

Skin Friction topology on generic objects by 3D particle tracking

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ABSTRACT

A framework is presented to determine time-averaged skin friction and surface-flow topology around three-dimensional objects using Lagrangian particle-tracking. The approach introduces a surface-compliant discretization and binning, whereby velocity samples are collected following the local orientation of the object surface. A nonlinear fit is introduced that models the near-wall velocity distribution using the arctangent function. The latter is shown to accommodate for the nonlinearity of near-wall velocity profile, while remaining robust to the near-surface data gap caused by a void of tracers. Comparison with flat surface-adapted binning (coins) indicates that the arctangent model most frequently yields physically consistent wall-shear estimates than linear or quadratic fits, owing to its ability to represent the steep velocity profile and complying with an external asymptote.

Time-averaged data from 3D particle-tracking of the flow around a wall-mounted cube at $Re = 8 \times 10^4$ is examined. The resulting skin-friction distribution reproduces the canonical features of the cube flow, including the horseshoe vortex, sharp separation at the leading-edges, arch vortex and the associated critical points.

The proposed framework provides a generalized methodology for skin friction analysis around objects of arbitrary shape based on large-scale 3D particle tracking experiments.

1. Introduction

The use of surface flow visualization to infer the flow topology is frequently practiced to understand the flow arrangement around complex objects. A widely used technique for qualitative inspection of the near-surface flow is oil flow visualization (OFV, Merzkirch & Gersten, 1987), whereby patterns are produced that reveal separation and attachment lines, transition regions and the footprints of vortical structures adhering to the surface or extending into the outer flow (Depardon et al., 2005). The technique has been practiced and perfected from art to science and to art again in the past century delivering fluid dynamic insight and inspiration (Van Dyke, 1982). However, OFV remains limited to stationary objects and mostly horizontal surfaces, being hindered by gravity or inertia force. Furthermore, comparing OFV to other measurements or CFD predictions needs careful scrutiny to translate the oil flow pattern into a quantifiable flow property.

Quantifying near-surface flow through the distribution of skin friction is challenging, as it requires high-resolution measurements taken in close proximity to the wall. Oil flow interferometry (OFI) offers a direct means of estimating wall shear stress under canonical flow configurations, such as turbulent boundary layers over a flat plate or flows past a backward-facing step (Tanner & Blows, 1976, Jovic & Driver, 1994). However, the application of OFI is limited, as it typically provides only local measurements of skin-friction magnitude at one or a few discrete locations in boundary-layer studies (Örlü & Vinuesa, 2020).

Planar particle image velocimetry (PIV) has also been used to estimate the skin-friction along the intersection of the laser light sheet and the object. High spatial resolution is demanded to resolve the velocity gradient in the linear layer at the wall as demonstrated in the studies of Kähler et al. (2006) and Kempaiah et al. (2020) among others. By combining numerous views, the skin friction along the curved line of an airfoil could be obtained by Fujisawa et al. (2009). A pioneering application of planar PIV for the description of the surface flow topology and the two-dimensional distribution of skin-friction over a wall mounted cube is documented in the work of Depardon et al. (2005), where the measurement plane is carefully placed at a small distance and parallel to the wall. The approach reproduced the details of the near wall flow topology and an indication of high and low velocity regions. It required however the combination of several measurements around the cube surfaces with the careful alignment of the light sheet and the imager. The generalisation to arbitrary objects featuring curved surfaces is considered impossible for planar PIV, based on the light sheet concept.

Three-dimensional particle tracking has recently become the option of choice for volumetric measurements, especially after the introduction of the Shake-The-Box algorithm for combined treatment of particle image detection and motion tracking (STB, Schanz et al., 2016). The latter has been used for measurements around objects of arbitrary shape using coaxial volumetric velocimetry (Schneiders et al., 2018), such as car mirrors (Kim et al., 2020) or even a full scale cyclist (Jux et al., 2018) employing and combining up to 450 views. The cyclist database delivered a quantitative visualization of the flow streamlines in close proximity (at approx. 1 \ cm distance) of the object and enabling the compact rendering of surface flow topology, for instance indicating stagnation, separation and reattachment points/lines, recirculatory regions and vortices. The analysis of skin friction magnitude was hampered by many factors, namely the spatial resolution and the mismatch between the Cartesian grid for the velocity field representation and the inclined and curved object surface. Such mismatch strongly affects the near wall velocity estimation yielding coarse approximations of the velocity gradient close to the wall. For time-averaged flow measurements, the spatial resolution can be improved by accumulating snapshots to enable smaller interrogation volumes (bins) as demonstrated by Miozzi et al. (2024), who recorded up to

50,000 snapshots to reproduce the velocity field and the skin friction distribution around a wall-mounted, transparent cube in a water tunnel.

A second challenge related to 3D-PTV measurements around a 3D object is related to illumination and imaging blockage. The problem was approached by Hysa et al. (2023) by two directions of illumination and a redundant set of viewing cameras. The latter was partitioned into sub-groups, which data needs to be later stitched, similarly to the robotic PIV technique (Jux et al., 2018). Introducing the object in the particle reconstruction algorithm has proven pivotal to advancing the technique. The object aware particle reconstruction technique (Wieneke & Rockstroh, 2024) disables specific regions of the measurement domain from views obstructed by the object, thereby eliminating the need to partition the imaging system into subgroup. As a result, 3D reconstruction of particles can be performed with a monolithic approach, to the benefit of the number of cameras participating the triangulation. Placement of the object geometry and in-situ correction of its position is referred to as object registration (Hendriksen et al., 2024) and the precise placement of the fluid-solid interface paves the way to a consistent evaluation of wall shear from the near surface velocity measurements.

