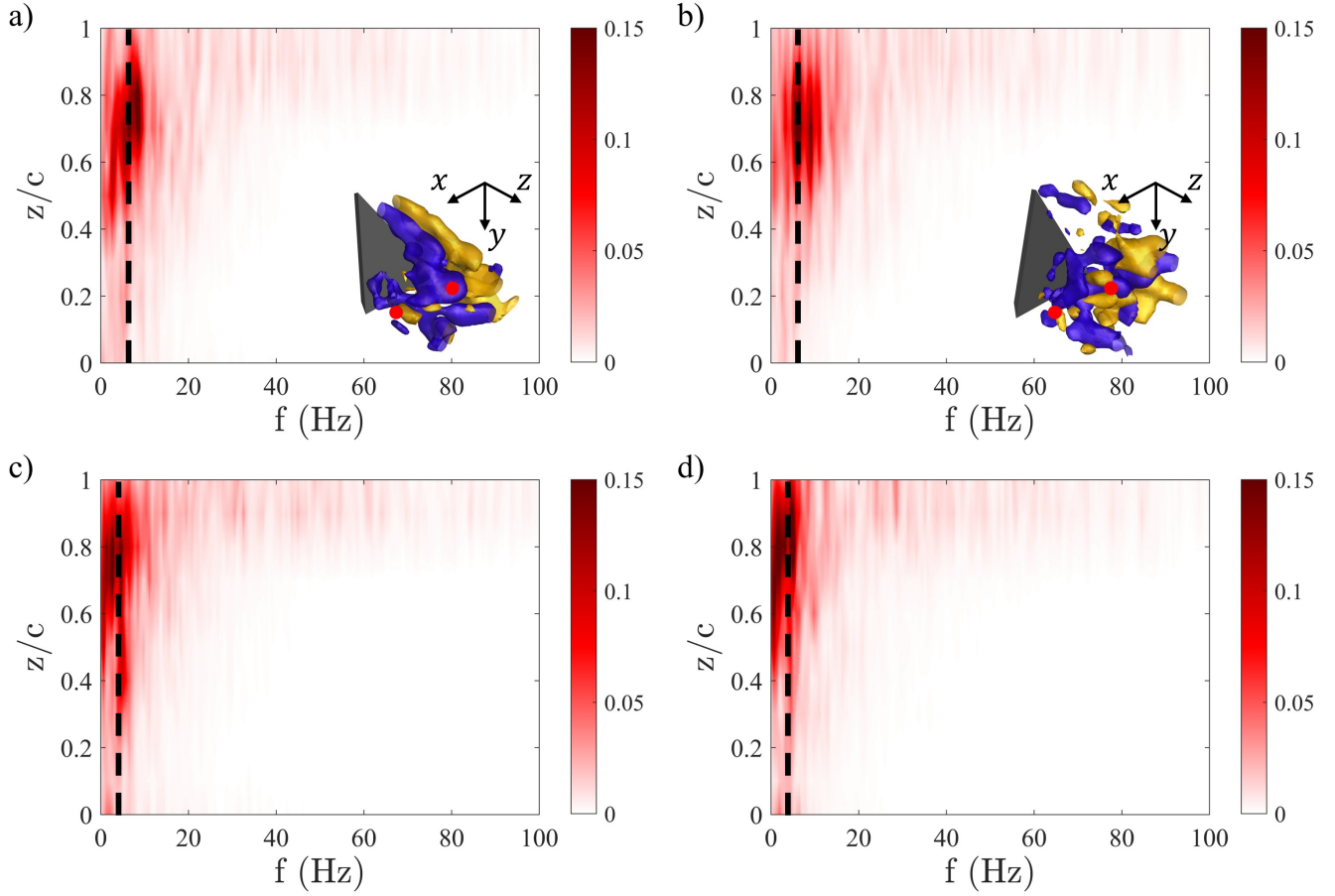


Figure 4. Spatial variation of FFT with frequency for 60° (a,b) and 75° (c,d). The FFT is performed at $\frac{x}{s/2} = -0.5$ and $\frac{y}{c} = 1.5$ (a,c), and $\frac{x}{s/2} = 0.5$ and $\frac{y}{c} = 1.5$ (b,d). The black dashed line in the figures show the dominant frequency for different α and locations. Inset in (a,b) shows the instantaneous flow field at 60° , and 75° . The red dots show the x and y locations where the velocity was probed to perform FFT.

