

Measurements of the full instantaneous dissipation rate & dissipation transport in a high-Reynolds number turbulent round jet

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ABSTRACT

An experimental facility capable of measuring and resolving the dissipation rate of the turbulent kinetic energy as well as the dissipation transport balance (i.e., the ε -equation) using particle tracking velocimetry is presented. A detailed discussion and comparison of the resolution requirements to the system resolution capabilities is presented. The long-term objective of the measurements is to study turbulence at different degrees of non-equilibrium to test the range of validity of the classical theories as well as to form the basis for developing new, improved models. Measurements have been carried out in a round turbulent jet, which is the ideal and in fact the only viable test-bed for these investigations, since the jet is the only flow that has all the necessary properties altogether; The stationary jet is very near equilibrium and can thus in a practical, arbitrary and calibrated manner be pushed out of equilibrium by modulating the jet exit velocity in time. Furthermore, the smallest relevant scales in the jet are large and slow enough that they can be resolved, while the flow also evolves over distances that are practical on a laboratory scale. In the present study, the Reynolds number ($Re_D = 10^4 - 10^5$) as well as the degree of mean shear, in the form of radial distance from the jet centreline ($r/D = \{0, 1.1, 2.2\}$, where D is the jet exit diameter), have been varied at a downstream distance of 30 jet exit diameters from the jet nozzle exit. Initial results of the dissipation rate, displaying expected intermittency behaviour, are presented and discussed. The results are preliminary, as noise plays a key role in the measurements, not least for spatial velocity gradient based estimates. This will be dedicatedly targeted in future efforts.

1. Introduction

The turbulent kinetic energy dissipation rate, ε , is one of the central quantities in the theory, modelling, and measurement of turbulent flows. It represents the rate at which turbulent kinetic energy is irreversibly converted into internal energy by viscous action. Its importance extends from phenomenological descriptions of the energy cascade to practical turbulence closures, where ε ,

or closely related quantities, enter directly into Reynolds-averaged models such as the $k-\varepsilon$ model and indirectly into the $k-\omega$ family of closures. The accuracy with which ε is represented therefore affects predictions of mixing, entrainment, scalar transport, and momentum exchange in a wide range of wall-bounded and free-shear flows.

Classical turbulence theory places dissipation at the end of the cascade process and links it to the smallest length and time scales of the flow, whereas turbulence modelling requires its mean value, spatial distribution, and transport to be represented accurately across inhomogeneous and non-equilibrium regions (Tennekes & Lumley, 1972; Pope, 2000).

Beyond the dissipation rate itself, the transport of dissipation provides a more stringent and physically informative description of turbulence dynamics. The dissipation transport equation (DTE) contains production, transport, pressure-Hessian, viscous and nonlinear small-scale interaction terms. These terms determine how dissipation is generated, redistributed, and destructed in space and time, and they are particularly relevant in flows where local equilibrium assumptions are not expected to hold.

In shear flows, such as turbulent jets, dissipation is not merely a passive sink of turbulent kinetic energy, but a dynamically evolving quantity influenced by mean shear, turbulent transport, intermittency, and the coupling between large scale motions and small scale velocity gradients. This is even more prominent in turbulent highly non-equilibrium flows. Direct experimental access to the full dissipation transport balance is therefore essential for assessing assumptions commonly embedded in turbulence closures and for developing models that remain valid outside the idealized equilibrium conditions often invoked in classical turbulence theory (Tsinober, 2009).

Despite this importance, direct experimental measurements of ε remain exceptionally demanding. The exact expression for the instantaneous dissipation rate requires all components of the velocity gradient tensor, $\partial u_i / \partial x_j$, to be accurately measured simultaneously with sufficient spatial resolution to resolve the dissipative scales. In practice, many experimental studies have relied on one-dimensional or two-dimensional velocity information, surrogate estimates, local isotropy assumptions, or Taylor's frozen-turbulence hypothesis. These approximations have enabled valuable progress, but they introduce uncertainties that are difficult to quantify in strongly inhomogeneous or anisotropic regions, such as the shear layer of a turbulent jet. The challenge becomes more severe at high Reynolds number, where the separation between the integral scale and the Kolmogorov scale increases, and where the measurement volume must be large enough to capture meaningful turbulent statistics while remaining sufficiently resolved to evaluate velocity gradients accurately.

This difficulty has motivated the development of specialised high-resolution optical arrangements, such as the long-range micro-PIV measurements of Fisaletti et al. (2014), who resolved the small-scale structure of a high-Reynolds-number turbulent round jet by combining high magnification with a long working distance. Comprehensive reviews and budget measurements in turbulent

round jets have shown that dissipation and its transport remain among the most difficult quantities to determine experimentally without invoking simplifying assumptions (Lai & Socolofsky, 2019).

Particle image and particle tracking techniques have progressively improved access to three dimensional turbulent flow information. In particular, time resolved volumetric Particle Tracking Velocimetry (PTV), and the Shake-The-Box (STB) algorithm (Schanz et al., 2016), have made it possible to reconstruct dense Lagrangian particle trajectories at high particle image densities. This development provides a route towards simultaneous three component velocity measurements in a three dimensional domain, from which the full velocity gradient tensor can be estimated. However, the use of particle tracking data also introduces new methodological requirements. The velocity field is sampled at irregular particle positions, and gradient estimation must therefore be performed using local reconstruction or data assimilation methods rather than standard finite differences on a structured grid. In addition, measurement noise, finite particle density, track length, and local seeding variability can strongly affect derivative estimates. These issues are particularly relevant for dissipation and dissipation transport measurements, because first, second, and higher order spatial derivatives appear in the relevant expressions.

The present work addresses these challenges by developing and applying a highly resolved volumetric particle tracking measurement strategy for a turbulent round jet. The experiments are performed in the recently established facilities of the DTU Turbulence Research Laboratory, using a round jet of exit diameter $D = 100$ mm issuing into a large confinement of cross section of approximately $4 \text{ m} \times 5 \text{ m}$. The large test-section dimensions reduce confinement induced effects, allowing the jet to be studied under conditions close to a momentum conserving axisymmetric free jet, a configuration that has long served as a canonical reference flow for turbulence measurements (Hussein et al., 1994).

Measurements are acquired in the self-similar region at 30 jet exit diameters downstream of the jet exit. Three radial positions are investigated; the jet centreline, the maximum shear region and in the outer region of the jet. The Reynolds number based on the jet exit conditions is also varied, with a measurement volume and optical magnification adapted to resolve sub-Kolmogorov length scales and to provide sufficient particle track densities for accurate velocity gradient estimation.

The objective of the current work is to demonstrate how the experimental configuration, imaging strategy, and particle tracking methodology enable direct access to the full instantaneous dissipation rate in a high-Reynolds number turbulent round jet. The longer term aim is to extend this capability to the complete dissipation transport equation, including the higher order derivative terms required by the exact balance, see Ribergaard (2025); Tsinober (2009).

We first introduce the theoretical quantities of interest, with emphasis on the derivative orders required for measuring the dissipation and dissipation transport balance. The experimental arrangement and spatial resolution are then described in relation to the local Kolmogorov scale. The procedure used to estimate spatial velocity gradients from the particle tracking data is summa-

rized, with detailed methodological validation deferred to the companion manuscript on gradient estimation methods, see Certini et al. (2026). Finally, preliminary measurements of instantaneous dissipation rate fields and their radial and Reynolds number variation are presented, establishing the basis for future experimental evaluation of the dissipation transport balance in high-Reynolds number turbulence.

