

On the aspect ratio of ellipsoidal bubbles at different viscosities

P. Kováts^{1,*}, K. Zähringer¹,

¹: Laboratory of Fluid Dynamics and Technical Flows, Otto-von-Guericke-Universität Magdeburg, Germany

*Correspondent author: peter.kovats@ovgu.de

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ABSTRACT

Bubble shape is one of the key parameters governing hydrodynamics, mass transfer, and residence time in bubble column reactors. Accurate prediction of bubble aspect ratio is therefore essential for reactor design, scale-up, and computational fluid dynamics (CFD) simulations. Most available correlations for bubble shape have been developed from single-bubble experiments and may not be directly applicable to bubble swarms encountered in industrial processes. However, experimental data on bubble swarms in liquids of higher viscosity are still limited.

In the present study, the aspect ratio of spherical and ellipsoidal bubbles rising in water-glycerol mixtures was investigated experimentally using high-speed shadowgraphy in a laboratory-scale bubble column. Measurements were performed in pure water and in 30, 40, 50, and 60 wt.% glycerol solutions, covering a Morton number range from $3.9 \cdot 10^{-7}$ to $2.3 \cdot 10^{-11}$. Air bubbles were generated using nozzles with different diameters and gas flow rates, resulting in equivalent sphere diameters (ESD) between approximately 0.65 and 8.5 mm. Bubble size, shape, and velocity were extracted from image sequences and statistically processed to obtain mean aspect ratios as a function of bubble diameter with a resolution of 50 μm . The results show a strong dependency of bubble shape on the liquid viscosity. Small bubbles remained nearly spherical, exhibiting aspect ratios close to unity, whereas increasing bubble size led to a rapid transition towards ellipsoidal shapes. Increasing glycerol concentration shifted this transition to larger bubble diameters, indicating a stabilizing effect of viscosity on bubble deformation. For sufficiently large bubbles, the aspect ratio approached a nearly constant value of approximately 0.50–0.55, independent of the investigated liquid. Analysis using Grace and Tripathi regime diagrams confirmed that the measured bubbles were located predominantly within the spherical and spheroidal deformation regimes and that increasing viscosity reduced Reynolds and Galileo numbers while suppressing oscillation and breakup tendencies. The newly acquired data were used to evaluate and refine a previously proposed empirical correlation relating bubble aspect ratio to Eötvös and Morton numbers. The revised correlation provides improved prediction of both, the transition region and the asymptotic aspect-ratio limit. The results contribute to closing an important data gap for bubble swarms in higher-viscosity liquids and provide a basis for improved bubble-shape modelling in multiphase reactor simulations.

1. Introduction

In wide fields of industry – e.g. chemical, biotechnological, pharmaceutical or petrochemical – multiphase reactors are used for different reaction processes and mixing. In bubble columns the liquid phase contained in the column is the continuous phase, while the gas phase is introduced into the liquid through a gas distributor. The design of these reactors is task-specific, therefore the careful design is crucial. All system properties – e.g. vessel volume, flow direction, sparger

type, or operation mode – influence the chemical or biological, and also the economic efficiency. To this end, during the reactor scale-up process, several steps are necessary from laboratory-scale bubble columns, through pilot-reactors up to the full size industrial one. For the design, also Computational Fluid Dynamics (CFD) is used, in order to save time and finances. These simulations are difficult, because of the complex hydrodynamics and mass transfer in the bubble swarm. Due to the interfacial forces acting on the bubbles, the bubble shape is one of the most important parameters of a bubbly flow. The shape determines bubble motion, has therefore an influence on the residence time and also on mass transfer processes. For this reason, a quantitative description of the bubble shape is essential for the design and numerical simulations of bubble columns. Unfortunately, most empirical equations for the characterization of bubble shape are based on single bubble experiments, which are not necessarily applicable in bubble swarms. For this reason, new bubble swarm oriented experiments and equations are demanded. In our previous study on bubble shape in different media we have found, that the shape of ellipsoidal bubbles of a certain size can be predicted using only the physical properties of the liquid (Kováts & Zähringer, 2026). This study also showed, that from a certain bubble diameter, the naturally generated bubble shape stays constant. This shape remains constant until the bubble can be described as oblate spheroid. The proposed equation had been validated with literature data, but only few reliable bubble data exists for bubble swarms in higher viscosity liquids. For this reason, we performed new measurements in order to supplement the incomplete literature data. This work aims to perform shadow imaging measurements in a laboratory-scale bubble column reactor and measure bubble size, shape and velocity. The experiments are carried out in different water-glycerol mixtures to cover a wide range of viscosity.