In the previous section, it was discussed that a coherent and attached vortex exists for $\alpha = 45^\circ$, whereas for 60° and 75° , multiple vortices are formed and shed, potentially associated with different shedding frequencies. As such, further discussion regarding the flow physics will focus on a comparison between $\alpha = 60^\circ$ and 75° .

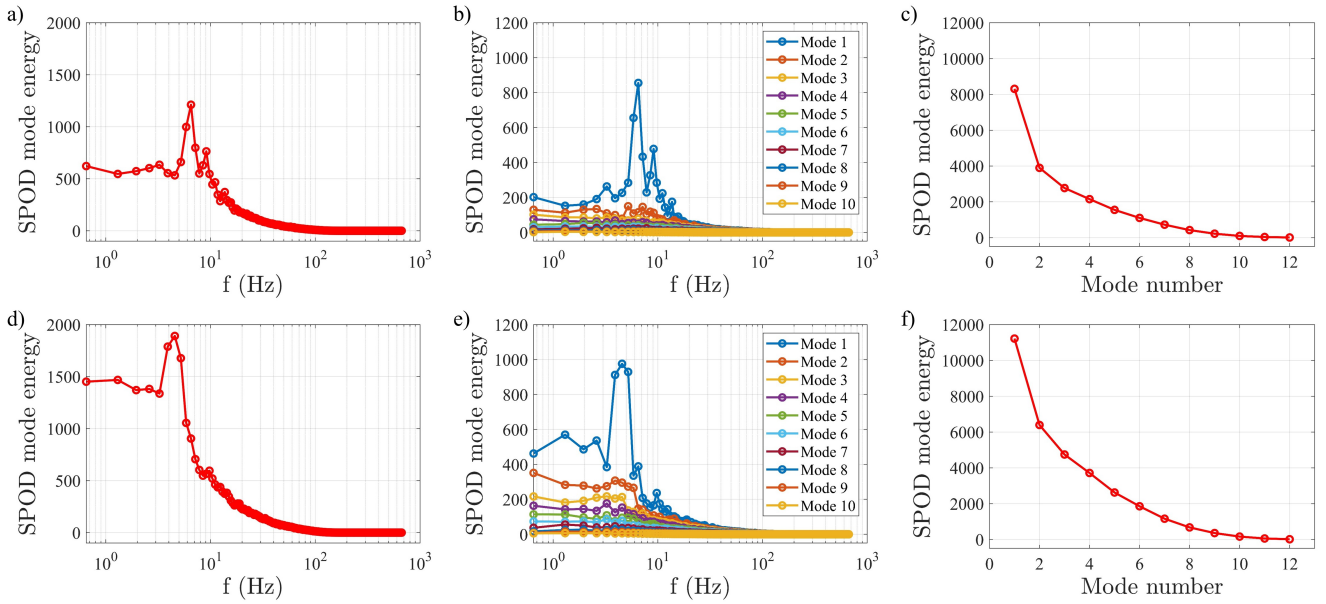


Figure 5. (a,d) Variation of total SPOD mode energy with frequency. (b,e) Variation of SPOD mode energy for the first 10 modes with frequency. (c,f) Variation of total SPOD mode energy with mode number. In the figure, (a,b,c) is for $\alpha = 60^\circ$ and (d,e,f) is for $\alpha = 75^\circ$.

To determine the frequencies and the spatial structures associated with them, spectral proper orthogonal decomposition (SPOD) was performed on the 3D flow field in the wake of the delta wing. Details regarding the SPOD methodology are provided by Towne et al. (2018). Fig. 5 illustrates the variation of SPOD mode energy with frequency and mode number for $\alpha = 60^\circ$ and 75° . In Fig. 5(a,d), it is observed that most of the energy is concentrated in the lower frequencies, with frequencies exceeding 50 Hz contributing negligibly to the overall flow field for both α . Consistent with the frequency trends observed in Fig. 4, Fig. 5(a,d) show that an increase in α results in a decrease in the dominant frequency. Examining the individual contributions of different modes, the first mode contributes the most energy to the overall flow field at the dominant frequencies in both cases (Fig. 5(b,e)). Furthermore, Fig. 5(c,f) indicates that the first mode is the primary energy contributor, and beyond the 10th mode, the contribution of higher modes becomes negligible for both cases. Based on the insights gained from this modal energy variation, further analysis will be restricted to the dominant frequency for both α cases.

