

Local cluster-based identification of flow dynamics

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ABSTRACT

This work presents a novel framework for the analysis and modelling of fluid-flow dynamics from time-resolved velocity fields measured using image velocimetry techniques. First, the time-resolved flow snapshots are partitioned into kinematically coherent subdomains through manifold learning applied to the time history of the measurements. Then, the resulting spatial manifold is clustered to identify flow regions exhibiting coherent kinematical behaviour. Cluster-based modelling is subsequently employed to derive a local model of the dynamics of each subdomain. This method is referred to as space-time cluster-based network modelling. The framework is validated with particle image velocimetry measurements of an experimental jet flow in water. The results show the ability of the proposed framework to identify regions such as vortex shedding and vortex pairing, which global models do not clearly capture. Consequently, the resulting compound models reproduce the dynamics of a multi-regime flow case with a small number of degrees of freedom as compared to global methods, while maintaining physical interpretability.

1. Introduction

The rapid development of experimental measuring techniques has significantly increased the ability to measure complex and turbulent flow fields with high spatial and temporal resolution. Techniques such as particle image velocimetry (PIV) or particle tracking velocimetry (PTV) enable the acquisition of full flow field velocity measurements containing millions of vectors, resulting in highly dimensional datasets. Thus, the extraction of meaningful and physically interpretable information from these measurements entails a challenge. The continuous advancements of measurement techniques is opening new pathways to develop models that can effectively uncover and represent the underlying flow dynamics. Addressing this challenge remains a central objective in experimental fluid mechanics.

The availability of interpretable dynamical models for complex flows is essential for flow control and design applications. The development of reduced-order models (ROMs) remains particularly

challenging for flows characterised by multiple interacting spatial and temporal scales, strong nonlinearities, and the coexistence of distinct dynamical regimes. Traditional analysis tools such as Proper Orthogonal Decomposition (POD) (Lumley, 1967; Sirovich, 1987) provide a global statistical description of the flow to identify coherent structures in terms of energy content. The temporal POD modes have often been used to identify ROMs. However, in highly nonlinear or turbulent flows, energy is distributed across a large number of modes, reducing both the compactness and interpretability of the resulting representation.

Projection-based ROMs based on the Navier-Stokes equations constitute one of the most established approaches for flow modelling. These methods project the governing equations onto a reduced basis obtained from POD or related decompositions (Rempfer & Fasel, 1994; Holmes, 2012; Noack et al., 2011). Their main limitation is the reliance on a global low-dimensional representation. Complex flows typically present multiscale behaviour, thus requiring a large number of modes in the model to accurately capture the dynamics, leading to increased model complexity, computational cost, and reduced physical interpretability. More recently, cluster-based network models (Kaiser et al., 2014; Li et al., 2021; Fernex et al., 2021; Deng et al., 2022; Hou et al., 2022, 2024) have been proven to identify quasi-attractors and transition processes by clustering the trajectory in time. Nonetheless, all the methods above rely on the identification of global models, which have limited flexibility and are not capable of resolving the dynamics of coherent structures that are localised.

Alternative approaches that emphasise dynamical content, such as dynamic mode decomposition (DMD) (Schmid, 2010), Spectral Proper Orthogonal Decomposition (SPOD) (Towne et al., 2018), and multiscale POD (Mendez et al., 2019), provide additional information regarding the temporal scales and frequencies. However, they remain fundamentally global descriptions of the flow and do not naturally capture spatially localised dynamical processes.

Partitioning the data into spatial subsets and constructing local models has emerged as a promising alternative. Colanera & Magri (2025) proposed a phase-space partitioning strategy, which is combined with intrusive methods. Geelen & Willcox (2022) introduced a non-intrusive framework based on clustering the temporal history of flow snapshots. Furthermore, recent studies (Farcas et al., 2024; Xu et al., 2025) have shown that spatially local models have better generalisation capabilities compared with global approaches. Additionally, domain partitioning strategies have long been employed in computational fluid dynamics to reduce the computational cost and improve scalability. Recent advances in manifold learning offer new opportunities based on data-driven partitioning, i.e., without having to necessarily assume a prior, which may bias the results.