2. Experimental set-up

The shadow imaging experiments were carried out in a laboratory-scale bubble column reactor (Figure. 1) made of acrylic glass which was already used for different experiments in our group before, and was already described in detail in previous papers (Kováts, Thévenin, & Zähringer, 2017, 2018, 2020; Zähringer & Kováts, 2021). The core of the bubble column is an acrylic pipe with a diameter of 140 mm and a length of 750 mm, which is surrounded by a rectangular jacket. This jacket was used to reduce the optical distortions due to the curved surface of the pipe and was always filled with the same liquid as investigated inside the column. As gas inlet four nozzles, placed in-line, were used with three different inner diameters – 0.25 mm (small nozzle SN), 2.5 mm (medium nozzle MN) and 5 mm (large nozzle LN) – to ensure a wide range of

naturally generated bubble sizes. The volume flow rate of the pressurized air is controlled by a mass flow controller. Additionally, magnetic valves were connected to each nozzle to ensure the flawless bubble generation.

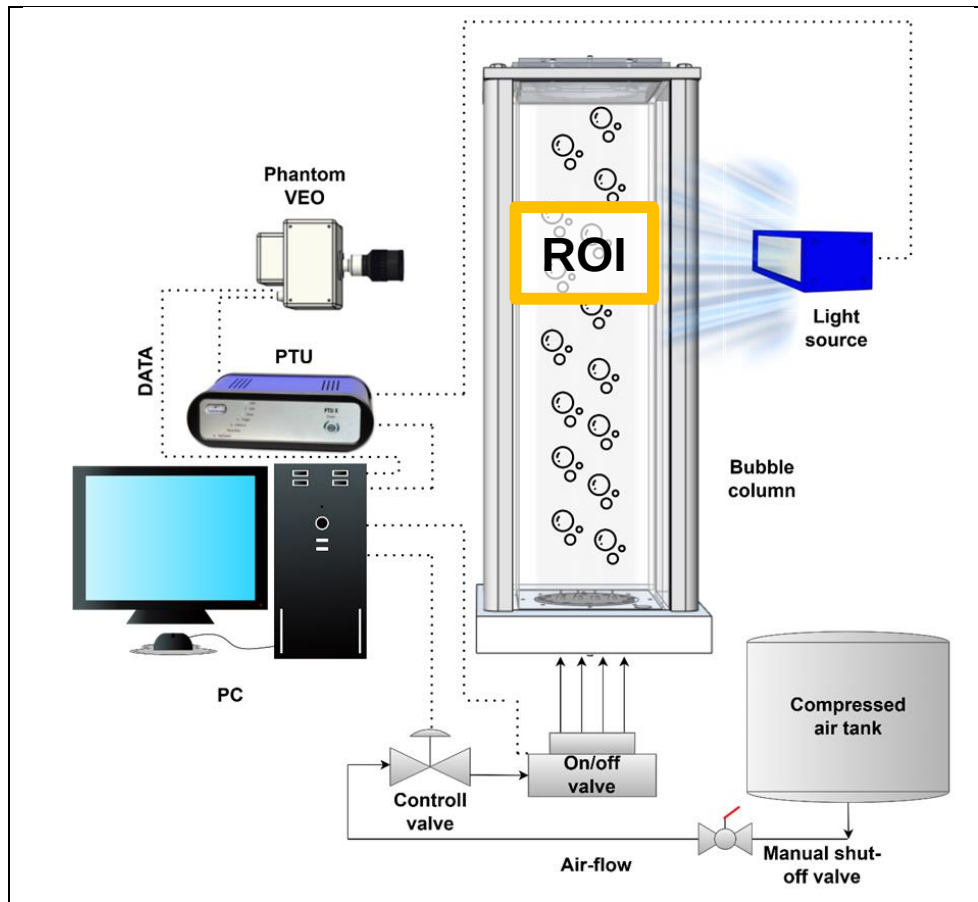


Figure. 1 Experimental setup.

The experiments were carried out at three different gas volume flow rates of 10, 30 and 50 l·h⁻¹ in five different water-glycerol mixtures, namely in pure water and 30, 40, 50 and 60 w% glycerol mixtures. The surface tensions and viscosities of the mixtures were measured in-house using the pendant drop method for surface tension and a Malvern Kinexus pro+ rotational rheometer for viscosity. The physical properties of the applied liquids are listed in Table. 1.