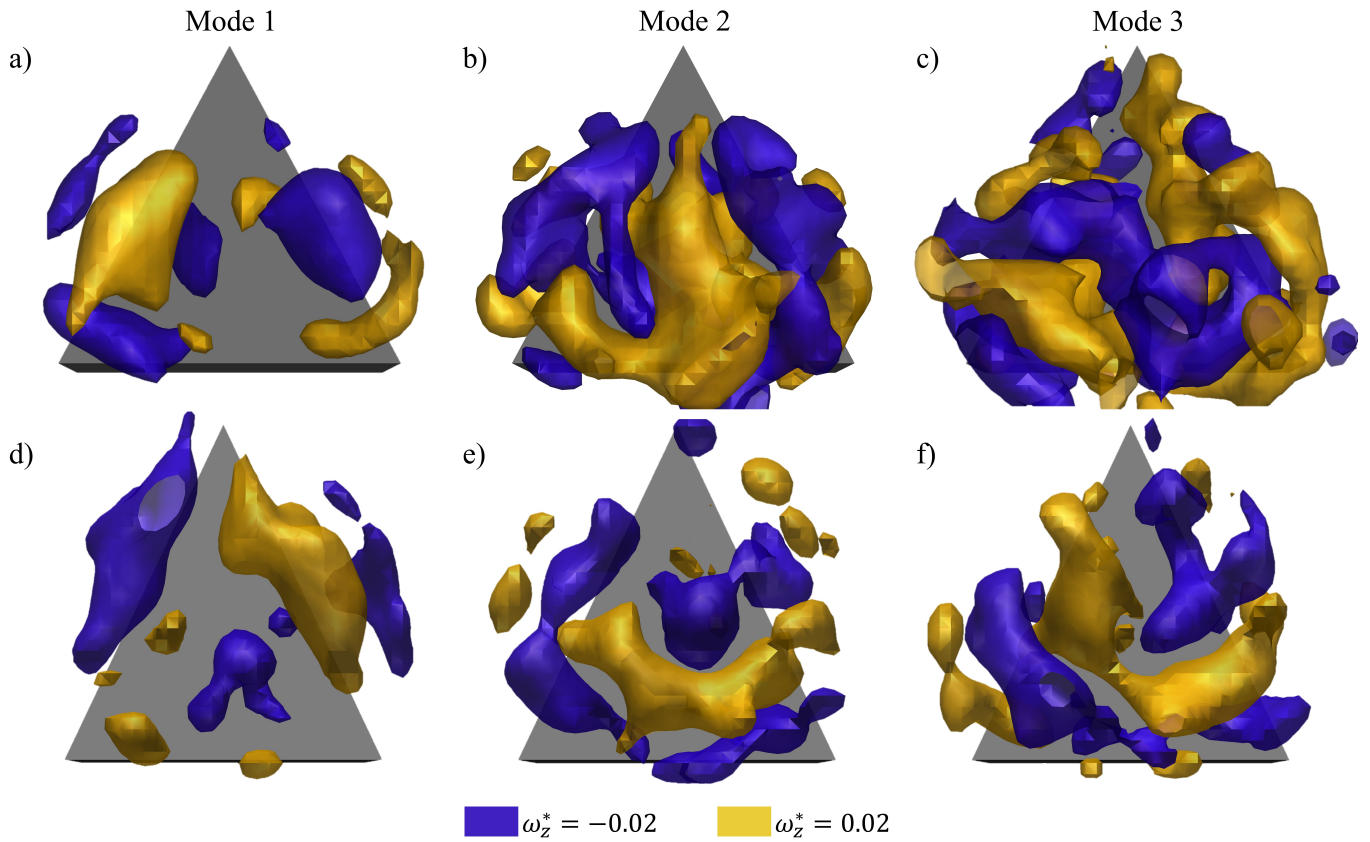


Figure 6. First three SPOD spatial modes for 60° (a,b,c) at $f = 6.5$ Hz and 75° (d,e,f) at $f = 4.5$ Hz. The flow structures are represented by iso-surfaces of positive (yellow) and negative (violet) ω_z^* .