Under the manifold hypothesis, high-dimensional flow measurements are assumed to lie on a lower-dimensional nonlinear manifold that captures the intrinsic organisation of the flow dynamics. Techniques such as isometric mapping (ISOMAP) (Tenenbaum et al., 2000) preserve geodesic distances and can reveal hidden structures that are not apparent in the original measurement

space. These methods have recently shown considerable promise for analysing experimental flow datasets and identifying coherent dynamical behaviour from high-dimensional observations (Farzaminik et al., 2023; Marra et al., 2024).

In this work, we present a data-driven framework for the data-driven analysis and modelling of experimental flow fields based on manifold learning and local dynamical decomposition. The methodology first constructs a spatial manifold from non-time-resolved measurements, enabling the identification of dynamically coherent subdomains through unsupervised clustering. These subdomains provide an automated partition of the measurement domain based solely on the observed flow dynamics. Subsequently, local reduced-order models are identified within each region, yielding a compact description of the global flow evolution.

The proposed methodology serves two main complementary purposes. First, it provides an objective tool for extracting coherent regions and dynamical interactions from large measurement datasets. Second, it generates low-dimensional representations that can be employed for flow reconstruction, reduced-order prediction, and future data-assimilation applications. The framework is demonstrated using an experimental dataset, consisting of the time-resolved PIV measurements of a transitional water jet flow. The results show that the proposed approach identifies physically meaningful flow regions, isolates local dynamical mechanisms, and provides compact representations capable of reproducing the dominant behaviour of complex flows based on the dominant features in each subdomain.

2. Methodology

The proposed local cluster-based model takes as input non-time-resolved and spatially-resolved snapshot data from experimental measurements, for example, PIV, or from direct numerical simulations. Figure 1 illustrates the overall methodology. First, the flow field snapshots are partitioned through a clustering strategy based on similarity of the local kinematics, resulting in the identification of spatially coherent subdomains. Subsequently, for each subdomain, a local cluster-based network model (CNM) is constructed for each subdomain, yielding a set of local Markov models that describe the dynamics within each region.

2.1. Kinematic partitioning

The flow domain is partitioned into subdomains to identify regions exhibiting similar local kinematic behaviour. Such spatial partitioning is done in a feature space, hereafter referred to as the *spatial manifold*, where spatial locations with similar kinematics are mapped close to one another. The manifold is constructed by applying ISOMAP to the vorticity fields denoted by $W \in \mathbb{R}^{N_s \times N_t}$, where N_s is the number of grid points and N_t is the number of snapshots. ISOMAP consists of a