2. Theoretical background

The quantity of interest in the present work is the turbulent kinetic energy dissipation rate and its transport. We consider an incompressible Newtonian fluid, neglect body forces, and use Einstein summation notation throughout. The instantaneous velocity is decomposed into a mean and fluctuating component according to

$$U_i = \langle U_i \rangle + u_i, \quad (1)$$

where angular brackets denote an ensemble or time average, and lower-case symbols denote fluctuations. The fluctuating strain-rate tensor is defined as

$$s_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (2)$$

For incompressible turbulence, the instantaneous dissipation rate associated with the fluctuating velocity field may then be written as

$$\varepsilon = 2\nu s_{ij} s_{ij}, \quad (3)$$

and the corresponding mean dissipation rate is

$$\langle \varepsilon \rangle = 2\nu \langle s_{ij} s_{ij} \rangle. \quad (4)$$

Equations (3) and (4) immediately show why dissipation is experimentally demanding: their direct evaluation requires all nine components of the fluctuating velocity-gradient tensor, $\partial u_i / \partial x_j$, to be measured simultaneously and with sufficient spatial resolution to resolve, say, 99% of the dissipation rate. The resolution requirements is discussed in Section 4. In contrast to estimates based on one or two velocity components, see, e.g., George & Hussein (1991), the basic definition of the dissipation rate does not require local isotropy or local homogeneity.

While Equation (4) defines the magnitude of the dissipation rate, its spatial and temporal evolution is governed by the dissipation transport equation. Following the derivation in Ribergaard (2025), equivalent but differently expressed to the formulation reported by Tsinober (2009), we write the transport equation for the strain-rate invariant as

$$\begin{aligned} \frac{D \frac{1}{2} \langle s_{ij} s_{ij} \rangle}{Dt} = & - \langle u^k s_{ij} \rangle \frac{\partial \langle S_{ij} \rangle}{\partial x^k} - \left\langle s_{ij} u^k \frac{\partial s_{ij}}{\partial x^k} \right\rangle - \left\langle s_{ij} \frac{\partial \langle U^k \rangle}{\partial x^j} \frac{\partial u^i}{\partial x^k} \right\rangle - \left\langle s_{ij} \frac{\partial u^k}{\partial x^j} \frac{\partial \langle U^i \rangle}{\partial x^k} \right\rangle - \\ & \left\langle s_{ij} \frac{\partial u^k}{\partial x^j} \frac{\partial u^i}{\partial x^k} \right\rangle - \frac{1}{\rho} \left\langle s_{ij} \frac{\partial^2 p}{\partial x^i \partial x^j} \right\rangle + \nu \langle s_{ij} \nabla^2 s_{ij} \rangle \end{aligned} \quad (5)$$

Here, $\langle S_{ij} \rangle$ is the mean strain-rate tensor,

$$\langle S_{ij} \rangle = \frac{1}{2} \left(\frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \langle U_j \rangle}{\partial x_i} \right), \quad (6)$$

and the material derivative is taken with respect to the mean flow,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \langle U_k \rangle \frac{\partial}{\partial x_k}. \quad (7)$$

The terms on the right-hand side of Equation (5) represent, respectively, production by interaction between fluctuating strain and mean strain gradients, turbulent transport of fluctuating strain intensity, interaction between fluctuating strain and mean velocity gradients, interaction between fluctuating velocity gradients and mean strain, nonlinear self-interaction of the fluctuating velocity gradients, a Reynolds-stress inhomogeneity term, the pressure-Hessian contribution, and the viscous diffusion/destruction term. This form is useful for the present experiment because it makes explicit which velocity derivatives are required and where the experimental difficulty enters.

The derivative order required for each quantity is central to the measurement strategy. The dissipation rate itself, Equation (4), requires only first-order spatial derivatives of the three fluctuating velocity components. The transport equation is more restrictive. Terms containing $\partial s_{ij} / \partial x_k$ require second order spatial derivatives of velocity, while the viscous term $\nabla^2 s_{ij}$ requires third order spatial derivatives. The pressure Hessian term is also non trivial, since pressure is not measured directly in the present experiment. It may be related to velocity quantities through the incompressible Navier-Stokes equations,

$$-\frac{1}{\rho} \frac{\partial^2 p}{\partial x_j \partial x_i} = \frac{\partial}{\partial x_j} \left(\frac{D u_i}{Dt} \right) - \nu \frac{\partial}{\partial x_j} (\nabla^2 u_i), \quad (8)$$

which again introduces higher order velocity derivatives, including third order spatial derivatives through the viscous contribution. The same point is noted in the derivation of Ribergaard (2025), where the pressure term is identified as the principal term requiring reformulation when pressure is not available experimentally.

The present measurements should satisfy a hierarchy of resolution requirements. Reliable evaluation of ε and $\langle \varepsilon \rangle$ requires accurate first order velocity gradients at the dissipative scale. Evaluation of the full dissipation transport balance further requires robust second and third order derivatives, or an equivalent reconstruction capable of estimating the pressure Hessian and viscous terms without excessive noise amplification. This derivative hierarchy motivates the use of highly resolved volumetric particle tracking and also motivates the companion methodological study, where the accuracy of velocity gradient estimation from irregularly sampled particle data is assessed in detail.