For the dominant frequencies, spatial modes were extracted to determine the associated flow structures. Fig. 6 illustrates the first three modes at 6.5 Hz for 60° (a,b,c) and 4.5 Hz for 75° (d,e,f). In this figure, the flow structures are represented by iso-surfaces of positive (yellow) and negative (violet) ω_z^* . A clear distinction in flow structures is evident between the first mode for 60° (Fig. 6(a)) and 75° (Fig. 6(d)). At 60°, structures form at both the leading and trailing edges of the delta wing and exhibit opposite-signed vorticity. Conversely, at 75°, the flow structures originate predominantly from the leading edges. Additionally, the first spatial mode for 60° does not show flow structures forming at the wing apex. The second mode for 60° (Fig. 6(b)) reveals flow structures throughout the field except near the apex, suggesting this mode captures the convection of structures from the edges. For 75°, the second mode captures shedding from the trailing edge (Fig. 6(e)), a feature further identified in the third mode (Fig. 6(f)). The third mode for 60° displays structures forming and shedding from the apex as well as the wing edges. These spatial modes demonstrate that the flow structures associated with the dominant frequencies at 60° and 75° are distinct, with different modes capturing formation and shedding of vortices at various locations along the wing edges.

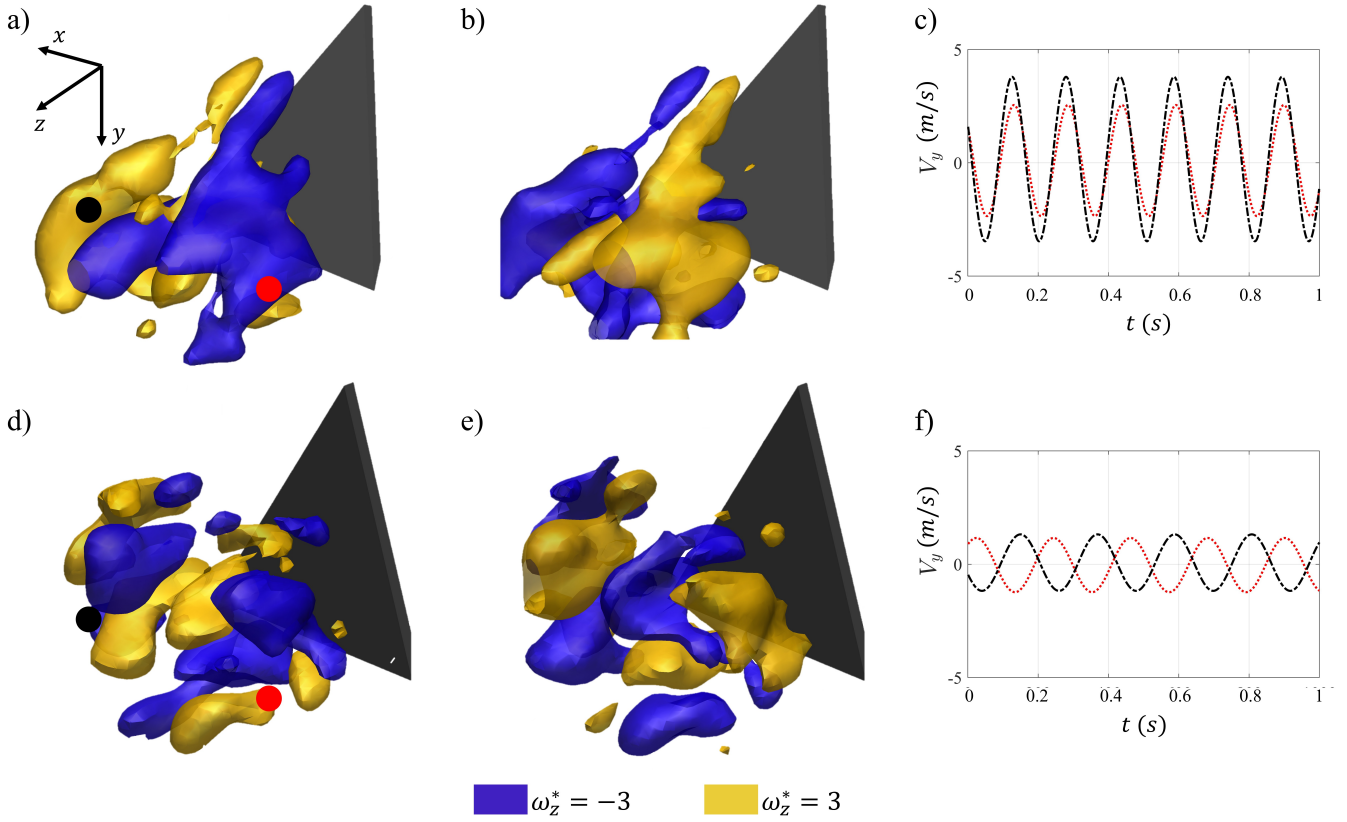


Figure 7. Reconstructed flow field at (a,b) $f = 6.5$ Hz for 60° , and (d,e) $f = 4.5$ Hz for 75° . Black and red dots in (a) and (d) represent the points at which the velocity signals are studied. (c,f) Temporal variation of the normal velocity (V_y) at $y/c = 0.25$, $z/c = 0.5$, and $x/(s/2) = -0.5$ (black dashed line), and $y/c = 0.25$, $z/c = 0.5$, and $x/(s/2) = 0.5$ (red dashed line) for 60° and 75° , respectively.