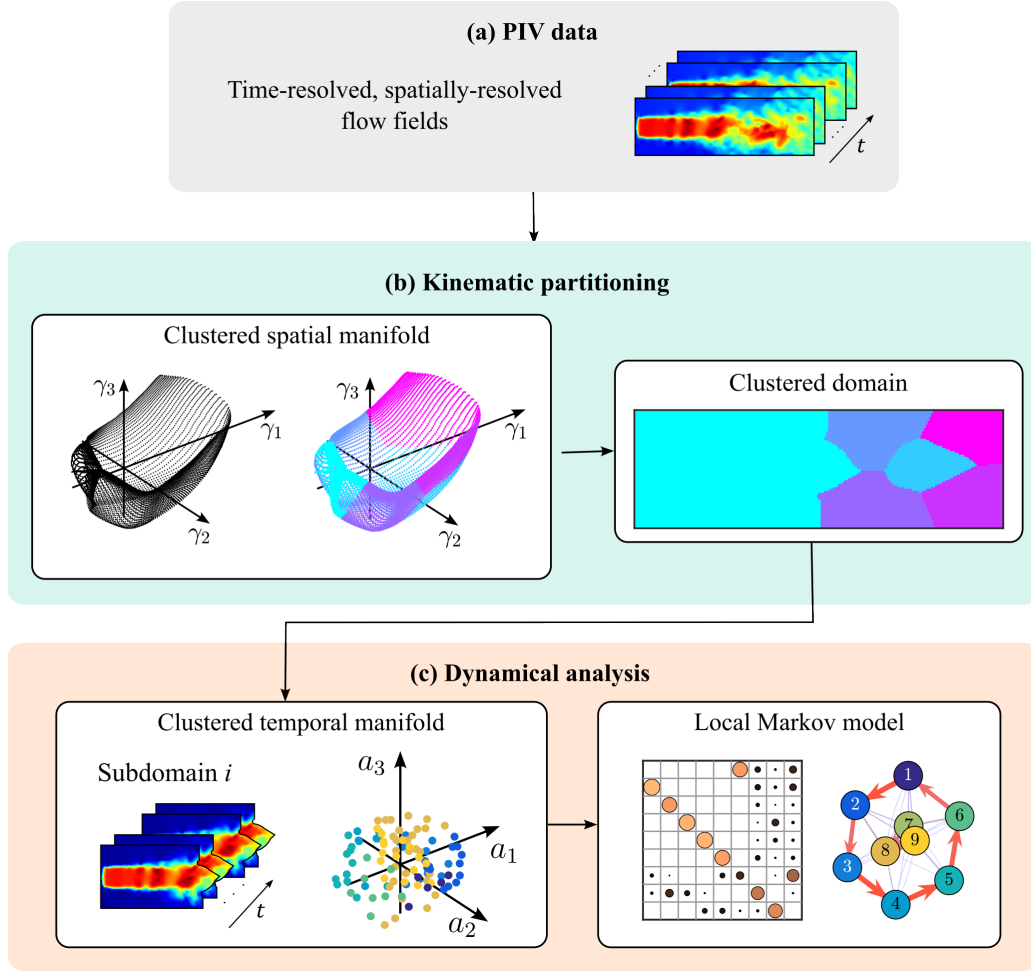


Figure 1. Schematics of the local cluster-based model methodology. (a) The input of the model is time-resolved velocity field snapshot measurements. (b) Domain partitioning into kinematically coherent subdomains by clustering of the *spatial manifold* spanned by the γ_i manifold coordinates. (c) Cluster-based dynamic models are obtained for each subdomain i , where a_i are the temporal modes of the flow fields in each subdomain i .

nonlinear data decomposition technique that extracts a low-dimensional manifold to describe the dataset while preserving the geodesic distance between data points. Additional details regarding its application to fluid-flow datasets can be found in Farzamn et al. (2023); Marra et al. (2024). The number of coordinates retained for the low-dimensional representation is selected based on the minimal residual variance R_v , defined as:

$$R_v = 1 - R^2(\text{vec}(\mathbf{D}_G), \text{vec}(\mathbf{D}_n)), \quad (1)$$

where $R(\cdot, \cdot)$ is the correlation coefficient, $\text{vec}(\cdot)$ is the vectorisation operator, and $\mathbf{D}_G$ and $\mathbf{D}_n$ are the matrices of geodesic and Euclidean distances respectively. The geodesic distance is the distance on the manifold, as approximated by the ISOMAP algorithm.

The flow domain is partitioned into subdomains by treating each spatial point as an observation and its corresponding time series as the feature vector. The partitioning is done using the

k -means++ algorithm (Arthur & Vassilvitskii, 2007) which identifies N_c subdomains within the spatial manifold. The optimal number of subdomains is determined automatically through a two-line fit (TLF) method as proposed by Brindise & Vlachos (2017). This approach evaluates a broad range of candidate values of N_c and selects the one corresponding to the breakpoint that best approximates the explained-variance curve with two linear segments, thereby balancing model complexity and clustering performance. The explained variance for N_c subdomains is computed as:

$$E_v = 1 - \frac{WCSS(N_c)}{WCSS(1)}, \quad (2)$$

with $WCSS(N_c)$ being the within-cluster sum of squares as a function of N_c clusters:

$$WCSS(N_c) = \sum_{k=1}^{N_c} \frac{1}{|\mathcal{S}_k|} \sum_{\mathbf{z} \in \mathcal{S}_k} \|\mathbf{z} - \mathbf{c}_k\|^2, \quad (3)$$

where $\mathbf{c}_k$ is the centroid of subdomain $\mathcal{S}_k$.