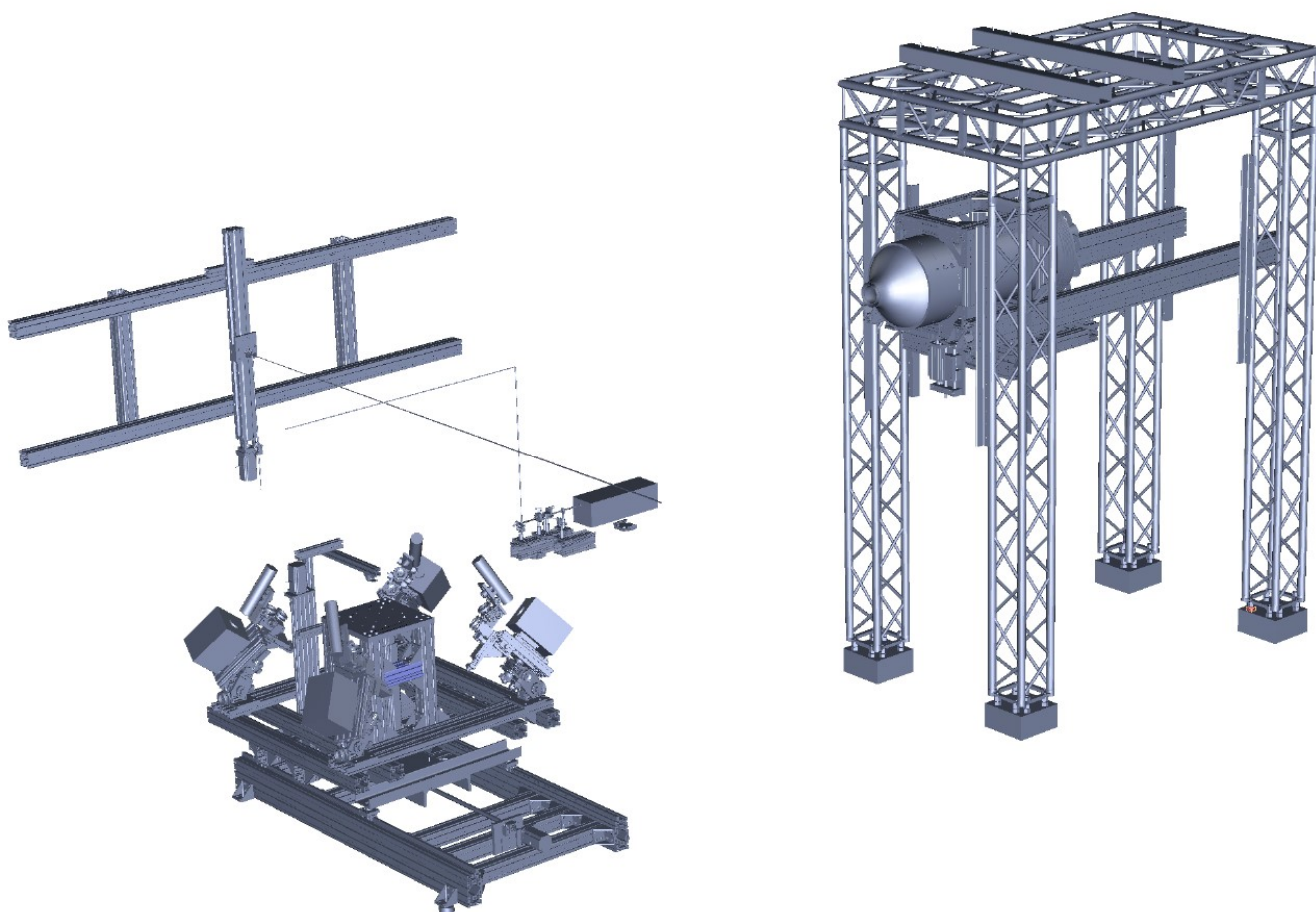


Figure 1. Experimental setup for direct measurements of the instantaneous dissipation rate. The jet issuing from the jet flow generator to the right is being measured at different downstream and radial positions using a 40 W laser and 4 v2640 high-speed cameras, as seen to the left. The measuring volume is cylindrical with a diameter of $\sim 2\text{ mm}$ and length $\sim 20\text{ mm}$.

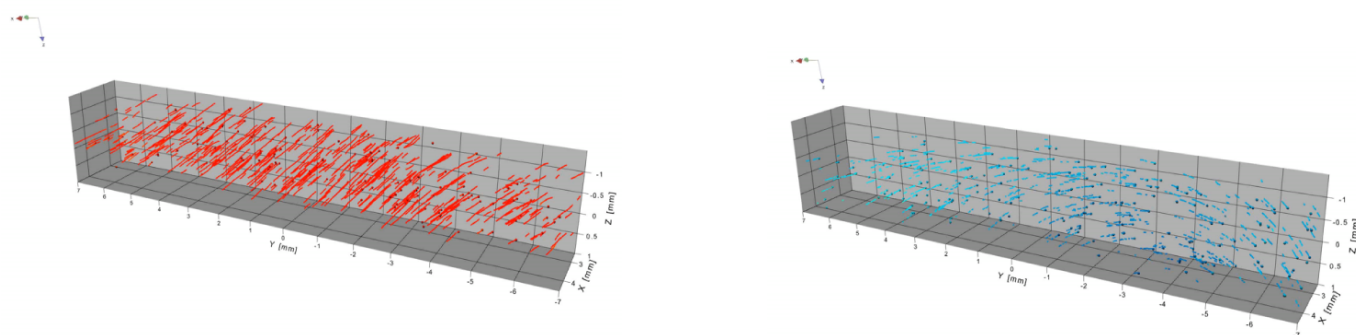


Figure 2. Sample particle tracks (Representative low track density, ~ 300 tracks) used for computing the dissipation rate and the dissipation transport equation balance. Measurements at (left) the centreline and (right) in the maximum shear region at 30 jet exit diameters downstream from the jet exit.

3. Experimental setup

The experiments were conducted in the DTU Turbulence Research Laboratory using a round turbulent air jet specifically designed to provide optical access to the dissipative scales of high-Reynolds-number turbulence (Ribergaard, 2025). The experimental setup is shown in Figure 1, and representative low-track-density reconstructions at two radial positions are shown in Figure 2. The objective of the experimental configuration is twofold: first, the flow must provide a canonical, well documented turbulent shear flow in which dissipation rate statistics can be interpreted with reference to established round jet measurements and second, the measurement system must resolve the full three component velocity field with sufficient spatial and temporal resolution to estimate the velocity gradient tensor, and eventually the higher order derivatives entering the dissipation transport equation.

The flow facility consists of a round jet of exit diameter $D = 100$ mm, issuing into a large enclosed test cell. The jet is operated at exit velocities in the range

$$U_0 = 1.51\text{--}15.1 \text{ m s}^{-1}, \quad (9)$$

corresponding to jet-exit Reynolds numbers

$$Re_D = \frac{U_0 D}{\nu} = 1.0 \times 10^4 - 1.0 \times 10^5. \quad (10)$$

where ν is the kinematic viscosity of the working fluid, in this case air. The measurements are being carried out in a cross-section at a primary downstream position located at $x/D = 30$, where the jet is fully turbulent, self-similar and developed and the local small scale structure can be investigated. This downstream location also permits comparison with classical high Reynolds number round jet measurements, in particular the momentum conserving jet configuration of Hussein et al. (1994).

A central design requirement of the facility is to reduce confinement induced modification of the jet. Following the analysis of Hussein et al. (1994), the jet enclosure cross section was chosen sufficiently large to limit the momentum loss associated with return flow along the walls (Ribergaard, 2025). The jet issues into an enclosure with a cross sectional area of approximately

$$A_{\text{cell}} \simeq 4 \text{ m} \times 5 \text{ m}, \quad (11)$$

so that the measurement region at $x/D = 30$ conserves momentum well and thereby remains close to the conditions of a free jet, while also not being affected by surrounding disturbances. In addition, the facility includes a dedicated recirculation and suction system, designed to remove the downstream flow from the test cell in a non-intrusive manner while reducing large scale backflow contamination in the measurement region. This aspect is important for the present purpose because residual mean flow distortion would enter directly into both the mean strain rate field and the spatial derivatives required for the dissipation transport equation.

The jet was selected as a test case, because it combines optical and physical accessibility and well-defined radial inhomogeneity, among many other reasons based on the non-equilibrium flow investigations to be conducted. Measurements were performed at several radial positions as measured from the jet centreline, including at the centreline itself, the region of large mean shear, and the outer intermittent region. The measurement locations are written as

$$(x/D, r/D) = (30, \{0, 1.1, 2.2\}), \quad (12)$$

This radial traverse of measurement points is essential, because the dissipation rate and the dissipation transport balance are expected to vary strongly across the jet (Hussein et al., 1994): the centreline provides a reference region of reduced mean shear, the half-width region is associated with high turbulent production and large velocity gradients, and the outer region probes the decay of turbulent activity and the sensitivity of gradient estimation at lower advection velocities.