For shadow imaging of the rising bubbles a Phantom VEO 640L (2560x1600 pixels) high-speed camera was used in combination with a LED illumination from the rear of the column. For every investigated case, 4 times 1000 images were recorded with 0.1 kHz image rate in the top third (400-600 mm) of the column (**Range Of Interest** on Figure. 1). This height ensures that the bubbles reach their terminal velocity and are not influenced by the outlet nozzle, but only the swarm. The geometrical calibration was performed in the whole measurement area with a 3D calibration plate, which results in a spatial resolution of 18 pixels/mm.

Table. 1 Investigated cases.

Temperature [°C]	Mass fraction glycerol	Density of the water- glycerol mixture [kg · m ⁻³]	Dynamic viscosity [mPa · s]	Surface tension [mN · m ⁻¹]	Morton number	log(<i>Mo</i>)	Volume flow [l · h ⁻¹]	Gas holdup [%]
20	0	998.05	0.965	71.611	2.32E-11	-10.63		0.04-0.3
20	0.3	1072.5	2.413	68.977	9.45E-10	-9.02		0.05-0.4
20	0.4	1099.1	3.843	67.67	6.28E-09	-8.20	10-50	0.05-0.24
20	0.5	1126.1	6.040	66.442	3.95E-08	-7.40		0.05-0.31
20	0.6	1153.4	10.669	65.655	3.89E-07	-6.41		0.05-0.33

3. Data processing

The recorded images were processed and the bubbles were recognized in the DaVis 8.4 software (LaVision) and the raw data was further processed in Matlab and MS Excel. With the bubble recognition process bubble parameters e.g. their small and big axis, their position or velocity were extracted for each recognized bubble. The shape of a bubble can be described quantitatively with the bubble aspect ratio E , or its inverse, the eccentricity χ :

$$E = \frac{2y}{2x} , \quad 0 \leq E \leq 1 \quad (1)$$

$$\chi = \frac{2x}{2y} = \frac{1}{E} , \quad \chi \geq 1 , \quad (2)$$

where x is the semi-major axis and y the semi-minor axis of the bubble. From the raw data, which contains the bubble's minor and major axes, the equivalent sphere diameter (ESD) can also be calculated for further analysis:

$$ESD = \sqrt[3]{\frac{3V_B}{4\pi}} \cdot 2 , \quad (3)$$

$$V_B = \frac{4}{3}\pi x^2 y , \quad (4)$$

where V_B is the volume of the spheroid.