Yet, the task of producing the near-surface flow visualization and skin friction estimation from three-dimensional measurements remains far from trivial. Cartesian data reduction from scattered PTV datapoints within cubic bins (Agüera et al., 2016), results in a voxelated representation of the 3D object. The noncompliance with the object surface position and orientation stands as a major limitation to the accuracy and resolution of the near surface flow reconstruction. Jux et al., 2018 circumvented the problem by dilating the object surface and interpolating the velocity distribution onto such surface. This method guarantees robust visualizations of near-surface streamlines (friction lines) but cannot capture the skin friction magnitude.

The present work addresses the latter objective developing a framework generalisable to 3D objects. The study examines first the problem of data domain discretization, introducing a surface-compliant approach. Furthermore the treatment of the near wall velocity data, in presence of finite gaps and limited resolution is approached with nonlinear modelling of the velocity profile. Chapter 2 describes the principles and the implementation of such skin friction estimation method. A detailed numerical as well experimental assessment is carried out to indicate the boundaries of applicability. The experiments over a wall-mounted cube are used to demonstrate the proposed technique (chapter 3) and compare its behaviour with respect to simpler methods reported in past works. The physical analysis of the flow 3D topology and skin friction around the surface mounted cube (chapter 4) completes the discussion.

2. Skin friction estimation

The skin friction generated by a fluid stream over a solid surface produces a tangential force on the surface, contributing to aerodynamic drag, often referred to as viscous drag. The wall shear stress, τ_w is the product of fluid viscosity μ and wall-normal derivative of the tangential velocity $\partial V/\partial y$ at the surface ($y = 0$). The skin-friction coefficient, C_f , is obtained by normalising the shear stress with a reference dynamic pressure as follows:

$$\tau_w = \mu \left(\frac{\partial V}{\partial y} \right)_{y=0} \quad (1)$$

$$C_f = \frac{\tau_w}{1/2 \rho V_\infty^2} \quad (2)$$

The estimation of C_f is challenged by the high-resolution required to measure the shear rate close to the wall. As a result of viscous and turbulent diffusion the velocity profile across the boundary layer is highly non-linear, often exhibiting maximum shear at the wall. The velocity vector is measured at a discrete location off the surface and numerical differentiation also requires the distance between such vector and the wall to be known. Alternatively, a linear fit of the velocity distribution has been extensively practiced, to estimate the wall shear from the linear sub-layer (Kempaiah et al., 2020, Dacome et al., 2024) amongst others. The latter approach is only valid, however, for the specific case of a developed turbulent boundary layer.

The problem complexity escalates when the object-fluid interface is inclined with respect to the Cartesian mesh of data, or even curved (Figure 1 **Error! Reference source not found.**) and with edges as occurring in many relevant engineering problems (prisms, bluff-bodies, airfoils, vehicles, living subjects etc.). The following discussion defines some general conditions to deal with three-dimensional objects.

2.1 Cartesian data and cubic bins

Cartesian grid representation/reduction (CGR) is considered the current best practice for particle tracking data reduction and visualization. A common approach to CGR is that of collecting an ensemble of randomly occurring measurements of particle velocity data falling within a specified bin (cube of side length l_c). The corresponding velocity vector can either be estimated as the ensemble average value or using a functional fit as explained in the work by Agüera et al. (2016). Cubic bins of side length l_c and spaced by a separation s_c retain a uniform spatial resolution along

the three coordinate directions and throughout the domain, to ease data post processing and visualization. Partial overlap of neighbouring bins by a factor 0.5 or 0.75 leads to a finer separation between velocity vectors.

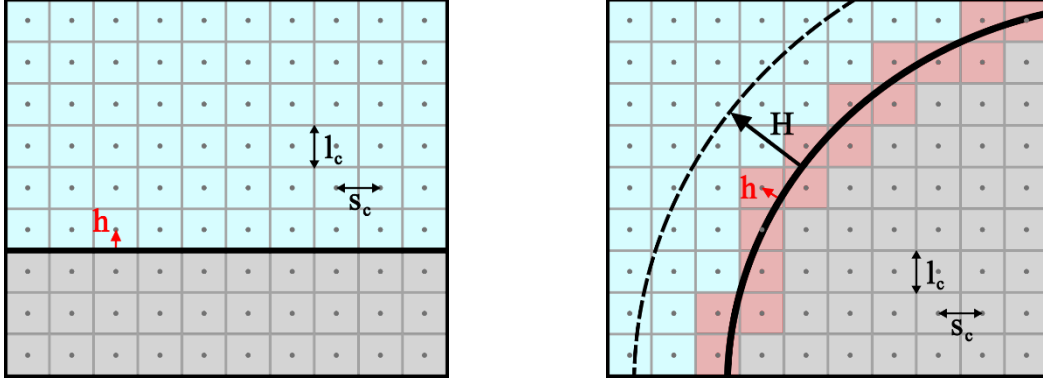


Figure 1: Cartesian domain discretization using cubic bins for ensemble averaging of particle tracking data around a flat (left) and curved (right) object. Bins in the fluid domain (blue), interface (red) and solid object (grey). Object surface (solid black line) and its dilated version (dashed line) by an amount H are shown, following the method applied by Jux et al. (2018).

In the case of a flat object, the cartesian grid representation can easily be made compatible with the local surface in order to simplify the extraction of the velocity gradient at the wall. This is illustrated in the left of Figure 1 where the interface between the fluid (blue) and object (grey) domain is aligned with the bin interface, leading to a constant spacing h between the object surface and closest available velocity vector without bins overlapping the fluid-solid interface.