2.2. Dynamical analysis

The dynamics of each of the subdomains are analysed through CNMs (Li et al., 2021). First, the snapshot sequences are clustered in time. Then, for each subdomain, the direct transition probability matrix, $\mathbf{Q}$, and the transition time matrix, $\mathbf{T}$, are subsequently computed. In the temporal clustering stage, the observations correspond to the N_t snapshots and the features are the temporal POD coefficients a_i of each subdomain. For subdomain i , the dimension of the feature space is $2 \times N_{s,i}$. The resulting temporal clusters are referred to as subsequences and denoted $\mathcal{C}_i$.

The transition probability matrix $\mathbf{Q}$ and the transition time matrix $\mathbf{T}$ are computed as:

$$\mathbf{Q}_{ij} = \frac{n_{ij}}{n_j}, \quad (4)$$

where n_{ij} is the number of snapshots belonging to cluster $\mathcal{C}_j$ followed by snapshots in cluster $\mathcal{C}_i$ in the original time sequence. The normalisation factor n_j corresponds to the total number of departures from cluster $\mathcal{C}_j$. The mean transition time from $\mathcal{C}_j$ to $\mathcal{C}_i$ is given by:

$$T_{ij} = \left\langle \frac{\tau_i + \tau_j}{2} \right\rangle, \quad (5)$$

where τ_j is the residence time associated with cluster $\mathcal{C}_j$, and $\langle \cdot \rangle$ denotes the ensemble average of the time over all observed transitions between $\mathcal{C}_j$ to $\mathcal{C}_i$. A synthetic flow-field sequence is reconstructed by sampling a sequence of clusters from the transition probability matrix, yielding the cluster trajectory $(\mathbf{c}_{k_0}, \mathbf{c}_{k_1}, \mathbf{c}_{k_2}, \dots)$, where k_n denotes the index of the temporal cluster at step n . The

associated time stamps are obtained from the transition time matrix such that $t_0 = 0$, $t_1 = T_{k_1 k_0}$, $t_2 = t_1 + T_{k_2 k_1}$, and so forth.

For $t \in [t_n, t_{n+1}]$, the reconstructed velocity field is obtained through linear interpolation between consecutive cluster centroids,

$$\hat{\mathbf{u}}(\mathbf{x}, t) = \alpha_n \mathbf{c}_{k_n}(\mathbf{x}) + (1 - \alpha_n(t)) \mathbf{c}_{k_{n+1}}(\mathbf{x}), \quad \alpha_n = \frac{t_{n+1} - t}{t_{n+1} - t_n} \quad (6)$$

The quality of the reconstruction is assessed by comparing the autocorrelation functions of the reference and reconstructed velocity fields (Protas et al., 2015). The autocorrelation function is defined as:

$$R(\tau) = \frac{1}{T - \tau} \int_{\tau}^T \int_{S_i} \mathbf{u}(\mathbf{x}, t - \tau) \cdot \mathbf{u}(\mathbf{x}, t) d\mathbf{x} dt, \quad \tau \in [0, T] \quad (7)$$

3. Cluster-based modelling of PIV measurements in a jet flow

The method is evaluated on an experimental dataset of planar PIV measurements of a transitional jet flow (see figure 2). The experiment is carried out in the water tank facility of the Aerospace Engineering department at the Universidad Carlos III of Madrid. The jet nozzle diameter is $D = 0.03\text{m}$ and is run at a $Re = 3300$ based on the bulk velocity $U_b = 0.11\text{m/s}$ measured at the exit of the nozzle. Velocity fields are captured with an Andor Zyla sCMOS camera, which has a sensor resolution of $2160 \times 2560\text{px}$ (pixel size of $6.5\mu\text{m}$). The seeding used for the measurements is $56\mu\text{m}$ polyamide particles, illuminated with a 5W laser. The field of view spans $8D$ streamwise in the direction of the jet flow, and $2.6D$ in the vertical direction, perpendicular to the direction of the jet flow ($x/D \in [0, 8]$ and $y/D \in [-1.3, 1.3]$ centred at the jet nozzle exit and the centreline of the jet, respectively).