Using the first 10 spatial modes (as modes beyond the 10th contribute negligible energy to the overall flow field) and their corresponding temporal coefficients, the flow field was reconstructed at 6.5 Hz for 60° and 4.5 Hz for 75° , as shown in Fig. 7(a,b) and (d,e), respectively. In Fig. 7(a), large-scale flow structures of opposite signs can be seen shedding from the two leading edges of the delta wing. Fig. 7(b), captured exactly one cycle later, displays nearly identical flow structures but with opposite vorticity magnitudes, suggesting simultaneous shedding. To confirm this, the normal velocity (V_y) was probed at two locations equidistant from the centerline (indicated by black and red dots in Fig. 7(a)) and plotted as a time-series in Fig. 7(c). The signals are nearly in-phase, with a phase difference of $\approx 10^\circ$ (determined by performing a coherence test between the two signals), confirming that vortices at 60° shed simultaneously from the leading edges. In contrast, the 75° case exhibits smaller-scale flow structures. Across the centerline, it is difficult to qualitatively determine if similar structures are shedding, but a clear difference is visible in the subsequent cycle shown in Fig. 7(d) and (e). The V_y time-series for 75° (Fig. 7(f)) clearly shows that the velocity signals are out-of-phase, with a phase difference of $\approx 160^\circ$. This confirms that at 75° , vortices shed alternately from the wing edges. These observations corroborate the findings of Rediniotis et al.

(1993), where it was reported that simultaneous shedding near $\alpha = 60^\circ$ and alternate shedding for $\alpha = 75^\circ$ occurs. In addition to confirming the observations made by Rediniotis et al. (1993), the current study also has shown the spatio-temporal of these flow structures under different α conditions at the dominant frequency.