An arbitrarily shaped object does not comply with the Cartesian discretization as shown in the right of Figure 1. The latter, instead produces bins fully immersed in the fluid domain (blue), within the object (grey) and overlapping the interface (red) resulting into a pixellated representation of the surface. Furthermore, the varying distance h between the position of a velocity vector and object surface affects the estimation of the velocity gradient at the wall. The solution proposed by (Jux et al., 2018) considers an outwards displaced surface (dashed black line) by an amount H , where the velocity is evaluated. From that point, a linear model is considered bringing the velocity to zero at the wall. The distance H must account for the bin size l_c , ($H \geq \sqrt{2}l_c$) as to exclude data from bins that intersect the surface. This technique has proven robust for the estimation of the near-wall topology and to identify separation, reattachment and stagnation lines. However, it largely underestimates the skin friction magnitude due to the highly nonlinear velocity profile in proximity of the surface (i.e. boundary layer).

2.2 Surface compliant binning

A surface compliant bin can be defined by selecting a point at the object surface and considering a region that extends outwards by an amount H and sideways by l_s . The main result is that the bin lies closest to the surface, but without intersecting it.

A set of accumulated velocity vectors around a 3D object is sketched in 2D in the left of Figure 2. The measured particle velocity $\vec{V}_m$ is indicated with the black arrow and the object is included as dark grey.

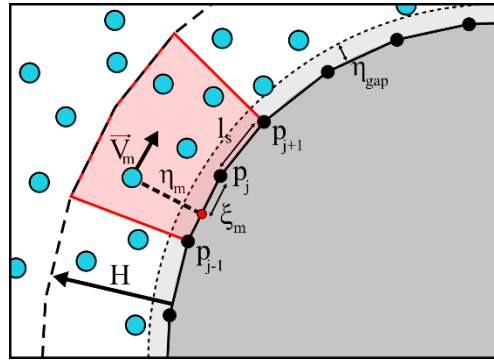


Figure 2: Local system of reference compliant with object-flow boundary and particles velocity vectors near the surface. Generalisation to a discretised 3D interface and corresponding bin.

The object surface is commonly represented as a discretized (faceted) surface formed by a set of connected vertices (p_j , black points) as typical for the STL format. The bin is centered around the vertex (or node) with the base area, defined by the STL discretization (vertex spacing l_s), spanning up to neighbouring vertices as shown in Figure 2. Such surface compliant bins are only defined in the vicinity of the object up to a distance from the surface H , indicated by the dashed line. Within each bin, the positions of data points are represented by local coordinates: (ξ, η) with η indicating the wall-normal distance from the surface and ξ the tangential offset from the bin vertex. Despite bins extending in the wall-normal direction from $\eta = 0$ at the surface to $\eta = H$, the finite size of the tracers prevents measurements within a thin region confined to the object surface as indicated by the dotted and shaded region η_{gap} .

2.3 Modelling

Similar to the approach by (Agüera et al., 2016) the velocity distribution inside a bin can be modelled by a Taylor expansion, typically of 1st or 2nd order. A first simplification is proposed here whereby the velocity inside the bin is considered a sole function of the wall-normal position and the variation along the other directions is neglected. This hypothesis may be justified by the separation between the small diffusive scale of the boundary layer and the relatively larger scale at which the pressure gradient drives convection.

For the velocity distribution within a surface compliant bin the following conditions are hypothesized:

- | | |
|--|--|
| i) $V(\eta) = 0$ when $\eta = 0$. | The velocity at the wall vanishes (no-slip condition) |
| ii) $\frac{\partial V}{\partial \eta} > 0$. | Monotonic increase of velocity along the wall normal direction |
| iii) $\left(\frac{\partial^2 V}{\partial \eta^2}\right)_{\eta=0} = 0$ | Constant gradient (i.e. linear velocity) at the wall |
| iv) $\lim_{\eta \rightarrow \infty} \left(\frac{\partial V}{\partial \eta}\right) = 0$. | Vanishing gradient outside of the outside of the viscous layer |

Such conditions are representative of the velocity distribution in a boundary layer, whereas conditions i, iii and iv may not apply strictly everywhere, when diverse flow patterns occur in separated three-dimensional flows.

A linear function complies with conditions i, ii and iii. In particular, the slope of a linear fit is highly dependent upon the height along which the fit is performed, which is a result of violating iv. A second order polynomial (parabola) also violates condition iv, but it offers the advantage of a decreasing shear rate moving away from the wall, until the apex of the parabola. Yet, sensitivity to the choice of H remains. A function that complies with the four conditions above whilst still offering an analytical formulation is that of an arctangent. Given the above, the arctangent function is considered here as a suitable candidate to mimic the near surface velocity profile. In synthesis:

- | | | |
|---------------|--|--------------------------|
| Linear fit | $V(\eta) = a\eta + b$ | (with $b = 0$, no slip) |
| Parabolic fit | $V(\eta) = a\eta^2 + b\eta + c$ | (with $c = 0$, no slip) |
| Arctan fit | $V(\eta) = a \cdot \arctan(b\eta) + c$ | (with $c = 0$, no slip) |

3. Experimental assessment

Experiments are performed in a low speed wind tunnel (W-tunnel) at the aerospace engineering laboratories of TU Delft. The W-tunnel features an open test section, equipped with a $60 \times 60 \text{ cm}^2$ cross section and operated at a free stream velocity of 10 m/s . A flat plate of 1.5 m long with elliptical leading edge followed by a transition strip is placed at the wind tunnel exit. A 3D printed cube of 12 cm side length is wall-mounted at a distance of 80 cm from the leading edge (Figure 3), resulting in a Reynolds number of 80,000. Despite its geometric simplicity, the cubic obstacle produces a variety of complex flow structures around its sides and surrounding the juncture (Martinuzzi & Tropea, 1993).