A set of 20000 snapshots with a time separation of $\Delta t = 0.011\text{s}$ is used to validate the method. The snapshots are processed with a multi-frame approach based on the 5-frame stencil proposed by Scarano & Moore (2010). A three-pass cross-correlation analysis is applied with a final interrogation window of $32 \times 32\text{px}$ with a 75% overlap. Then, a second-order polynomial Savitzky-Golay filter with a $5 \times 5 \times 5$ kernel in space and time is used to reduce the measurement noise. The dataset is subsampled by a factor of 4 to alleviate the computational load while ensuring statistical convergence.

Figure 3 shows the results of the partitioned domain of the jet flow, computed with 6 spatial clusters, corresponding to the optimal number of subdomains according to the two-line fit method. It is possible to distinguish two main regions. The first is the initial region, which spans the length of the potential core (subdomain S_1). The second region consists of several clusters (subdomains S_2, S_3, S_4, S_5, S_6) and corresponds to the transitional region.

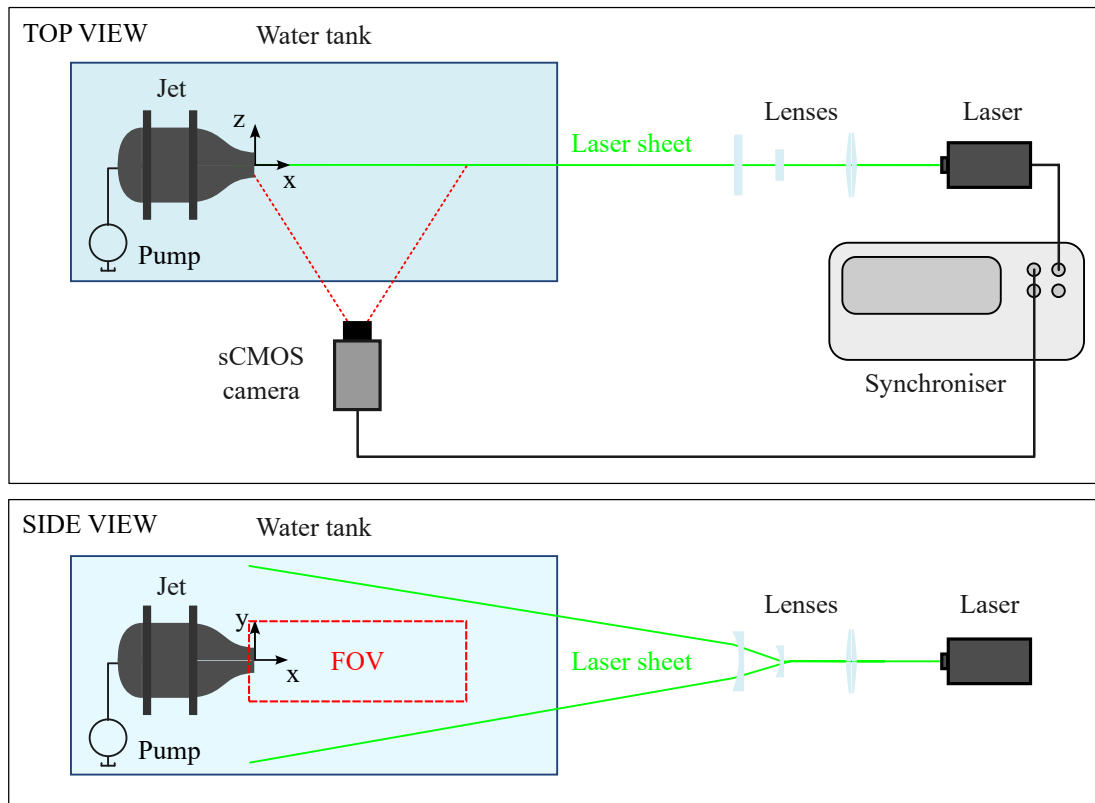


Figure 2. Schematics of the planar PIV setup used for the water jet flow experiment. The top and front views of the experimental arrangement are shown from top to bottom. The laser sheet generated by the optical system is indicated in green, while the PIV camera field of view is highlighted in red. Figure adapted from Franceschelli et al. (2025), under license CC BY 4.0.

The regions obtained from the partitioning are well established in the literature for submerged free round jets (Abramovich et al., 1984; Yule, 1978). The initial region spans along the potential core, where the centreline velocity remains approximately equal to the nozzle exit velocity due to the limited penetration of the surrounding shear layer. The potential core typically extends over approximately $4 - 6D$ for this type of jet flow configuration, while the shear layer grows inwards from the nozzle lip towards the jet centreline.

Around the potential core, a mixing layer originates at the nozzle edge as a result of the strong velocity gradient between the jet and the quiescent ambient fluid. This velocity difference generates Kelvin-Helmholtz instabilities that roll up into coherent vortical structures, forming vortex rings in the case of an axisymmetric jet. These structures promote turbulent mixing and entrainment of ambient fluid into the jet, causing the shear layer to thicken and progressively reduce the potential core until it vanishes downstream. Note that due to the strong dynamic coupling between the potential core and the shear layer, these regions are identified as a unique subdomain.

Moreover, beyond the initial region, the transitional regions generally extend over the following $5 - 10D$. Within these regions, partially observed in our field of view, the jet continues evolving