In order to obtain an average aspect ratio for each bubble diameter, the raw bubble data has been reduced. To this end, two steps were necessary. First the obviously erroneous recognitions were removed from the data. To this end, a velocity filter was applied to remove non-moving bubbles. An aspect ratio filter was also used to eliminate bubbles with an extremely low aspect ratio. Secondly, the bubble data was reduced to relevant unique bubble shapes based on the major and

minor bubble axes and mean parameters were calculated for these unique shapes. The whole process was already described in detail previously (Kováts, 2021; Kováts & Zähringer, 2026) and an exemplary representation of the raw and filtered mean aspect ratios and *ESDs* are represented in Figure. 2. The difference in the present study is the threshold of the occurrence filter, where bubble occurrence is defined as the number of recognized bubbles for each unique shape. In this study, the occurrence threshold was set between 5 and 10 counts depending on the total number of recognized bubbles, which was considerably lower than in the previous case.

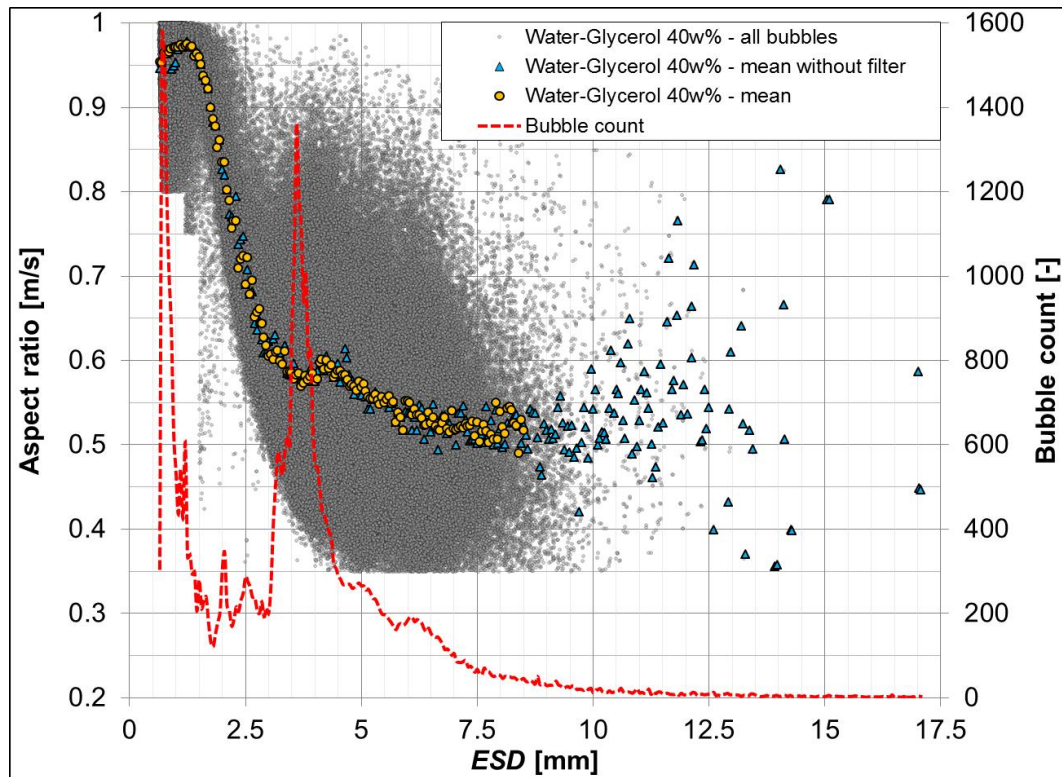


Figure. 2 Raw and reduced bubble data with bubble occurrence for the statistical analysis.

After these processing steps, from all experimental data mean aspect ratios calculated for each *ESD* in 50 μm size steps were calculated.

4. Results

The mean results are plotted in Figure. 3 for all glycerol contents at all investigated volume flows. The plotted data represent a clear inverse relationship between *E* and *ESD* and the strong influence of the glycerol concentration on bubble shape is obvious. The small bubbles below 1-2 mm, depending on the glycerol concentration, remain more round, therefore the mean aspect ratio is around 0.95. The higher the glycerol concentration the longer bubbles remain spherical. As the bubble size increases further, the aspect ratio suddenly drops and the bubbles become more elliptical. After the bubble size reaches a certain value, the aspect ratio become nearly

constant around 0.5-0.55. This value depends again from the glycerol concentration, namely with increasing glycerol concentration this value is shifted towards higher aspect ratios. Higher viscosity of the mixture stabilizes the bubble's shape and it delays the transition of the spherical bubbles to ellipsoidal ones.