The velocity measurements were performed using time-resolved volumetric particle tracking velocimetry based on the Shake-The-Box algorithm (Schanz et al., 2016). The jet enclosure was seeded with DEHS (Di(2-ethylhexyl) sebacate, sebacic acid) droplets generated by a Laskin nozzle seeding system, creating a uniform seeding since the jet air was taken from the seeded air from the outlet of the enclosure. The droplet size distribution exhibits a peak at approximately $1 \mu\text{m}$. The seeding particles were chosen to be sufficiently small to follow the air motion over the investigated range of operating conditions. With a nominal particle diameter $d_p = 1.0 \mu\text{m}$, the particle response time is estimated as $\tau_p = \rho_p d_p^2 / (18\mu) \simeq 2.8 \times 10^{-6} \text{ s}$. Using the jet exit time scale, D/U_0 , this corresponds to a Stokes number range $St = \tau_p U_0 / D \simeq 4.2 \times 10^{-5} - 4.2 \times 10^{-4}$, indicating that the seeding particles behave as faithful tracers. The seeding particles provide adequate scattered light intensity, with an average Signal to Noise Ratio around 10.

The illumination was provided by a pulsed 532 nm laser beam (Q-switched water cooled 40 W Innolas Photonics Blizz laser with maximum repetition rates of 400 kHz, $M2 < 1.4$ TEM00 at 532 nm) expanded to form a cylindrical measurement volume.

The effective measurement volume has streamwise length $L_m \simeq 20 \text{ mm}$, and transverse dimension $d_m \simeq 2 \text{ mm}$. The laser-beam thickness defines the small dimension of the measurement volume, whereas the camera field of view defines the observed portion of the illuminated beam.

The reconstructed particle trajectories provide a direct visual indication of the spatial coverage achieved within the cylindrical measurement volume. Figure 3 shows an example instantaneous Shake-The-Box reconstruction containing more than 2.2×10^4 tracks. The tracks fill the elongated illuminated volume over its full streamwise extent, demonstrating the level of Lagrangian information available for subsequent estimation of the three-component velocity field and its spatial derivatives.

The imaging system consists of four Phantom high-speed v2640 cameras (288 GB) arranged around the measurement volume, see Figure 1. This geometry keeps all cameras predominantly in side

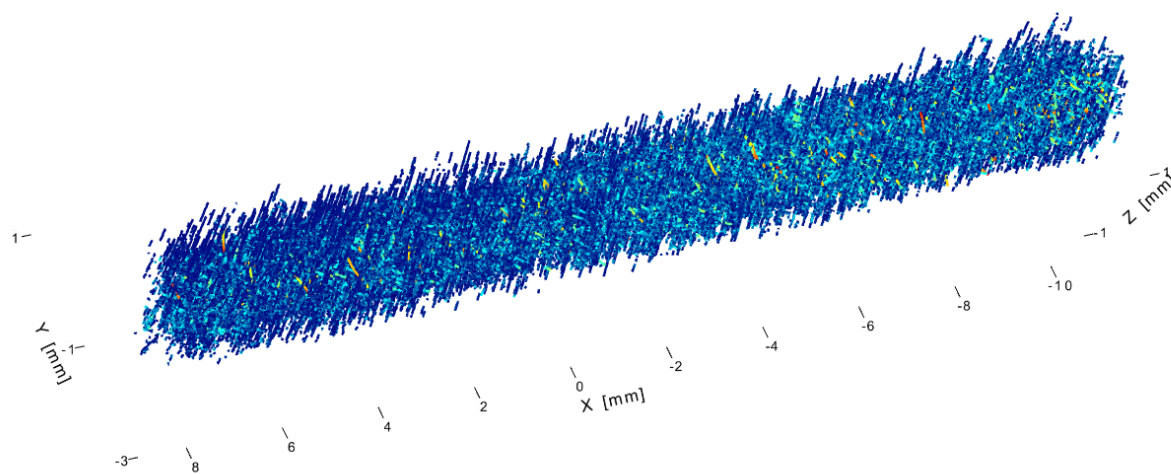


Figure 3. Example volume of interest reconstructed with Shake-The-Box particle tracking velocimetry. The instantaneous field contains more than 2.2×10^4 particle tracks within the elongated measurement volume of approximate streamwise length 20 mm. The colour represents the local particle-track velocity magnitude. This high-track-density example illustrates the spatial coverage available for estimating the three-component velocity field and the velocity-gradient tensor required for dissipation-rate evaluation.

or forward scatter, improving particle image quality for the DEHS droplets. Each camera was equipped with long working distance optics providing high magnification while keeping the cameras outside the region of aerodynamic influence. The acquisition was performed with a reduced sensor window of $N_x \times N_y = 2048 \times 400$ pixels and a sampling frequency of $f_s = 20$ kHz. At this acquisition rate, each recording contains approximately $N_t = 40$ frames per camera, corresponding to 0.002 s of time-resolved particle track data. The laser repetition rate was matched to the camera acquisition frequency. This temporal resolution is required not only for trajectory reconstruction, but also for estimating temporal derivatives and for assessing the temporal content of the dissipation rate signal.

The amount of Lagrangian information available from the Shake-The-Box reconstruction was assessed from the track statistics reported by DaVis. Figure 4 summarises the average number of instantaneous tracks across the Reynolds-number range and radial measurement locations considered in the present study. The vertical bars indicate the minimum-to-maximum variation of the instantaneous track count within each recording, while the marker size represents the average track length in time steps. This diagnostic provides a compact indication of both the local track density and the temporal persistence of the reconstructed trajectories, which are central for subsequent velocity-gradient and dissipation estimates.

Next, the resolution requirements and the attainable resolution will be explored.

4. Evaluating the effective resolution

We will first evaluate the resolution criterion based on the scales of the turbulence and model spectra. Then quantify the attainable resolution in the current experiments with a following discussion on how spatial resolution in PTV measurements can be defined.

4.1. Quantifying the scale resolution requirements

The requirements for resolving the turbulent kinetic energy dissipation rate, ε , are substantially higher than those for resolving the turbulent kinetic energy. One manner to illustrate this is to compare the spectrum for the turbulent kinetic energy, $E(k)$, to the spectrum of the corresponding dissipation rate classically formulated as, $2\nu k^2 E(k)$. The dissipation spectrum is therefore shifted to higher wavenumbers $\sim k^2$ higher than the energetic scales.

Contrary to popular belief, and many textbooks in turbulence, the Kolmogorov length scale is *not* the smallest scale of turbulence, but merely a characteristic length scale for the dissipative scales. The long, flat tail of the spectrum does not suddenly end at the Kolmogorov scale, which would be quite unphysical, but rather stretches down to the continuum limit. Resolving 100% of the turbulent kinetic energy or its dissipation would therefore be excruciatingly difficult and with