3.3. Interaction between multiple frequencies

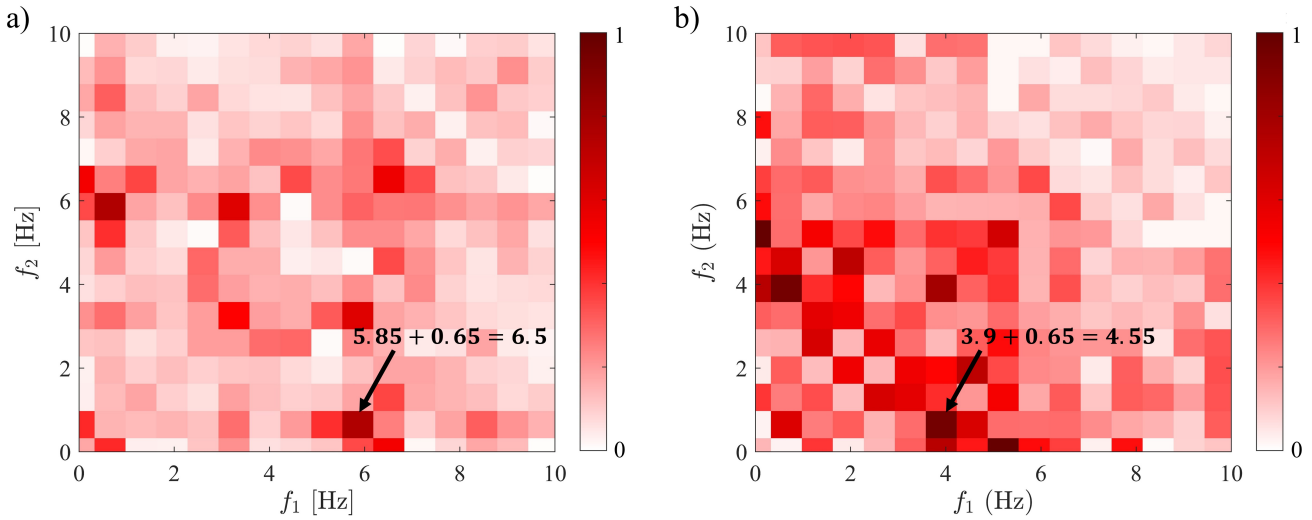


Figure 8. Bispectrum magnitude for (a) 60° and (b) 75° . The arrows in (a) and (b) correspond to the dominant frequencies for 60° and 75° .

As observed in Fig. 5(a) and 4, the wake of the delta wing contains multiple frequencies. These frequencies can interact, facilitating energy transfer that strengthens or weakens specific frequencies and their associated flow structures depending on the direction of that transfer. To determine the magnitude of these interactions, the bispectrum was computed. The bispectrum is a higher-order statistical tool uniquely capable of identifying whether two frequencies, f_1 and f_2 , are non-linearly interacting to generate a third frequency, f_3 (Kim & Powers, 2007; Schmidt, 2020; Biswas & Buxton, 2024). However, for the bispectrum to be finite and non-zero, the following conditions must be met:

$$f_3 = f_1 + f_2 \quad (1)$$

$$\theta_3 = \theta_1 + \theta_2 \quad (2)$$

where θ_1 , θ_2 , and θ_3 correspond to the phases of the respective frequencies. The frequencies that meet this condition are known as a triad. In general, the bispectrum serves as a measure of the

magnitude of the nonlinearity present in the Navier-Stokes equations, where a bispectrum value of zero indicates that the flow is purely linear. The bispectrum is mathematically defined as follows:

$$B(f_1, f_2) = E[\hat{X}(f_1)\hat{X}(f_2)\hat{X}^*(f_1 + f_2)] \quad (3)$$

where $\hat{X}(f)$ is the complex Fourier coefficient of the signal at frequency f , $\hat{X}^*$ denotes the complex conjugate, and $E[\cdot]$ represents the expectation operator. The bispectrum magnitude is shown in Fig. 8(a,b) for 60° and 75° , respectively. In the figure, the x - and y -axes represent f_1 and f_2 , respectively, and the contour color denotes the bispectrum magnitude. Notably, the bispectrum magnitude in the figure is normalized using its maximum value.

Comparing the bispectrum magnitudes between 60° and 75° reveals clear distinctions. For 60° , as shown in Fig. 8(a), high bispectrum magnitude is localized to specific frequencies. In contrast, for 75° , higher bispectrum magnitude is spread across multiple frequencies, as seen in Fig. 8(b). This suggests that multiple flow structures associated with various frequencies interact over a broader range for 75° compared to 60° . Focusing on the dominant frequency for both α , the bispectrum magnitude is closer to one for 75° , whereas it is significantly lower for 60° . As indicated by the arrows in the Fig. 3.3(a), for 60° , frequencies of 5.85 Hz and 0.65 Hz interact to generate $f_3 = 6.5$ Hz, the dominant frequency. Conversely, for 75° , 3.9 Hz and 0.65 Hz interact to produce the dominant frequency of 4.55 Hz (indicated by the arrows in Fig. 3.3(b)). These observations suggest that, at the respective dominant frequencies for both α , nonlinear interactions between parent frequencies are considerably higher for the delta wing at 75° .

The bispectrum has established that strong nonlinear interactions exist between two lower frequencies to create a third frequency, which is the dominant frequency. To understand the direction of this energy transfer, the triadic energy transfer was computed at the previously mentioned triads for 60° and 75° . Triadic energy transfer measures the rate at which energy is exchanged between the interacting frequencies in a specific triad. This transfer originates from the nonlinear convective term in the Navier-Stokes equations. In the frequency domain, the rate of energy transfer due to this nonlinearity is expressed as follows:

$$T(f) = Im\left[\sum_{f_1+f_2=f} \hat{u}^*(f) \cdot \mathcal{N}(\hat{u}(f_1), \hat{u}(f_2))\right] \quad (4)$$

where $\mathcal{N}$ represents the Fourier-transformed convective term (Domaradzki & Rogallo, 1990). The triadic transfer of energy summed over the entire spatial domain can be written as follows Biswas & Buxton (2024):