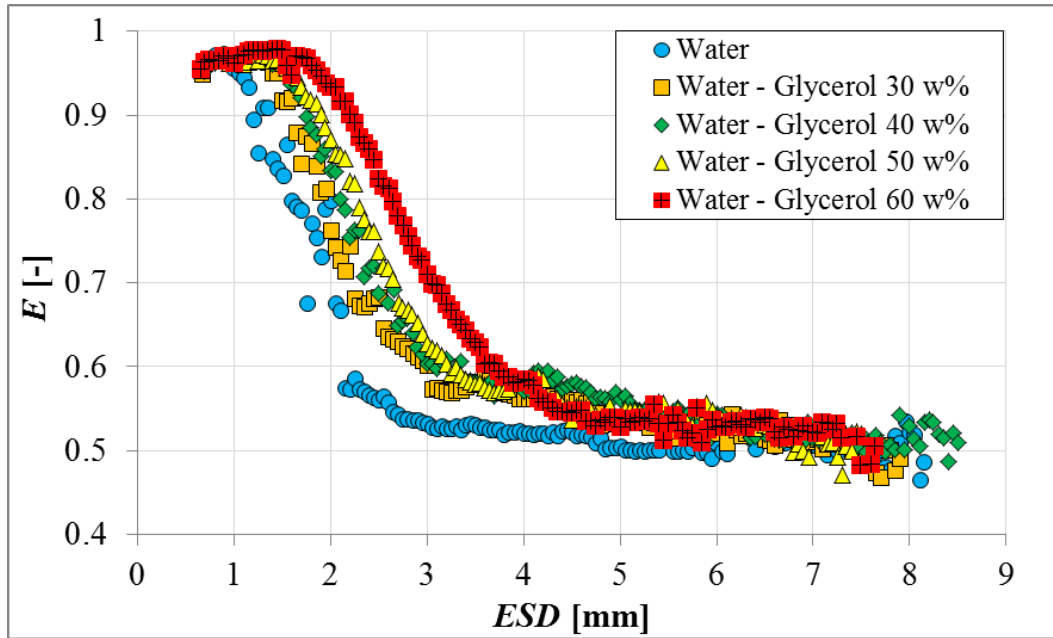


Figure. 3 Relationship between E and ESD for all investigated cases.

Cioncolini and Magnini (Cioncolini & Magnini, 2025) described very well the reasons of that behaviour. The small spherical bubbles are influenced mainly by the viscous and buoyancy forces, they remain longer round with increasing viscosity. With increasing bubble size the surface tension force and inertial forces become more dominant against the viscous forces. For this reason, the aspect ratio drops. Increasing further the bubble diameter, the aspect ratio reaches the 0.5 - 0.55 value, which is the maximum stable deformation state, still affected by the surface tension, buoyancy and inertial forces. Cioncolini and Magnini also considered a third state, where the bubble diameter further increases and the rise velocity is sufficiently large. In that case also the surface tension force becomes negligible, and the bubble is affected only by the inertial and buoyancy forces. Accordingly, the elliptical bubble shape turns to spherical cap shape. In the present measurements such spherical caps were not produced.

To visualize the above mentioned forces and their effects dimensionless numbers can be utilized. For the ratio of inertial and viscous forces the Reynolds number, Re ; for the ratio of buoyancy and surface tension forces the Eötvös number, $Eö$; for the ratio of gravitational and viscous forces the Galilei number, Ga ; and for the fluid properties the Morton number, Mo . According to

these dimensionless numbers the results can be represented with two well-known plots. The Grace-diagram (Grace, 1973) (Figure. 4 left) represents the relationship between the Reynolds, Eötvös and Morton numbers, while the shape and behaviour of the bubbles can be represented with the plot from Tripathi et al. (Tripathi, Sahu, & Govindarajan, 2015) (Figure. 4 right) using Galilei and Eötvös numbers.