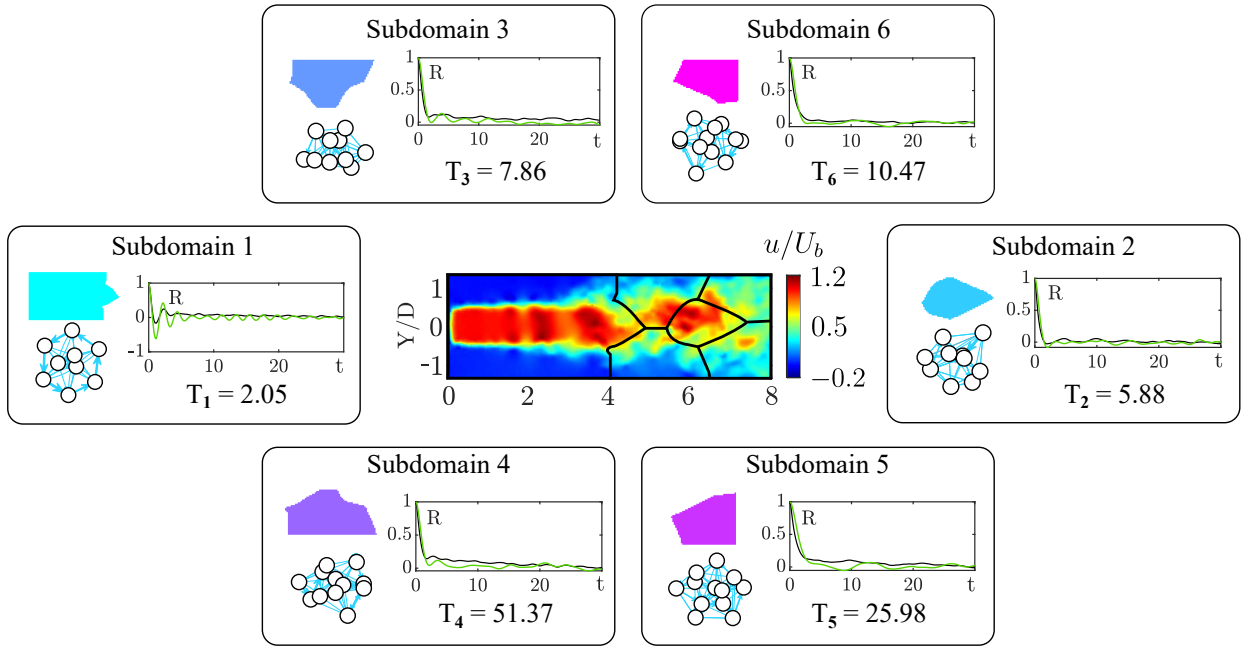


Figure 3. Spatial clustering of the jet flow measurements, partitioned into 6 subdomains. In the centre, a sample normalised velocity field with the boundaries of the subdomains. Each subdomain presents the local cluster-based network model represented in terms of the centroids (nodes), the autocorrelation functions versus time nondimensionalised with the convective time (in black, line the original data, and in green line the reconstructed data), and the time of the first zero of the autocorrelation function of the reconstruction.

towards self-similarity, although the velocity profiles have not yet reached a fully developed state. Therefore, the transitional region acts as an adjustment layer between the potential-core region and the fully developed jet. Distinct clusters can be identified along the streamwise direction, each corresponding to a different stage of turbulent mixing and momentum redistribution. Note that, as the flow evolves downstream, large coherent structures progressively break down into smaller scales. Thus, increasing the number of clusters can yield finer partitioning of this region, allowing these stages of flow development and mixing to be captured in greater detail.

For each of the aforementioned regions, the corresponding spatial domain is depicted with the temporal cluster network and the comparison of the autocorrelation functions of the original data and the reconstructed data (see figure 3).