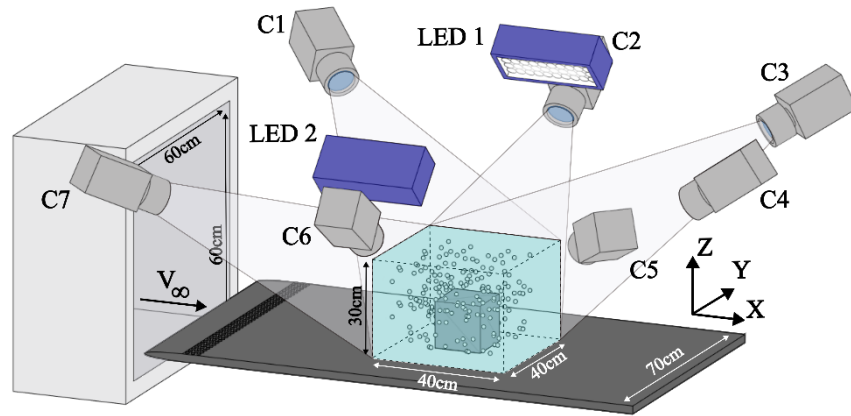


Figure 3: Measurement setup consisting of 7 cameras and 2 directions of illumination. The wall-mounted cube is exposed to unconfined stream (open jet) at the exit of the wind tunnel.

3.1 PIV system

Helium Filled Soap Bubbles (HFSB) with a mean diameter of $400 \mu\text{m}$ are generated in the settling chamber by a seeding rake comprising 200 nozzles (Saiz et al., 2022) yielding a spatial concentration of approximately 0.5 bubbles/cm^3 . Two LaVision *flashlight-300* LED arrays are directed towards the cube from the opposite sides. Seven Photron *FASTCAM* CMOS cameras (1 Mpx, 5400 fps max) surround the object with angular interval of approx. 45° . The resulting measurement domain is approximately $40 \times 40 \times 30 \text{ cm}^3$ centered around the object. The seeding, imaging and illumination setup are shown in the sketch of Figure 3. Image to object 3D calibration is obtained imaging a planar calibration plate with fiducial markers in six positions. The pinhole model is adopted for calibration, yielding sub-pixel accuracy. The latter is further refined using the volume self-calibration technique (Wieneke, 2008), bringing the triangulation errors below 0.1 pixels.

The image acquisition rate is set to 3kHz (free-stream particle displacement of 10 pixels) for a total of 5000 images. Time resolved particle tracking is performed using the Shake-The-Box algorithm (Schanz et al., 2016) available in the LaVision *DaVis 11* software. Particle tracks are regularized by a 2nd order polynomial regression over 5 timesteps with positional and velocity uncertainty being provided from the fit residuals (Janke & Michaelis, 2021), which amount to 10 mm and 1 cm/s respectively.

3.2 Skin friction estimation from experiments

The velocity field in the symmetry plane is illustrated in Figure 4 by streamlines and velocity contours. The flow stagnates at the front face of the cube and develops a boundary layer over said face, which sharply separates at the upper edge. The separated flow over the top face features a complex recirculation, including a secondary separation towards the aft side. On the floor, the boundary layer approaching the front face undergoes separation and forms a well-known horse-shoe vortex, depicted later in its 3D features. As a result of the diverse flow conditions, also the near wall flow exhibits different velocity distributions. Three salient points are selected for an in-depth analysis of the proposed methods, in this case using experimental data that include large spatio-temporal fluctuations as well as a wall gap and a degree of uncertainty for the solid surface position. At point A, in the approaching boundary layer, a relatively high value of C_f is expected. In contrast, at location B, the reverse flow features a milder velocity gradient. Finally, at point C, the highly turbulent conditions of the wake produce large amplitude fluctuations and a relatively high skin friction in the reverse flow.

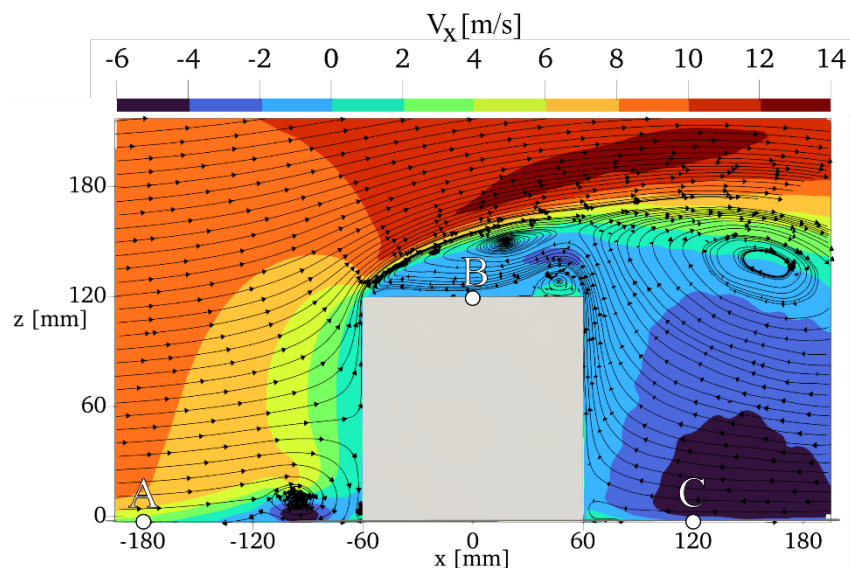


Figure 4: Time-averaged velocity field in the symmetry plane.

In these three locations, the velocity samples are represented in an interval from the wall till a height of 8 mm (Figure 5). A data gap with $h_{gap} \sim 0.5$ mm is evident from the plot, which is ascribed to the finite size of the HFSB tracers. An estimate of the velocity profile is obtained here using specifically adapted bins with a minimal height (1 mm) and extended in the other two directions (10 mm radius), such to capture a sufficient number of velocity samples. The resulting *coin-like* bins are intended to better resolve the velocity variation along the wall normal direction. Within the *coin* bin, the velocity samples are ensemble averaged. Neighbouring *coins* overlap by 50%, yielding data points spaced 0.5 mm. The value at the wall is enforced to zero for skin friction estimation. This approach is only applicable as a verification tool to the present case where the points are selected over a flat surface. Generalization of the *coin* bin is not regarded as applicable to curved surfaces and 3D objects in general.