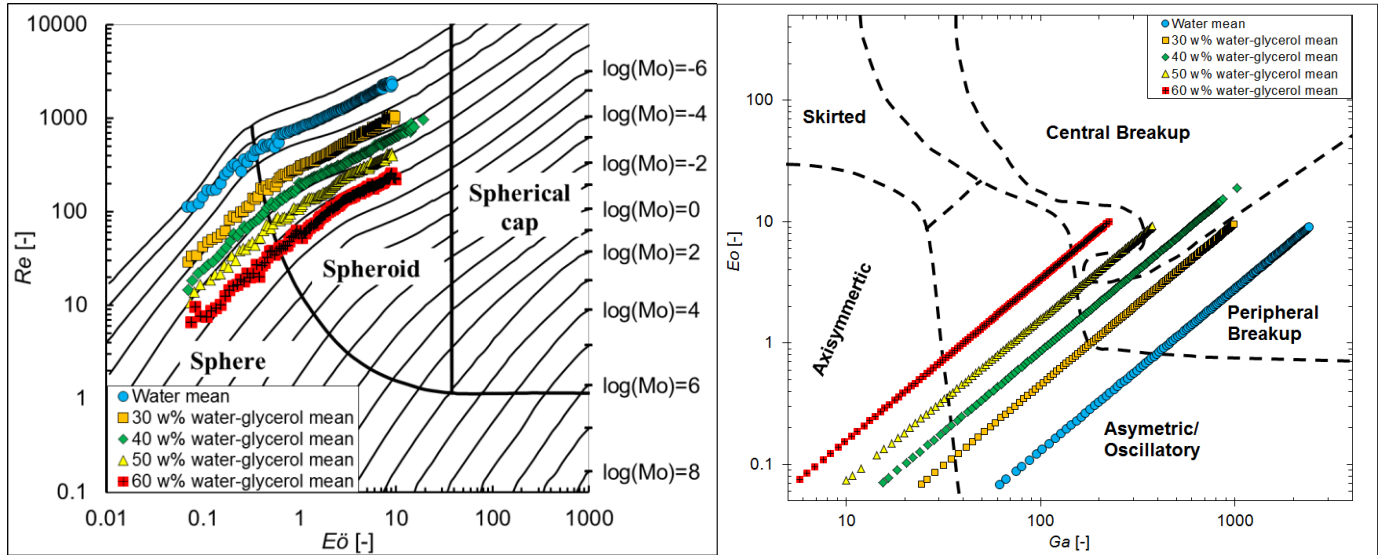


Figure. 4 Grace (left) and Tripathi (right) diagrams showing the mean result of the investigated cases.

The Grace diagram shows the consistency of the mean results of our measurements, the Morton-lines are followed by the data-sets for the different investigated water-glycerol mixtures. The investigated bubbles fall here in the spherical and spheroid (ellipsoidal) shape regimes. With increasing glycerol concentration, the Reynolds number decreases for comparable Eötvös number, since the viscosity increases, but the bubble size remains. The Tripathi diagram shows, that with increasing viscosity the Galilei number decreases. The highest values were reached with pure water. For similar Eötvös number the Galilei number at different mixtures shifts horizontally, which results in parallel lines for cases with the same Morton number. Increased viscosity decreases oscillation behaviour and suppresses breakup. Most of the investigated bubbles remain in the axisymmetric/oscillatory regimes. Nevertheless, but mostly at lower viscosities, some cases can be found in the peripheral breakup and central breakup regimes. However, during the experiments no breakup was recognized on the bubble images. This is not unexpected, since the Grace and Tripathi diagrams were based primarily on single bubble experiments. It is highly likely, that in bubble swarms, as in the present cases, the swarm has a stabilising effect and delays bubble breakup compared to single rising bubbles.