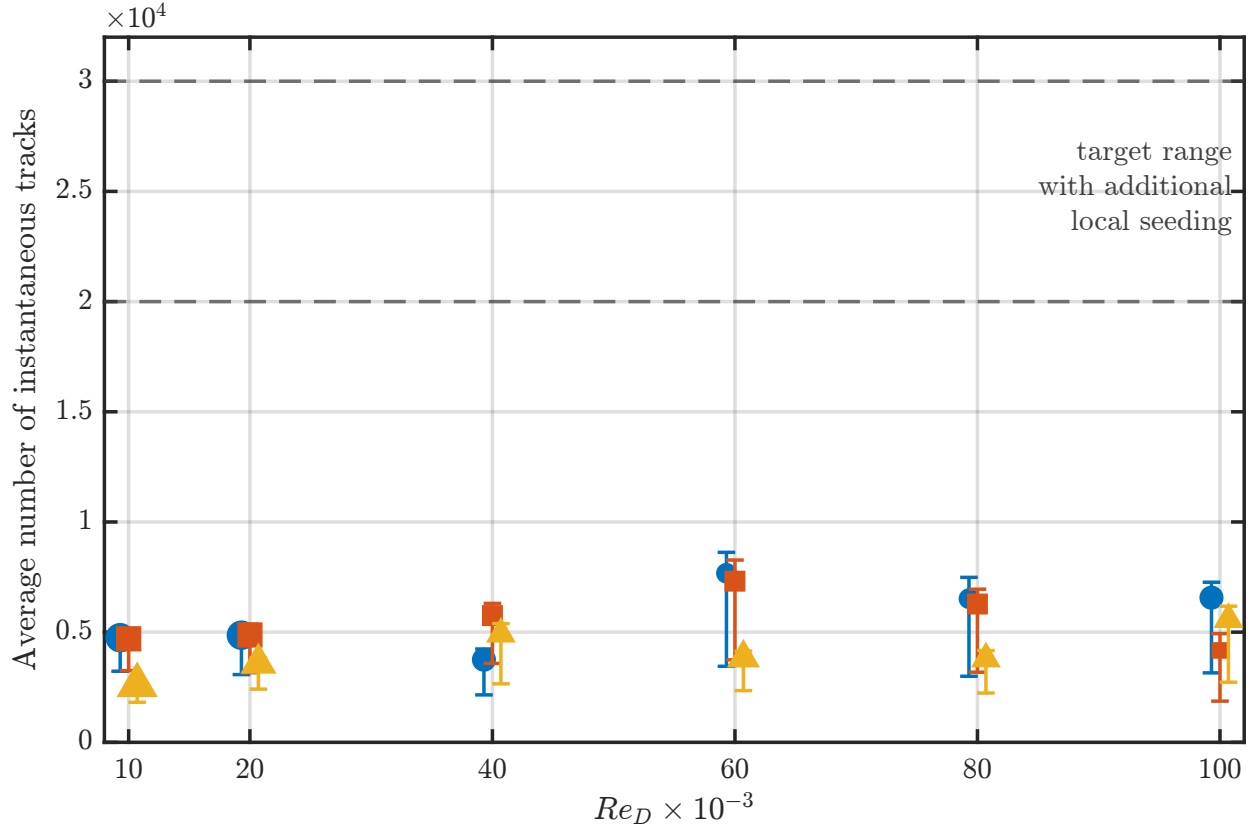


Figure 4. Particle-track statistics across the experimental matrix. The average number of instantaneous Shake-The-Box tracks is shown as a function of nozzle Reynolds number for the three radial measurement positions at $x/D = 30$. Vertical bars indicate the minimum to maximum range of instantaneous track counts reported by DaVis. Marker size represents the average track length in time steps. The plotted data were obtained using seeding introduced through the nozzle only. The two horizontal dashed lines indicate the target range of 2×10^4 – 3×10^4 instantaneous particles/tracks obtained with an additional local atomiser, used in the test cell together with the nozzle seeding.

only very little gain. Since both the energy and dissipation spectra beyond a certain limit have very long and flat tails with little integrated area (energy and dissipation, respectively) beyond, the resolution is no longer sensitive to the cut-off wavenumber or frequency. We will thus define a cut-off frequency for resolving 99% of the dissipation rate, ε , and the dissipation of the dissipation, $\varepsilon_\varepsilon = 4\nu^2 \int_{-\infty}^{\infty} k^4 E(k) dk$, respectively. We will thus note that this cut-off is only a number well within the flat and insensitive spectrum range of resolving a major portion of the respective considered quantity.

Furthermore, the estimation of the cut-off wavenumber will also be conservative, since we will be using a model spectrum to estimate this. Most model spectra, among other issues, overestimate the energy in the dissipation range, since the roll-off of the spectrum is does not cut back the spectral flux to a sufficient degree, e.g. the Dryden spectrum, the von Kármán spectrum, the Kaimal spectrum the Kovasznay spectrum and even the Pao-Corrsin spectrum.

The Lin-Hill spectrum is less over-predicting of the energy in the dissipation range, in particular as the Kolmogorov scale is approached:

$$E(k) = \alpha_{Kol} \varepsilon^{1/3} k^{-5/3} \left[1 + (k\eta_{Kol})^{2/3} \right] \exp \left\{ -2\alpha_{Kol} \left[\frac{(k\eta_{Kol})^{4/3}}{(4/3)} + \frac{(k\eta_{Kol})^2}{2} \right] \right\} \quad (13)$$

Using this to integrate the dissipation spectrum, $\varepsilon = 2\nu \int_0^{\infty} k^2 E(k) dk$ shows that contributions to the dissipation integral is in essence negligible above $k\eta_{Kol} = 1$. Or, for $k = \frac{2\pi}{\lambda}$ where λ is the wavelength and defining the resolution requirement in accordance with the Nyquist resolution criterion: $\frac{\lambda}{2} = \pi\eta_{Kol}$. The fact that the right hand side happens to equal to unity does not seem to have any particular physical explanation but rather happened by coincidence (George & Taulbee, 1992). The Kolmogorov length scale is thus a scale below which approximately 99% of the dissipation occurs, i.e. $2\nu \int_0^{1/\eta_{Kol}} k^2 E(k) dk \approx 0.99\varepsilon$.

For the dissipation of the dissipation, $\varepsilon_\varepsilon = 4\nu^2 \int_{-\infty}^{\infty} k^4 E(k) dk$, the analogous analysis shows that the resolution criterion is even sharper: $k\eta_{Kol} = 4$. Similarly, based on the Nyquist criterion, $\frac{\lambda}{2} = \frac{\pi}{4}\eta_{Kol} \approx 0.8\eta_{Kol}$.

4.2. Quantifying the attainable measurement resolution

It is visually evident from the measured tracks that the dissipative gradients are large compared to the track gradient resolution. We expect to be able to reach at least between 20.000–30.000 particles in the current measuring volume, corresponding to ~ 0.04 – 0.06 ppp. Furthermore, since the temporal and spatial resolutions are inherently coupled in PTV measurements, this ensures a sufficiently high resolution in space and time.

To properly quantify the spatial and temporal resolution requirements, we will be utilizing the classical (equilibrium) estimates for the Kolmogorov length and time scales, respectively $\eta_{Kol} = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$ and $\tau_{Kol} = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$. The kinematic viscosity, ν , is a known material property. In the case of the stationary round jet, measured dissipation rate measurements can be found from Hussein et al. (1994), Figure 21a. The locally axisymmetric dissipation is the most accurate for the round turbulent jet, yielding from similarity scaling: $\varepsilon \approx \frac{0.70 U_c^3}{2(x - x_0)}$ at the jet centreline, where $U_c \approx 0.2U_{exit}$ is the local centreline mean velocity at downstream distance $x/D = 30$ from the nozzle and with $x_0 \approx 4D$ being the distance from the nozzle to the jet virtual origin. The radial dissipation rate profile in the jet similarity region is approximately Gaussian, and hence the largest dissipation rate, and hence the smallest η_{Kol} , are found at the jet centreline. Now, the jet centreline happens to be one of the least interesting regions in the jet. The region of maximum shear, around the jet half width, is much more exciting to study, not least since this is the region in which production of turbulent kinetic energy is the highest.