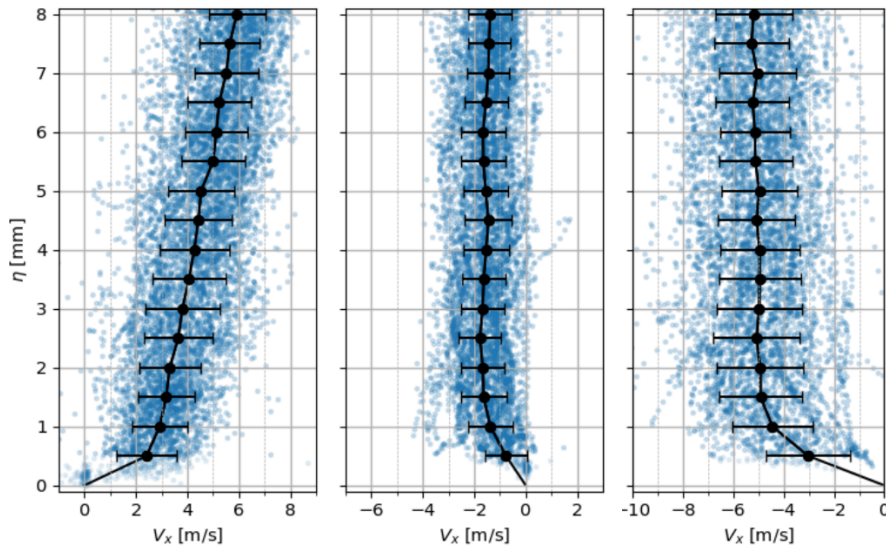


Figure 5: Clouds of velocity samples (blue markers) at locations A (left), B (middle) and C (right). Ensemble averaged velocity profile from coin-bins (black line and labels). Horizontal error bars indicate the velocity standard deviation within the bin.

Location A (Figure 5-left) lies within the incoming turbulent boundary layer of about 20 mm thickness. The large difference of slope between the first and the second segment indicates a potential bias (underestimation) error for the gradient at the wall. From the second segment onwards the velocity profile increases at a rather constant rate. Point B (Figure 5-middle) corresponds to a milder velocity profile, with lower levels of fluctuations. A lower level of skin friction is expected in B and the continuous variation of the gradient indicates a better resolved wall gradient. Finally in C (Figure 5) the nonlinear velocity profile is distributed over

approximately 4 segments. The large data scatter is due to the high turbulence intensity in the wake, which hampers the convergence of ensemble averaging.

In Figure 6, the parabolic and arctangent fit are examined with a bin height $H = 3 \text{ mm}$. The parabolic fit exhibits visible departure from the velocity profile based on coin bins, with a clear underestimation of the slope at the wall in locations A and C. The arctangent fit exhibits a good agreement with the coin bins and tends to predict a larger wall shear, consistent with the observations made in the previous sections. The near wall slope using the coin, polynomial and arctangent fit are most similar in location B, while they visibly deviate at locations A and C. This result is consistent with the relative velocity variations observed in locations B and C which will be referred to as the *maxmin* criterion. The *maxmin* criterion is taken as the ratio between the maximum (V_{max}) and minimum (V_{min}) velocity over the segment from $\eta = H$ down to $\eta = \eta_{gap}$. At point B a *maxmin* of approximately 3 is estimated. Comparatively, in A and C a *maxmin* of 1.7 and 1.9 is estimated, indicating a poorer velocity gradient resolution.

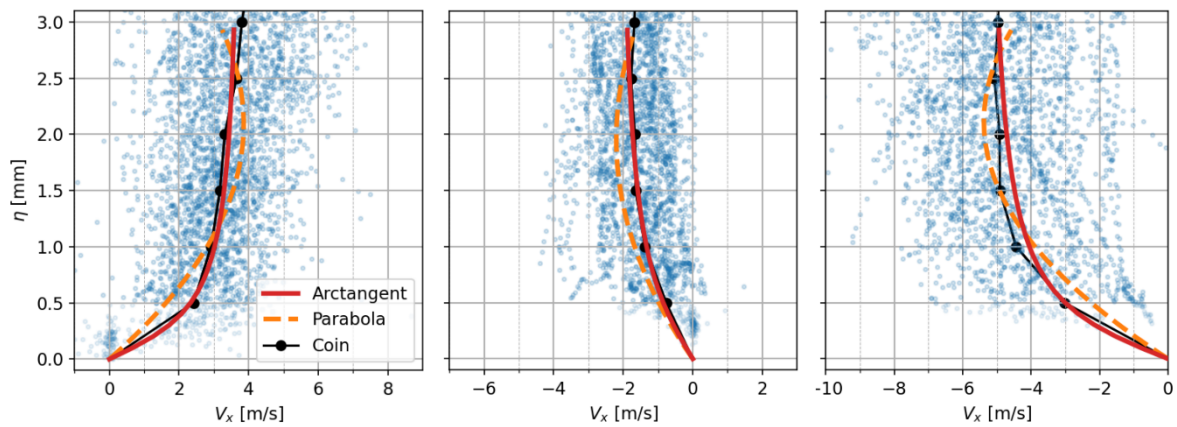


Figure 6: Arctangent and polynomial surface binning results at location A (left), B (middle) and C (right) with bin height $H = 3 \text{ mm}$. Velocity from coin-like bins included as reference.