5. Correlation for the determination of the bubble aspect ratio

Using the ESD and the properties of gas and liquid, a combination of $E\ddot{o}$ and Mo dimensionless numbers is calculated, that was found to correlate with the aspect ratio in our previous study (Kováts & Zähringer, 2026):

$$E\ddot{o}^{4/3} \cdot Mo^{-1/3} = \frac{g \cdot (\rho_l - \rho_g) \cdot ESD^{8/3} \cdot \rho_l^{2/3}}{\sigma^{1/3} \cdot \mu^{4/3}}. \quad (5)$$

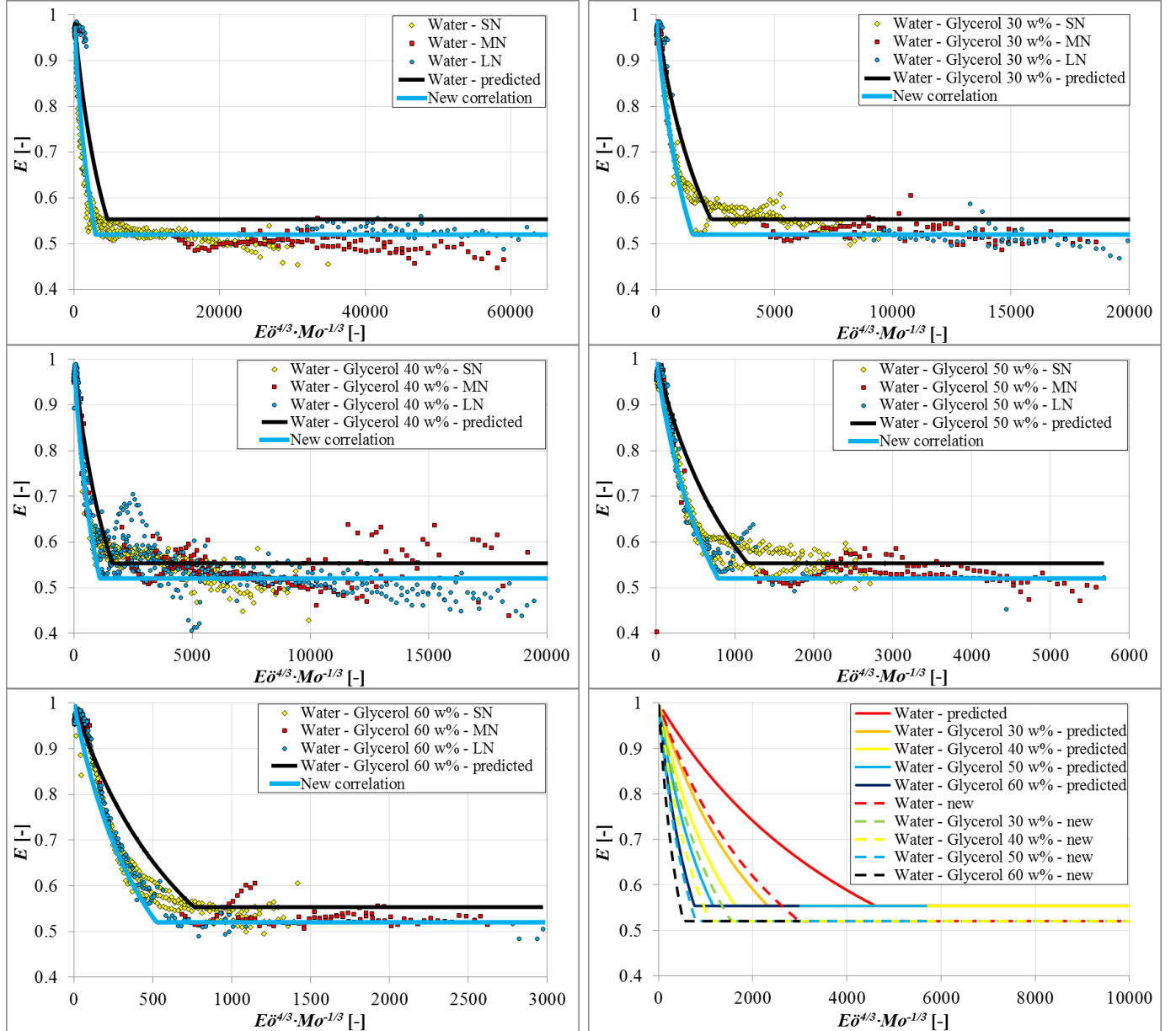


Figure. 5 Relationship between aspect ratio and $E\ddot{o}^{4/3} \cdot Mo^{-1/3}$. SN - small nozzle, MN - medium nozzle, LN - large nozzle.

The influence of the viscosity on bubble shape is clearly visible in Figure. 5. With increasing viscosity (increasing glycerol content) the curves are pushed to lower $E\ddot{o}^{4/3} \cdot Mo^{-1/3}$. The

average aspect ratio decreases for given ESD (or $E\ddot{o}^{4/3} \cdot Mo^{-1/3}$) with increasing glycerol concentration, that means with increasing viscosity. With increasing ESD (or $E\ddot{o}^{4/3} \cdot Mo^{-1/3}$) the aspect ratio decreases in all cases, as it was expected. As it has also been found in previous studies (Besagni & Inzoli, 2019; Gao et al., 2022; Kováts & Zähringer, 2026; Liu et al., 2021; T. Ziegenhein & Lucas, 2017; Thomas Ziegenhein, Lucas, Besagni, & Inzoli, 2018), from a certain ESD (or $E\ddot{o}^{4/3} \cdot Mo^{-1/3}$) the aspect ratio becomes nearly constant.

Our previous correlation Eq. (6), developed before (Kováts & Zähringer, 2026), and based on the liquid physical properties is represented in Figure. 5 by black solid lines. It predicts rather well the bubble's aspect ratios acquired within this new study, but can now be refined with the new data.

$$E = \frac{1}{\left[1 + 0.0164 \cdot \frac{E\ddot{o}^{4/3}}{Mo^{(1/3 - 0.1854)}}\right]} = \frac{1}{\left[1 + 0.0164 \cdot E\ddot{o}^{4/3} \cdot Mo^{-0.1479}\right]}; \quad (6)$$

