

On the estimation of the skin friction topology and magnitude from surface temperature by TSP

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ABSTRACT

A strong interdependence exists between the thermal and momentum boundary layers. This so-called Reynolds Analogy establishes a connection between heat transfer and fluid friction in bounded flows, asserting that the same flow-induced structures govern both processes. As a direct consequence, the behavior of the flow over a slightly heated surface can be inferred from the evolution of the wall temperature field, $T_w(x, z, t)$, where local temperature gradients are typically correlated with the distribution of wall shear stress. However, this assumption may induce misleading results in the experimental analysis of boundary layers, e.g., the detection of flow separation and reattachment, which do not necessarily occur at the minima and maxima of the time-averaged wall temperature: in fact, this kind of distribution can also originate as the consequence of other flow dynamical phenomena, such as laminar-turbulent transition. Such effects complicate the interpretation of thermal measurements and may lead to misinterpretations or spurious correlations. This misinterpretation can be avoided by considering the celerity of displacement of thermal perturbations of $T'(x, y, t) = T_w(x, y, t) - \bar{T}_w(x, y)$ (blobs) under the influence of τ_w . This representation does not rely, at least to a first-order approximation, on spatial temperature gradients $\partial T / \partial x_i$.

Recent advances in Temperature-Sensitive Paint (TSP) applications have greatly enhanced this technology, enabling non-intrusive measurements of the surface temperature distribution T_w with high-resolution in both space and time. This technique offers an innovative and visually compelling means of investigating near-wall fluid dynamical processes, allowing instantaneous observation of complex flow phenomena and their evolution.

Given this scenario, the present study deepens the relevance of using TSP in advancing the understanding of flows undergoing separation and, in general, of skin friction exerted by the flow on a bounding surface, focusing on a TSP-based diagnostics of the flow before and after a Backward Facing Ramp (BFR). A qualitative, at-a-glance interpretation of the surface flow by T'_w maps will be followed by the estimation of the $\kappa - f$ spectrum. The wall shear stress will be estimated using a state-of-the-art skin friction reconstruction algorithm based on the close correlation between the celerity of displacement of the temperature blobs and the friction velocity U_τ , as proposed in Miozzi et al. (2020, 2024) (TH method). Results will be compared with Shake-the-Box data regarding the identified reattachment location.

1. Introduction

There are at least two fundamental connections between the friction acting on the surface of a body—slightly warmer or cooler—and the temperature distribution on that surface when exposed to an incompressible flow.

The first stems from the energy equation, which links the evolution of wall temperature gradients to the influence of the skin-friction field. The second arises from the passive transport equation for temperature perturbations at the wall, revealing a direct relationship between the propagation celerity of thermal disturbances and the friction velocity.

The energy-equation-based approach relies on the empirical relationship, so-called Reynolds analogy, whose role is to fill the gap between the energy-equation itself and the momentum equation. An adapted version of this analogy (Reynolds-Colburn) has been used, for example, by Lemarechal et al. (2021) to determine the time-averaged skin friction field from TSP data in a laminar water channel, while its differential form, solved by Liu & Woodiga (2011) in analogy with the optical flow equation, has been applied, among the others, in Miozzi et al. (2016) to estimate instantaneous, normalized skin friction fields around a cylinder. In non-canonical flows, however, the Reynolds analogy fails, and methodologies relying on it cannot be applied.

One underlying assumption in applying these relationships is that all temperature variations, along with their associated gradients, are exclusively attributable to the shear stress exerted by the boundary layer—whether laminar or turbulent—on the body's surface. In practice, this assumption is generally not valid, and its experimental implementation faces significant challenges even under seemingly simple test conditions.

A more general approach, based on time-resolved surface temperature maps, has recently been developed and applied to determine skin-friction distributions in water flows over a hydrofoil in crossflow (Miozzi et al., 2019, 2020), a hydrofoil in the wake of a marine propeller (Miozzi & Costantini, 2021), and a flat plate with a wall-mounted cube (Miozzi et al., 2024). This method relies on observations of the behavior at the wall of scalar quantity fluctuations, temperature in our case, which is assumed to act as a passive scalar.

At the wall, the no-slip condition holds. The propagation velocities of T' blobs are referred to as celerities, because they displace information without material displacement, in analogy with wave motion. The propagation celerities of these fluctuations are related to the friction velocity (Hetsroni et al., 2004). In canonical flows, this relationship between at-the-wall propagation celerities and friction velocity is reported to be linear, whereas in non-canonical flows, it remains under investigation. If the relationship between friction velocity and the propagation celerity of temperature fluctuations at the wall is known, skin friction can be inferred from surface temperature measurements.