A data segment dependence analysis is carried out on the experimental data. The skin friction coefficient estimation is obtained varying the bin height H , with its functional model. Results are shown in Figure 7 for the three chosen locations. The coin bin result is indicated by a single value (orange label) at $H = 0.5 \text{ mm}$ corresponding to the coin middle point. The data labelled as CGR stands for the approach based on the Cartesian Grid Reduction as adopted by Jux et al. (2018, see section **Error! Reference source not found.**). The linear extrapolation towards zero velocity at the object surface from CGR largely underestimates the curvature of the profile resulting in a skin friction underestimation by a factor 2 to 5. The parabolic fit brings a substantial increase in the skin friction prediction, comparing favourably to the coin bins for bin height of 2 to 3 mm. The

monotonic behaviour towards asymptotic value of the arctangent function fits the velocity profile further away from the surface without sacrifice to the wall shear. The arctangent generally returns higher values of C_f compared to both the parabolic fit and even to the thin coin bins. Also in this representation, best agreement among methods is observed at location *B* compared to *A* and *C*. This follows from the milder velocity gradient at the surface, as also indicated by an increased $maxmin$.

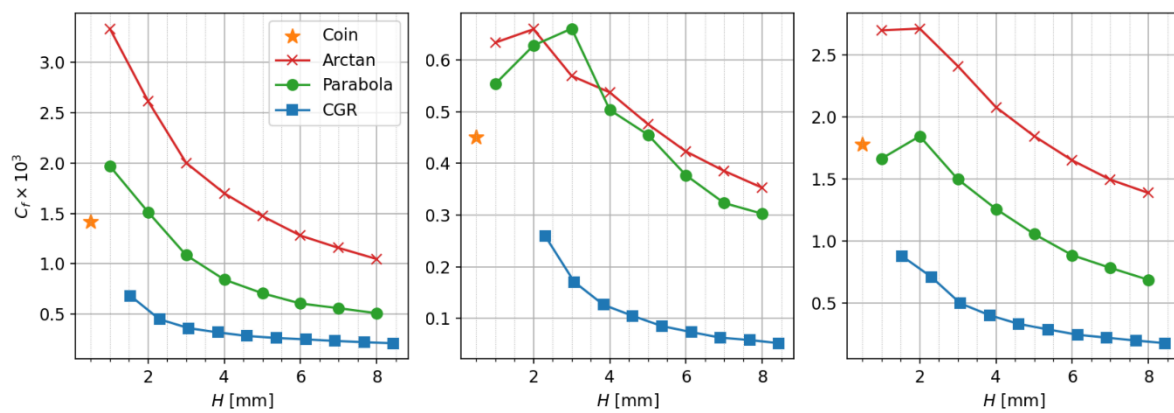


Figure 7: Skin friction estimation at locations *A*, *B* and *C* as a function of bin when changing H for different approaches.

4. Surface flow topology around the wall-mounted cube

The global skin friction analysis on the cube surface is discussed as obtained using a mesh with average vertex spacing of 4 mm and a bin height $H = 3$ mm. The 3D vortex organization around the obstacle leaves a distinct footprint in the skin-friction magnitude distribution and skin-friction lines.

The skin-friction distribution on the wall-mounted cube and surrounding floor is illustrated in Figure 8. The boundary layer advancing towards the obstacle thickens and undergoes separation under the influence of the adverse pressure gradient (see also Figure 4). As a result the wall shear decreases and vanishes at approximately half the cube height upstream of the front face. This point of stagnation is marked as S_1 as it features a saddle point from which skin friction lines are redirected laterally, around the obstacle. Flow passing over S_1 is also elevated around the horseshoe vortex, which develops in front and aside the cube aligning with the streamwise direction past it. The horseshoe vortex is highlighted with the iso-surface of λ_2 criterion (Jeong & Hussain, 1995) at a value of $-50,000 \text{ s}^{-2}$. The 3D visualization of such structure corresponds on the floor with S-shaped streamlines. A cross section of the streamlines identifies the complex flow

organization in the region of the horseshoe vortex (zoomed insert of Figure 8). Just in front of the cube, the horseshoe vortex induces a reattachment point in the form of a diverging node D_2 . Upstream of D_2 , the skin-friction lines indicate reversed flow with increased wall shear just below the horseshoe vortex as it recirculates high momentum flow close to the surface. Further upstream, a small secondary separation region is visible between saddle point S_2 and node D_1 . Depardon et al. (2005) reported the identification of these two distinct critical points to be sensitive to wall-normal resolution as for increased wall distance, they converge into a single saddle point. The characterization of the upstream flow topology by a saddle-node-saddle-node configuration moving downstream respectively follows the findings of Depardon et al. (2005) as well as the earlier observations of Martinuzzi & Tropea (1993).