if $E_{measured} \leq 0.5534$, $E = 0.5534$.

Especially the position of the bending point, where the decreasing aspect ratio transitions to a constant one was evaluated with the help of these new measurements, as well as the value of the constant part of the curve. The newly developed equation, Eq. (7) is changed in main form to fit better the curved slope of the experimental data:

$$E_{new} = \frac{1}{1 + 0.0249 \cdot e^{(0.413 \cdot \log Mo)} \cdot E\ddot{o}^{4/3} \cdot Mo^{-1/3}}; \quad (7)$$

if $E_{measured} \leq 0.52$, $E_{new} = 0.52$.

This new equation is presented with blue lines on Figure. 5. It fits better the slope of the measured data and also the constant value is more precisely reproduced. Unfortunately, the smooth transition from the slope into the constant value cannot yet be described with this equation. To this end, more data for bubble swarms will be needed, to find a better correlation between bubble shape and the liquid properties.

6. Conclusions

With this study we try to fill up a gap concerning literature values for aspect ratios of elliptical bubbles in different viscosity fluids. We therefore performed shadow imaging measurements of ascending air bubbles in a model bubble column with different glycerol-water mixtures at different nozzle sizes. This allowed us to acquire aspect ratios for a wide range of bubble

equivalent diameters ranging from 0.65 mm to 8.5 mm in total. The Morton number range of our study covers Morton numbers from $3.9 \cdot 10^{-7}$ to $2.3 \cdot 10^{-11}$.

With this data a previously determined empirical correlation, that allows for the calculation of the aspect ratio of a given bubble diameter from the material values of liquid and gas, was refined. Especially the constant part of this correlation and its starting point were specified in function of viscosity.

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