There are hence several reasons why our required resolution estimates are conservative:

- The resolution estimates are based on the highest dissipation rate along the radial jet profile, at the jet centreline, yielding the smallest Kolmogorov length scale across the radial jet profile.
- The Lin-Hill spectrum also over-predicts the dissipation rate, since it does not cut off the energy of the energy spectrum sufficiently rapidly in the dissipation range.
- Both the energy- and the dissipation-spectra in the considered region are very flat and the resolved energy or dissipation, respectively, is not sensitive to the exact cut-off in this high wavenumber range.

Since the measurements are carried out in the self-similar region at $x/D = 30$ at different Reynolds numbers, we list the relevant parameters for the tested Reynolds numbers at the jet centreline in Table 1.

The temporal resolution is defined by the time between pulses, $f = 20 \text{ kHz} \rightarrow \Delta t = 50 \mu\text{s}$. This is to be compared to the Kolmogorov timescale, ranging between 2 ms - 63 ms for Reynolds numbers between 10^4 to 10^5 . Even at the highest Reynolds number, the acquisition time step is 40 times smaller than the Kolmogorov timescale. The spatial resolution thus constitutes the most critical criterion.

As a first comparison, one could estimate the average inter-particle spacing in the measuring volume, ℓ_p . The cylindrical volume of approximate length 20 mm and diameter 2 mm yields a volume of $V \approx 62.8 \text{ mm}^3$. A track count of $N = 25.000 - 50.000$ particles should be attainable, corresponding to $\sim 0.05 - 0.10$ ppp. The average volume per particle $V_p = V/N \approx 0.00251 \text{ mm}^3$ for

Re_D	$\eta_{Kol, centreline}$ (μm)	$\frac{\lambda}{2} _{0.99\varepsilon} = \pi\eta_{Kol}$ (μm)	$\frac{\lambda}{2} _{0.99\varepsilon_\varepsilon} = \frac{\pi}{4}\eta_{Kol}$ (μm)
20.000	578	1816	454
40.000	344	1080	270
60.000	254	797	199
80.000	204	642	161
100.000	173	543	136

Table 1. Spatial resolution check across tested Reynolds numbers. The Kolmogorov length scales are estimated at the centreline, where the dissipation is the highest and the Kolmogorov length scale η_{Kol} is the smallest. The estimates are based on a Lin-Hill model spectrum that cuts back the energy in the dissipation range more sharply than most spectral models, yet is expected to over-predict the dissipation and hence under-predict the Kolmogorov length scale.

$N = 25.000$, yielding a characteristic spacing based on a cube cell estimate of $\ell_p \approx V_p^{1/3} \approx 136 \mu\text{m}$. Using instead a nearest neighbour estimate based on a random distribution yields slightly lower values, $\ell_{NN} \approx 75 \mu\text{m}$. Since the inter-particle spacing scales as $\sim N^{-1/3}$, the inter-particle spacing only reduces slowly with increased track density. For $N = 50.000$ particles, i.e., a doubling of the track counts, the average particle inter-spacing only reduces to $\ell_p \approx 108 \mu\text{m}$ or, respectively, $\ell_{NN} \approx 60 \mu\text{m}$.

With the inter-particle distance ℓ_p as a simple initial measure, the experiments do indeed resolve both the dissipation rate, ε , and the dissipation of the dissipation, ε_ε , beyond 99%, see Figure 5, despite the conservative dissipation estimates under-predicting the Kolmogorov length scale.

A local linear fit for first-order spatial velocity gradients requires a neighbourhood of order $\sim 2\ell_p$, whereas a quadratic fit required for higher-order spatial derivative estimates requires a support closer to $\sim 3\ell_p$. These values are of the same order as the estimated Kolmogorov length scale, indicating that the measurements are designed to resolve the dissipative range directly rather than infer it from large-scale surrogate quantities. This strategy follows the same physical motivation as the long range micro-PIV measurements of Fiscaletti et al. (2014), but extends the measurement concept to time-resolved volumetric particle tracking in order to access all three velocity components and all components of the velocity gradient tensor.

The measurement strategy is therefore matched to the derivative hierarchy introduced in Section 2. The dissipation rate requires accurate first order spatial derivatives of the fluctuating velocity field, whereas the dissipation transport equation additionally requires higher order derivatives, including terms involving $\partial s_{ij}/\partial x_k$, $\nabla^2 s_{ij}$, and the pressure Hessian. The experimental configuration combines high magnification, high speed imaging, dense particle tracking, and radial traversing of the measurement volume to provide the spatial and temporal resolution needed for these quantities. The detailed procedure used to compute velocity gradients from the irregularly distributed particle tracks and a full comparison of gradient estimation methods is provided in the companion paper (Certini et al., 2026).

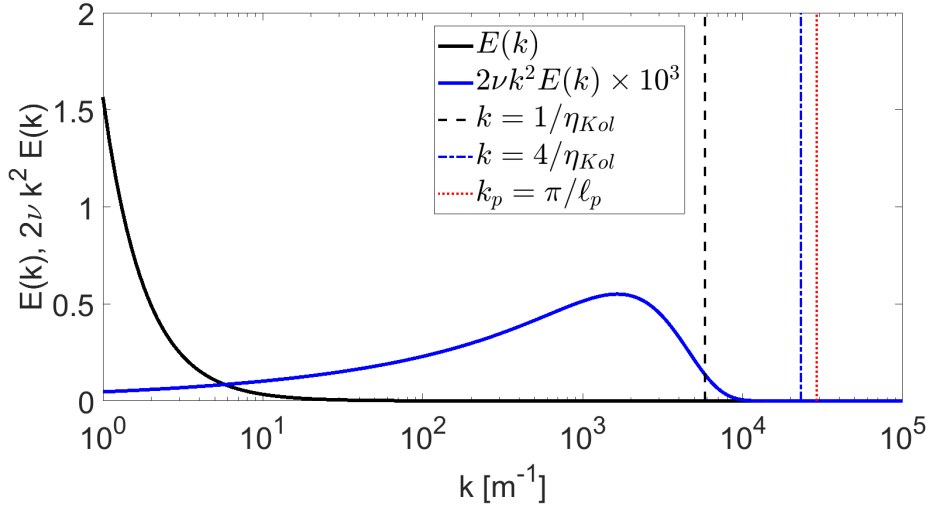


Figure 5. Lin-Hill turbulent kinetic energy spectrum, $E(k)$, and the corresponding dissipation spectrum, $2\nu k^2 E(k)$. The resolution criteria for resolving 99% of ε , $k\eta_{Kol} = 1$, and 99% of ε_ε , $k\eta_{Kol} = 4$, and the wavenumber $k_p = \pi/\ell_p$ corresponding to the inter-particle distance for $\ell_p = \lambda/2 = 108 \mu\text{m}$, respectively, are indicated by vertical lines. All quantities shown are relevant for the jet centreline at downstream distance $x/D = 30$ and $Re_D = 100.000$.