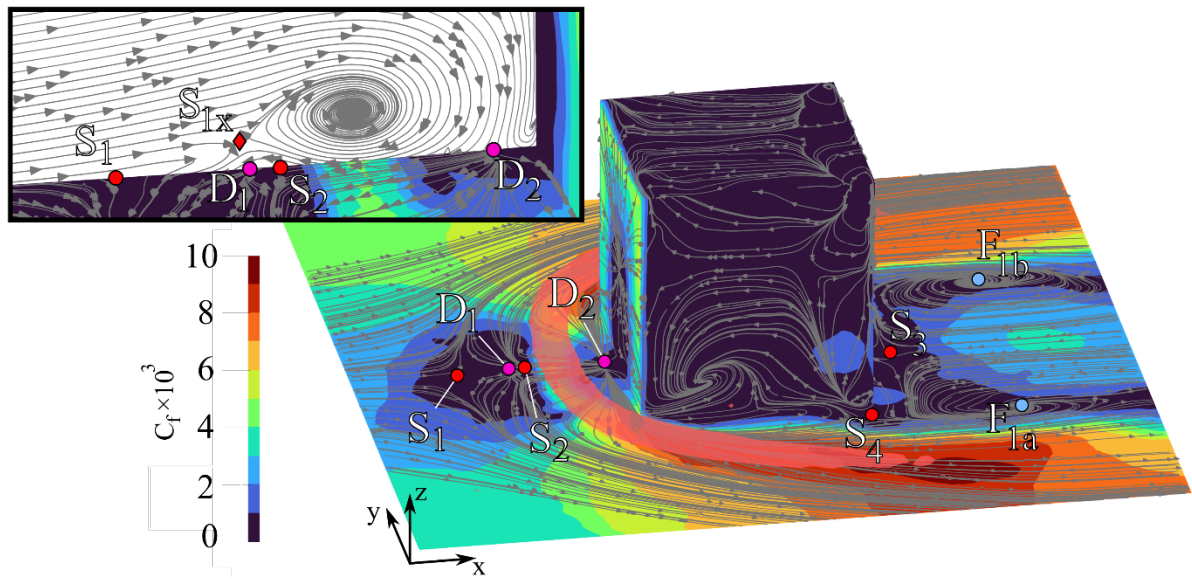


Figure 8: Surface flow topology on and around the wall-mounted cube visualized through skin-friction distribution and λ_2 criterion isosurface of the horseshoe vortex.

The flow over the horseshoe vortex stagnates at the centre of the front face (Figure 9) and a boundary layer develops radially towards the sharp edges of the cube where separation occurs. Given the relative boundary layer thickness ($\delta/L \approx 0.15$, with cube side length $L = 120 \text{ mm}$) the flow does not reattach at the top or side faces of the cube.

The wake behind the rear face is characterized by the presence of an “arch vortex” fed by the quasi semi-circular separated shear layer. The arch-vortex recirculates flow at the floor and produce a reverse motion in the symmetry plane towards the rear cube face as indicated by the skin-friction lines. Such reverse flow has relatively high momentum and skin-friction values generally larger compared to the region upstream of the cube. A local maximum is visible below the arch vortex which can be identified through its footprint featuring two foci (F_{1a} and F_{1b}). The reattachment at

the rear of the cube generally lies 1.5 - 2.5 side lengths aft of the cube rear face depending on the experimental conditions. Such point falls beyond the measurement domain in the current experiment. The reverse flow develops a boundary layer that approaches the rear face and separates, as marked by the saddle point S_3 , closer to the surface compared to the separation at the front of the cube. A spanwise elongated recirculating region is confined to the rear cube-to-floor corner upstream of S_3 , which also transports fluid spanwise outwards.

The separated shear layer creates a pocket of recirculating flow on the floor along each of the side faces of the cube. The point at which this side recirculating region, the shear layer and spanwise outwards moving flow aft of the cube meet coincides with saddle point S_4 . The horseshoe vortex wraps around the separated region on the sides and induces a downwash, which interacts with the accelerating flow around leading edges producing two elongated lobes with highest wall shear at the sides of the obstacle. The configuration of the aforementioned critical points on the surrounding floor follows those described by Martinuzzi & Tropea (1993) and Depardon et al. (2005) who performed measurements at $Re = 40\,000$ to $160\,000$ and incoming boundary layer thickness ($\delta/L = 0.2$).

Figure 9 shows the skin friction lines on the cube faces. Only one of the side faces is shown for conciseness.