The results show that subdomains S_1 and S_2 retrieve similar periods (first zero crossing points), meaning that the reconstruction of the dynamics in both subdomains is reliable. For brevity, only subdomain S_1 is further analysed. Figure 4 presents the results for the subdomain S_1 . The probability transition matrix reveals two distinct dominant cycles: $C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4 \rightarrow C_5 \rightarrow C_6$ and $C_7 \rightarrow C_8 \rightarrow C_9$ with periods $T_{1,1} = 2.29$ and $T_{1,2} = 1.84$ (non-dimensionalised with a characteristic time based on the bulk velocity and the jet diameter), respectively. The first one, characterised by the presence of two large vortical structures (with a size of the order of half a jet diameter), whereas the second cycle shows three smaller vortices. Both cycles are associated with the vortex shedding

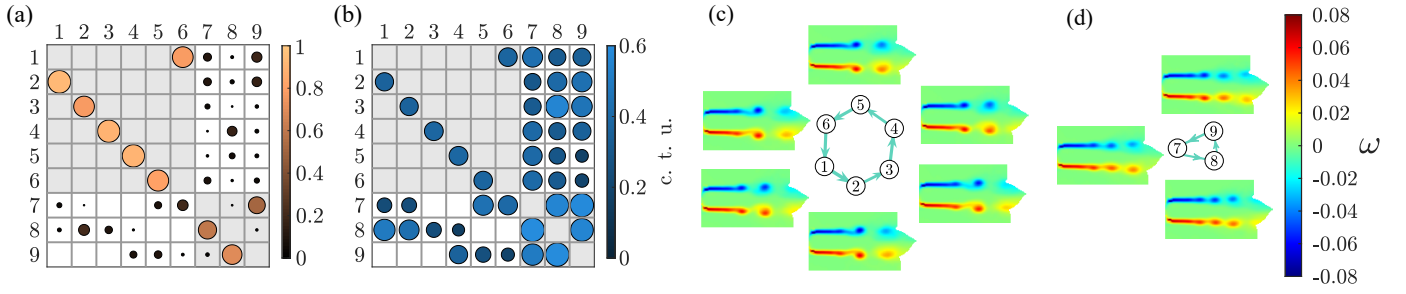


Figure 4. Cluster-based network model of subdomain $\mathcal{S}_1$. a) Transition probability matrix, highlighting the existence of two periodic trajectories ($\mathcal{C}_1 \rightarrow \mathcal{C}_2 \rightarrow \mathcal{C}_3 \rightarrow \mathcal{C}_4 \rightarrow \mathcal{C}_5 \rightarrow \mathcal{C}_6$ and $\mathcal{C}_7 \rightarrow \mathcal{C}_8 \rightarrow \mathcal{C}_9$). b) Transition time matrix in convective time units (c.t.u.). c,d) Periodic trajectories together with the corresponding nondimensional vorticity snapshots of the centroid state (mean). The first trajectory has a period $T_1 = 2.29$ and the second has a period $T_2 = 1.84$.