4.3. Defining the spatial resolution of PTV measurements

It is important to note, that for the Lagrangian methods, determination of the effective resolution may not be fully trivial. When evaluating the gradient of a velocity vector, $\nabla \mathbf{u}$, from practical data, one is in fact evaluating the gradients of a spatially filtered velocity field, $\nabla \mathbf{u}_{\Delta_{eff}}$. The estimated gradient therefore corresponds to a gradient field filtered across a filter scale Δ_{eff} .

Although it may seem most straightforward to equate the resolution to the average inter-particle spacing, for Lagrangian methods, the relevant length scale is rather the characteristic radius of the regression neighbourhood. At the same time, the resolution of the gradients is also characterized by the resolvable gradients of each particle trajectory. So the effective resolution varies with the local flow conditions compared to the capabilities of the optical diagnostics. For example, if the gradients do not vary significantly across the filtered volume, the resolution is not much diminished by the filter extent.

The Taylor microscale in e.g. the x-direction,

$$\lambda^2 = \frac{\langle u'^2 \rangle}{\left\langle \left(\frac{\partial u}{\partial x} \right)^2 \right\rangle}$$

where u' is the velocity fluctuation, can be used to evaluate the effective resolution. The denominator is dominated by scales of order λ and smaller. So if $\Delta_{eff} \ll \lambda$, the filter should have very little influence. If $\Delta_{eff} \sim \lambda$, the estimate becomes strongly biased. If $\Delta_{eff} > \lambda$, the measured gradient variance is no longer representative of the physical gradient field. One may thus suggest

a dimensionless resolution parameter:

$$\Pi_\lambda = \frac{\Delta_{eff}}{\lambda},$$

which should be $\ll 1$ for low filter influence. Or, correspondingly for the Kolmogorov length scale, imposing a sharper resolution criterion,

$$\Pi_{\eta_{Kol}} = \frac{\Delta_{eff}}{\eta_{Kol}}.$$

Important measures from turbulence theory and modelling, such as the dissipation rate ε , the Taylor microscale λ and the derivative skewness S , all depend strongly on filter scale.

5. Results: dissipation-rate and dissipation-transport diagnostics

A time series of the absolute value of the complete set of measured Cartesian spatial velocity derivatives and the resulting turbulent kinetic energy dissipation rate, ε , is presented in Figure 6. The dissipation rate is observed to display intermittency where high bursts of dissipation and the gradients contribute non-isotropically to the resulting dissipation rate. These results should however be considered preliminary, since the noise amplification of the gradients will have to be understood and alleviated, subject to future work.

Preliminary evaluation of the dissipation transport balance is shown in Figures 7–9. The measurement volume was subdivided into 24 smaller subvolumes, and the terms of the dissipation transport equation were evaluated in each subvolume. This representation provides a first spatially resolved assessment of whether the different terms of the balance can be estimated consistently from the particle tracking data.

Figure 7 compares the left-hand side of the dissipation transport equation with the sum of the right-hand-side terms. The two curves follow each other closely over the full set of subvolumes, including both the large negative excursions and the local positive maxima. This agreement is encouraging, because it indicates that the implemented processing chain preserves the overall structure of the dissipation transport balance at the level of the spatially averaged subvolumes. The strongest variations occur over a limited number of subvolumes, where the magnitude of the balance increases substantially, suggesting that the local dissipation dynamics are highly inhomogeneous within the measurement volume.

The individual right-hand-side contributions are shown in Figure 8, with an additional rescaled view excluding the pressure-Hessian contribution shown in Figure 9. The different terms exhibit markedly different amplitudes and spatial variability. Some terms remain close to zero across most of the domain, whereas others show large positive and negative excursions. The complete decomposition shows that the pressure-Hessian contribution is the largest term in the present preliminary

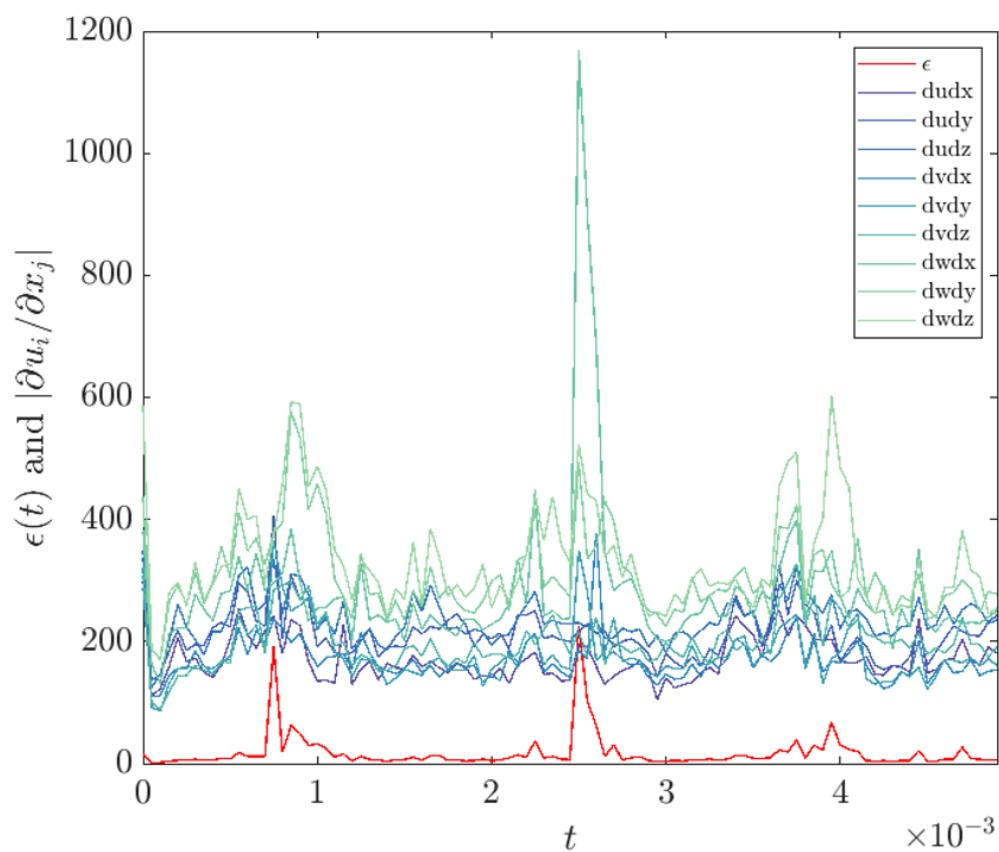


Figure 6. A time series of the absolute values of the complete set of measured Cartesian spatial velocity derivatives and the resulting turbulent kinetic energy dissipation rate, ϵ , measured at the jet centreline at $Re_D = 67.000$.

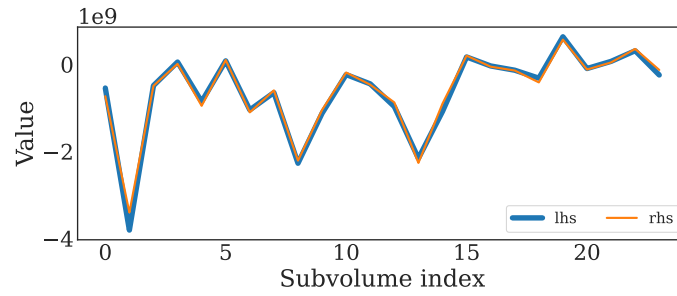


Figure 7. Comparison between the left- and right-hand sides of the dissipation transport equation along the subdivided measurement volume. The blue curve shows the left-hand side, corresponding to the material derivative of $\frac{1}{2}\langle s_{ij}s_{ij} \rangle$, while the orange curve shows the sum of the evaluated right-hand-side terms. This comparison provides a preliminary assessment of the experimental closure of the dissipation transport balance and highlights the sensitivity of the balance to the estimation of higher-order velocity derivatives.

evaluation. At the same time, the remaining contributions also exhibit appreciable spatial variability, indicating that the final right-hand side in Figure 7 results from the combination of several competing mechanisms. The large fluctuations observed in selected terms also highlight the sensitivity of the dissipation transport equation to derivative estimation, since several terms involve products of velocity-gradient quantities and higher-order spatial derivatives.