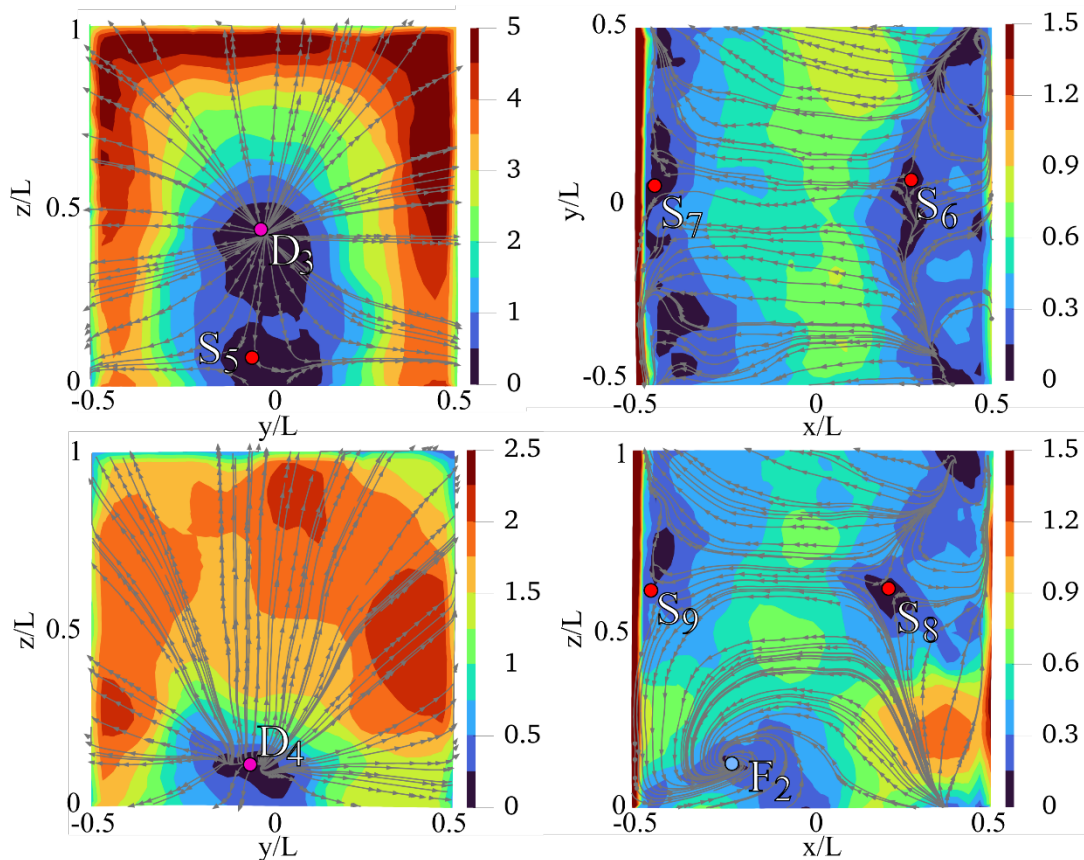


Figure 9: Skin-friction distribution on the cube faces. Color coded skin friction coefficient C_f (pre multiplied by 1,000).

At the front face (Figure 9-top-left) the flow stagnates slightly below the centre (node D_3) and develops radially towards the edges accelerating under the effect of a favourable pressure gradient, as indicated by the increasing values of skin friction. Since the flow is attached only on the cube front, the highest values for shear are encountered on this face. A small, elongated recirculating region is confined between point D_2 of Figure 8 and the upstream cube-to-floor corner. This vortex produces a small spanwise band with upward pointing skin-friction lines near the floor and the saddle point S_5 where this meets the outwards flow from the stagnation.

The flow pattern at the back face of the cube is illustrated in Figure 9-bottom-left. Here the reverse flow within the wake is brought towards the rear face under the induction of the arch vortex and a stagnation point appears considerably lower than that on the front face. The loss of momentum in the wake results in overall lower value of shear (approx. a factor 2, note the change of scale in the colormap) compared to the front despite the similar topology. Skin friction increase as the flow moves outwards from the stagnation point until separation at the trailing edges. Such secondary separation produce small recirculation bounding the trailing edges (sometimes referred to as a trailing edge vortex, da Silva et al., 2024). The flow pattern on the top face is shown in the top right insert of Figure 9. The reattachment line separating the main reverse flow and the trailing edge vortex appears to bend downstream towards the corners and a saddle point S_6 appears roughly in the center at approximately a quarter length from the cube trailing edge. The levels of skin friction are particularly low, approximately one order of magnitude below those on the front face. The boundary layer induced by the reverse flow advances towards the leading edge, encountering an adverse pressure gradient, which decelerates the flow until it separates. The spatial resolution of the measurement does not allow appreciating whether a tertiary structure is present at the leading edge, between the outer flow and the main reverse region. The friction lines patter suggest that such structure exists as a saddle point S_7 seems to indicate an additional flow reversal. Such organization was reported by Depardon et al. (2005) who identified a saddle-node-saddle combination in the vicinity of S_7 at $Re = 40,000$. Yet, for larger values of the Reynolds number these points appear to merge, similar to what indicated in the current experiment. A word of caution should be given when analysing regions close to the cube corners as the surface binning approach produces artefacts where the wall normal direction changes abruptly. The narrow band of high skin friction at the leading edge is ascribed to a numerical artefact.

Finally, the side face shown at the bottom-right of Figure 9 shows similar features as the top face, with the only marked distinction close to the floor. Here the upward shift of the position of the

saddle point S_8 is associated to the presence of a focus F_2 in the lower upstream corner. This focus indicates the presence of a corner vortex which spans across the junction of the floor and cube as also found by oil flow visualization in Martinuzzi & Tropea (1993). Additionally, a region of increased skin friction appears in the lower right corner where flow is transported upwards within the trailing edge vortex.

5. Conclusion

A surface-compliant binning approach has been developed to estimate the near-wall velocity distribution and skin-friction field from three-dimensional particle tracking measurements around arbitrary 3D objects. By enforcing the no-slip condition and modelling the wall-normal velocity profile with an arctangent functional, the method is able to capture the nonlinear behaviour of the near-wall flow more effectively than conventional linear or polynomial fits. The approach inherently accommodates the orientation and curvature of the surface, avoids the pixellation inherent to Cartesian grid reduction, and allows the extraction of both skin-friction magnitude and skin-friction lines over complex geometries.

Application to the flow around a wall-mounted cube demonstrates that the method reproduces known features of the surface-flow topology, including separation, attachment and foci points on the faces and surrounding floor, as well as the footprints of the horseshoe and arch vortex. Comparison with coin-based binning in selected locations shows that the arctangent fit consistently provides higher and more physically plausible skin-friction values, particularly where strong nonlinear gradients occur near the wall. The captured skin-friction patterns show good agreement with earlier surface-visualisation and planar-PIV studies, including the location of critical points along the centreline.

Accuracy in determination of the wall-shear depends on the dynamic velocity range resolved near the surface, expressed through the *maxmin* criterion. Regions of low skin friction generally exhibit sufficiently large *maxmin* values to ensure accurate reconstruction, whereas regions of high shear, where the velocity gradient is confined to a thin, experimentally inaccessible layer, yield smaller *maxmin* values and thus reduced quantitative reliability. This behaviour reflects an intrinsic experimental limitation associated with finite tracer size and wall-proximity gaps. Nevertheless, the method remains able to recover the qualitative topology of the flow, even in such challenging regions, and is expected to deliver improved quantitative accuracy in experiments featuring larger boundary layer thicknesses or smaller tracers.

Overall, the proposed surface-compliant binning and arctangent fitting framework provides a generalizable approach for reconstructing 3D skin-friction distributions over complex geometries from volumetric particle-tracking data. The approach enables time-averaged near-surface flow characterisation, including the identification of critical points and vortex footprints, and offers a pathway to quantitative skin-friction measurement in large-scale three-dimensional experiments.

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