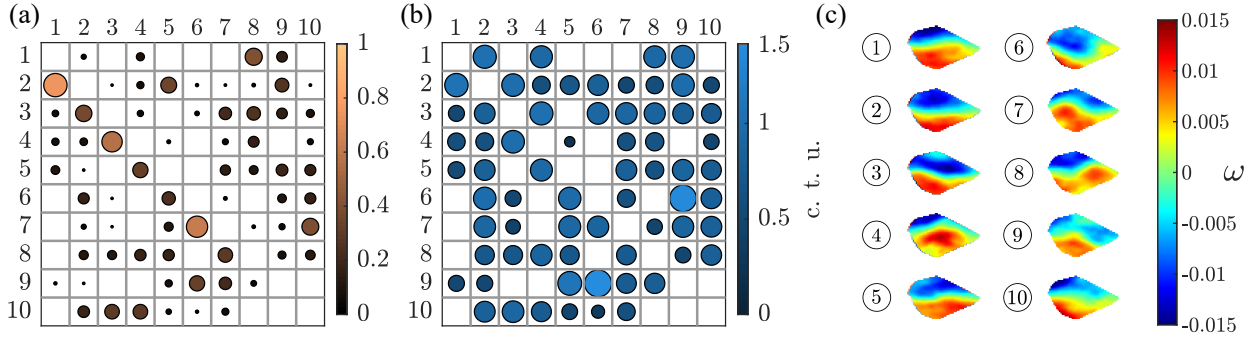


Figure 5. Cluster-based network model of subdomain $\mathcal{S}_2$. a) Transition probability matrix. b) Transition time matrix normalised in convective time units (c.t.u.). b) Shows the transition time matrix in convective time units (c. t. u.). c) Nondimensional vorticity snapshots of the centroid states (mean).

process at the jet nozzle exit; however, they capture different stages of its dynamics. In particular, the second cycle corresponds to the primary vortex shedding mechanism, while the first cycle is linked to the subsequent vortex pairing process.

The results for subdomain $\mathcal{S}_2$ are represented in figure 5, where a total of 10 subdomains are obtained. Different from subdomain $\mathcal{S}_1$, in this case, there is no clear dominant periodic trajectory but, instead, multiple interconnected cycles can be identified. This subdomain exhibits a switching and oscillatory behaviour, as illustrated by the vorticity fields of the centroid states. This switching mechanism is mainly driven by the interaction between the vortical structures generated along the centreline and the three-dimensional dynamics. Thus, the results show a recurrent temporal pattern that has the possibility of following different trajectories, corresponding to distinct cyclic sequences.

Note that in a global analysis, such an interpretation of the dynamics cannot be directly retrieved due to the lack of resolution of the prohibitively large number of modes required for the reconstruction of the dynamics.

4. Conclusions

The present work proposes a novel method for the identification and interpretation of local dynamics that global methods are not able to extract. The proposed local cluster-based network models receive as input flow field velocity snapshots. Then, the method obtains the so-called *spatial manifold* over which the spatial partitioning is computed. The spatial manifold is computed with ISOMAP of the vorticity fields using the temporal history as features. Then k -means is employed to cluster the domain, where the optimal number of subdomains is selected through a two-line fitting method. For each of the subdomains, a local cluster-based network model is computed, which reconstructs the corresponding dynamics. The method is evaluated on a PIV experiment of a jet flow, and results show that the method can retrieve local interpretations of the flow dynamics that a global analysis would not be able to describe.

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