Because the pressure-Hessian contribution is substantially larger than several of the remaining terms, Figure 9 shows the same decomposition with rhs7 omitted. This does not modify the evaluated dissipation-transport balance, but provides a clearer view of the spatial variation of the lower-magnitude contributions.

At this stage, the results should be interpreted as a preliminary consistency check rather than as a definitive quantitative closure of the dissipation transport equation. The close correspondence between the left-hand side and the summed right-hand side is promising, but the individual terms remain sensitive to noise, local particle density, fitting support, and the treatment of spatial derivatives. A systematic uncertainty analysis and a parameter-sensitivity study will therefore be required before firm conclusions can be drawn regarding the relative importance of the individual dissipation transport mechanisms. Nevertheless, these preliminary results demonstrate that the present experimental and processing framework can access the terms of the dissipation transport balance in a spatially resolved manner, thereby providing a basis for future measurements of non-equilibrium turbulence in the high-Reynolds-number jet.

6. Discussion & Conclusions

The current work presents a high-resolution PTV experimental facility capable of measuring and resolving the dissipation rate of the turbulent kinetic energy and even the dissipation transport balance. These measurements will be utilized to study these and other physical quantities under

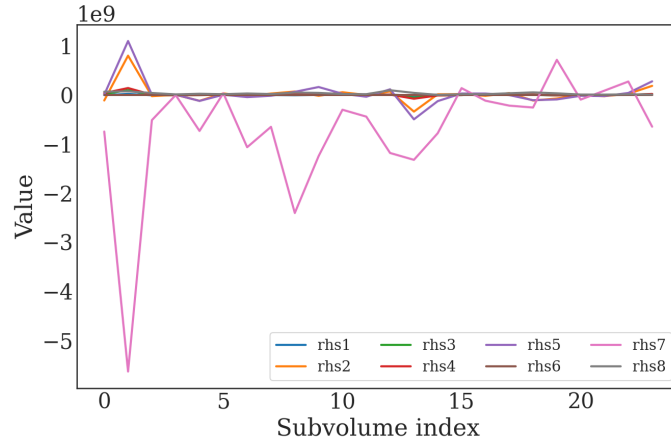


Figure 8. Individual contributions to the right-hand side of the dissipation transport equation along the subdivided measurement volume. The plotted terms correspond to production by interaction between fluctuating strain and mean strain gradients (**rhs1**), turbulent transport of fluctuating strain intensity (**rhs2**), interaction between fluctuating strain, mean velocity gradients, and fluctuating velocity gradients (**rhs3** and **rhs4**), nonlinear self-interaction of the fluctuating velocity gradients (**rhs5**), the Reynolds-stress inhomogeneity term (**rhs6**), the pressure-Hessian contribution (**rhs7**), and the viscous diffusion/destruction term (**rhs8**). The relative magnitude and spatial variation of the terms illustrate which mechanisms dominate the preliminary balance and where the evaluation is most sensitive to spatial derivative estimation.

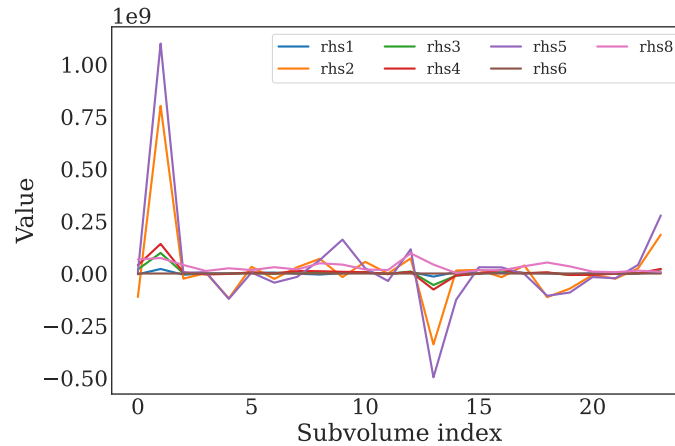


Figure 9. Individual right-hand-side contributions to the dissipation transport equation, excluding the pressure-Hessian term. The pressure-Hessian contribution (**rhs7**) is omitted from this figure because it is the dominant term and would otherwise compress the smaller contributions. This representation is therefore used to visualise the spatial variation of the remaining right-hand-side terms; the complete decomposition including **rhs7** is shown in Figure 8.

variation of parameters central to both theory and modelling of turbulence.

The results show intermittency in the dissipation rate time series and anisotropy in how the different gradients contribute to the resulting dissipation rate. These intermittent bursts are hypothesized to result from the passing of relatively sharp local shear layers. The idea is that two relatively large co-rotating structures, if sufficiently close to one another, can generate a thin shear layer with smaller structures in between. When this kind of structure passes the measurement region, this will temporarily result in higher values of the gradients, and thereby of the dissipation rate. The fact that the different gradients do not tend to contribute similarly to the dissipation rate supports the hypothesis that the small scales are coupled to the large anisotropic scale dynamics in local shear layers between relatively large structures. Hence, isotropy of the small scales may be hampered by the direct coupling between large and small scales. This idea remains to be properly tested and evaluated.

The preliminary evaluation of the dissipation transport equation further demonstrates that the present measurement and processing framework can access not only the dissipation rate itself, but also the separate terms entering its transport balance. When the measurement volume was subdivided into smaller subvolumes, the left-hand side of the balance was found to follow the summed right-hand-side contributions closely, including the main positive and negative excursions. This agreement is encouraging, as it suggests that the dominant spatial structure of the dissipation transport balance is preserved by the present processing chain. At the same time, the individual right-hand-side terms showed large differences in magnitude, with the pressure-Hessian contribution being dominant in the present preliminary evaluation. The decomposed terms therefore highlight both the physical richness of the balance and its sensitivity to derivative estimation, noise, local particle density and fitting support. The present result should consequently be interpreted as a first consistency check rather than as a final quantitative closure of the dissipation transport equation.

While the resolution has been shown to be sufficient for resolving both the dissipation rate of the turbulent kinetic energy and its transport balance, an outstanding challenge is to tackle the noise and how it impacts the signal when computing the gradients. Coming work will target the analysis of measured particle tracks from the present setup and how noise impacts the results and can be minimized. An important feature of the present measurement data is the high resolution compared to the measured scales. Since the turbulent kinetic energy, its dissipation rate and even the dissipation transport (including dissipation of the dissipation) are effectively resolved, the filtering effects should be negligible and therefore the noise effects can be isolated and tackled in a more straightforward manner. Measures are currently being taken with collaborators to adapt the experimental setup to better enable the noise reduction, as well as to find the most suitable analysis method to suppress these effects.

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