$$T_{f_1, f_2, f_3}(t) = -\frac{1}{2} \sum_{f_1, f_2} \tilde{u}_i^{f_3} \tilde{u}_j^{f_2} \frac{\partial \tilde{u}_i^{f_1}}{\partial x_j} \quad (5)$$

where f_1 , f_2 , and f_3 represent a triad. It should be noted that triadic transfer is non-negligible only when the frequencies satisfy the triad condition. As discussed previously, specific triads were identified for 60° and 75° ; consequently, the triadic transfer was investigated for these specific sets of frequencies, with the evolution of the triadic energy transfer shown in Fig. 9.

Comparing the two cases reveals a clear distinction between the α conditions. At 60° , the mean energy transfer between the frequencies is positive (Fig. 9(a)), whereas for 75° , the energy transfer is negative (Fig. 9(b)). This suggests that at 60° , the lower frequencies identified in the bispectrum ($f_1 = 5.85$ Hz and $f_2 = 0.65$ Hz) provide energy to the dominant frequency ($f_3 = 6.5$ Hz), which strengthens the flow structures forming and shedding at that frequency. Conversely, for 75° , the dominant frequency of 4.5 Hz provides energy to the lower frequencies ($f_1 = 3.9$ Hz and $f_2 = 0.65$ Hz), suggesting that the flow structures shedding at 4.5 Hz continuously lose energy to lower frequencies. However, as shown in the bispectrum in Fig. 9(b), nonlinear interactions at 75° are spread across multiple frequencies; thus, other frequencies may also be providing energy to the dominant frequency at 75° .

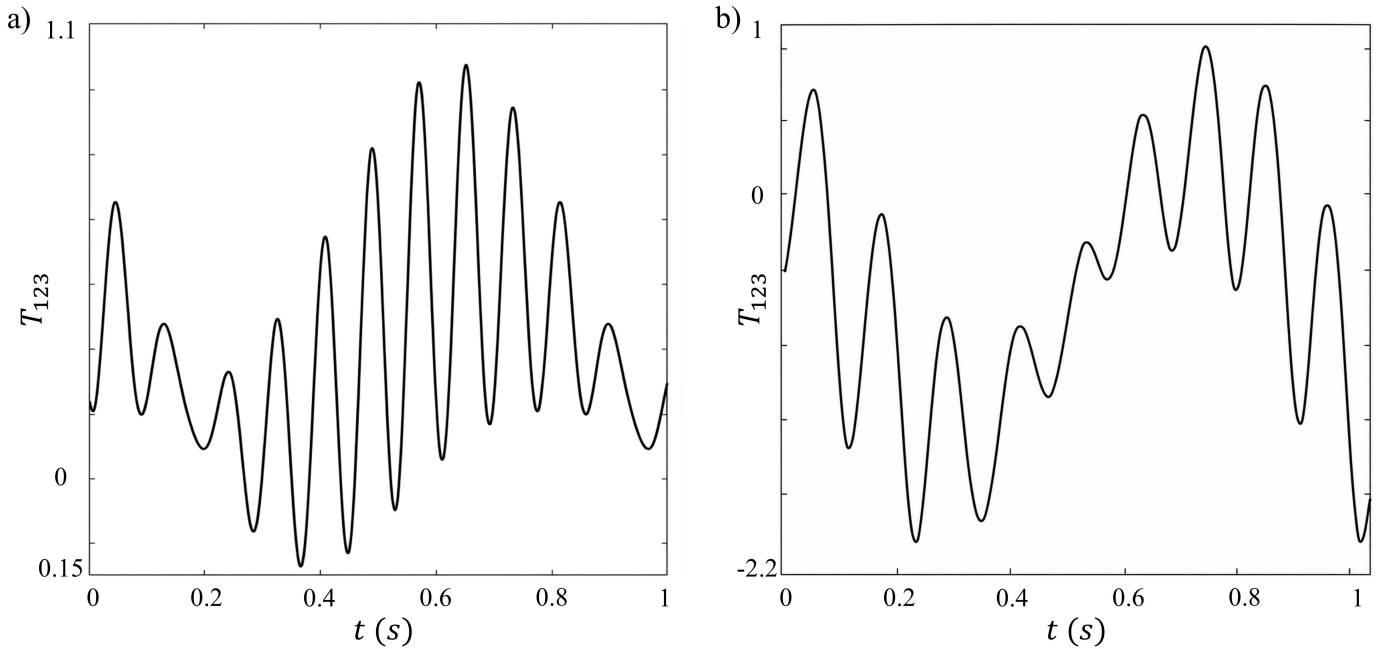


Figure 9. Temporal evolution of triadic energy transfer for (a) 60° and (b) 75° .

4. Conclusions

An experimental investigation was conducted to understand the effects of the angle of attack on the near wake of a non-slender delta wing. Three-dimensional particle tracking velocimetry (3D-

PTV) was used to quantify the spatio-temporal evolution of the flow field for three different angles of attack: 45° , 60° , and 75° . The instantaneous and averaged flow fields showed that an increase in the angle of attack from 45° to 75° resulted in the vortex shifting from an attached vortex regime to a vortex shedding regime. Furthermore, the flow field in the wake becomes increasingly chaotic as the angle of attack increases from 60° to 75° .

Power spectral density analysis of the wake showed that the shedding frequency decreased as the angle of attack increased. Because multiple frequencies were present in the flow field, spectral proper orthogonal decomposition (SPOD) was performed on the 3D flow field to conduct a modal decomposition. Based on the variation of modal energy, the first spatial mode was the most dominant for $\alpha = 60^\circ$ and 75° , while modes beyond the 10th contributed negligibly to the overall flow field. At the respective dominant frequencies for $\alpha = 60^\circ$ and 75° , the spatial modes showed significant variation. For $\alpha = 60^\circ$, the first mode suggested that flow structures shed from both the leading edges and the trailing edge of the delta wing. In contrast, for $\alpha = 75^\circ$, flow structures predominantly formed and shed only from the leading edges. Using the first ten spatial modes and their corresponding temporal coefficients, the flow field was reconstructed at the dominant frequency, and the wake velocity was probed to determine the vortex shedding pattern. A comparison and coherence test of these velocity signals showed that vortices are shed simultaneously from the edges of the delta wing at 60° , whereas they are shed alternately at 75° .

Due to the presence of multiple frequencies and associated structures, a bispectrum was computed to identify nonlinear interactions. The bispectrum magnitude revealed that non-linearity was localized to specific frequencies at 60° but spread across multiple frequencies at 75° . Similarly, at the respective dominant frequencies, the bispectrum magnitude was much stronger for 75° , suggesting stronger non-linear interactions between the specific triad of frequencies compared to 60° . Analysis of the triadic energy transfer for 60° showed that the dominant frequency acted as a sink, gaining energy from lower frequencies and strengthening the associated shedding flow structures. In contrast, the dominant frequency at 75° acted as a source, providing energy to lower frequencies in the overall flow field.

This study has provided a deeper understanding of the influence of the angle of attack on the specific flow structures in the delta wing wake. In addition to corroborating past literature regarding different vortex shedding regimes, the study has identified the flow structures associated with those regimes. Furthermore, modal decomposition has demonstrated that the first mode is the most dominant, suggesting the flow field is a low-rank system. This knowledge can be used in the future to formulate theoretical models for predicting the flow field in the wake of a delta wing.

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Nomenclature

B	Bispectrum [$\text{m}^3 \text{s}^{-3}$]
c	Delta wing chord [m]
f	Frequency [Hz]
s	Wing span [m]
T_{123}	Triadic transfer [$\text{m}^2 \text{s}^{-3}$]
t	Time [s]
U_∞	Freestream velocity [m s^{-1}]
$\hat{u}$	Complex Fourier coefficient of the velocity [m s^{-1}]
$\hat{u}^*$	Complex conjugate of the velocity [m s^{-1}]
$\tilde{u}_i^f$	Velocity fluctuation at frequency f [m s^{-1}]
$\hat{X}$	Complex Fourier coefficient of the signal [m s^{-1}]
V_y	Normal velocity [m s^{-1}]
$\hat{X}^*$	Complex conjugate of the signal [m s^{-1}]
E	Expectation operator
α	Angle of attack [$^\circ$]
ω	Vorticity magnitude [s^{-1}]
ω_z	Z-Vorticity [s^{-1}]
θ	Phase of bispectrum [$^\circ$]
$\frac{\partial \tilde{u}_i^{f1}}{\partial x_j}$	Spatial gradient of the velocity component [s^{-1}]

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