In the approach proposed in the aforementioned studies, the time-averaged propagation celerity of temperature fluctuations (U_T) is obtained from time-resolved TSP data by minimizing the discrepancy between the observed surface temperature evolution and that predicted by the Taylor hypothesis of “frozen turbulence”, following an analogy with the method introduced by (Del Álamo & Jiménez, 2009) for velocity fluctuation celerities. Applications of this celerity-based approach compared to the better-established Shake-the-Box (STB) technique (Schanz et al., 2016) can be found in Miozzi et al. (2024) to study the flow around a cube immersed in a turbulent boundary layer and, more recently, in Costantini et al. (submitted) for a turbulent boundary layer over a flat-plate configuration, which undergoes separation and reattachment as induced by a 25° backward-facing ramp. The examined configuration, here considered in deeper detail, is sketched in Figure 1. The experiment was performed at TU Braunschweig in the GWB water tunnel within the DLR project ADaMant at Reynolds numbers based on the ramp location x_h (measured from the model leading edge) in the range $Re_{x_h} = 1.2 - 2.7 \cdot 10^6$. The Europium-based TSP from Ondrus et al. (2015) was used in this case. A snapshot of surface temperature fluctuations (i.e., defined as the difference between instantaneous temperature maps and the time-averaged temperature map of the data series) measured via TSP at $Re_{x_h} = 1.2 \cdot 10^6$ is shown in Figure 2.

The associated time series has been used to feed the TH algorithm and excerpt (U_T). Example results are reported in Figure 2, where the topology of the temperature celerity overlaps the snapshot of T' , and in Figure 3 (top), where the profile of the spanwise-average U_T streamwise component is plotted for the region downstream of the ramp. Outcome from the TH algorithm clearly depicts a recirculation bubble, extending up to about $X = 40$ mm. This result is in interesting agreement with the data obtained via Lagrangian particle tracking by Shake-The-Box acquired in a companion experimental campaign (Schanz et al., 2025; Costantini et al., submitted). The focus of the final paper will describe in closer details the main features of the TH algorithm, its multiscale capabilities, and the possibility of a time-resolved extension, with the aim of providing a cooperative vision of the flow behavior close to the wall by integrating the TSP and the STB results.

2. Skin-friction from temperature fluctuations

2.0.1. The structure of the viscous sublayer

Eckelmann (1974) was the first to identify the existence of a *surface flow* of velocity fluctuations in a turbulent boundary layer. Using a dedicated oil-tank facility, he measured the fluctuating wall-normal gradient of the streamwise velocity $\partial U(Y, t) / \partial Y|_{wall}$ together with the velocity fluctuations $u(Y, t)$ at several wall distances Y . For increasing non-dimensional wall distances $Y^+ = YU\tau/\nu$, he observed nearly identical temporal behaviour up to $Y^+ < 10$, followed by progressive phase separation farther from the wall (see Eckelmann, 1974, Fig. 6). Although the mean velocity must vanish monotonically as $Y \rightarrow 0$ due to the no-slip condition, the velocity fluctuations prop-

agate as a wave moving faster than the surrounding flow. We therefore adopt the term *celerity* to denote the propagation speed U_q of fluctuations q of a generic passive scalar $Q = \bar{Q} + q$, like velocity, vorticity, temperature, and pressure. Using DNS of channel flow, Kim & Hussain (1993) examined the propagation celerity of velocity and vorticity fluctuations. From the space–time correlation peak, they demonstrated that for $Y^+ < 10$ the celerity of velocity disturbances scales linearly with the friction velocity:

$$\mathbf{U}_u = c_u \cdot \mathbf{U}\tau, \quad (1)$$

with $c_u = 9 \div 10$ (Kim & Hussain, 1993; Geng et al., 2015). Whether c_u is a scalar or depends on local flow conditions (e.g. Reynolds number, pressure gradient, etc.) remains an open question (Liu et al., 2024). Kim & Hussain (1993) also showed that U_u exhibits a scale dependence near the wall ($Y^+ < 10$), reflecting the influence of multi-scale coherent structures convected at different speeds by the local mean flow $U(Y)$. Since a complete description requires the dispersion relation in the (k, ω) plane, Del Álamo & Jiménez (2009) proposed a physically based method to estimate U_u by identifying the celerity that minimizes the mismatch between measurements and a generalized, relaxed form of Taylor’s hypothesis. This approach extends the applicability of the Taylor hypothesis even in regions close to the wall, as discussed in the next section.

2.0.2. Relaxation of the Taylor Hypothesis

In its classical form (Taylor, 1938), the Taylor Hypothesis assumes that turbulent structures are advected past a sensor without significant evolution, provided that the advection velocity is much greater than turbulent fluctuations. For a fluctuating quantity q , this yields

$$\frac{Dq}{Dt} = \frac{\partial q}{\partial t} + \mathbf{U}_q \cdot \nabla q = 0, \quad (2)$$

with $\mathbf{U}_q$ typically approximated by the local mean velocity. This assumption breaks down near the wall where shear production is strong, and the condition $|u/U| \ll 1$ is violated. Del Álamo & Jiménez (2009) relaxed the frozen-field assumption by defining $\mathbf{U}_q$ as the celerity that minimizes the discrepancies between the ideal Taylor hypothesis and its practical behavior. By placing this discrepancy at a minimum ε , a standard minimization problem follows:

$$\varepsilon = \overline{(\partial q / \partial t + \mathbf{U}_q \cdot \nabla q)^2}, \quad \frac{\partial \varepsilon}{\partial U_i} = 0. \quad (3)$$

For orthogonal directions $i = 1, 2$ on the wall-tangent plane, this yields (Liu et al., 2024):

$$\begin{cases} U_1 = \frac{-\overline{(\partial_2)^2}, \overline{\partial_1 \partial_t} + \overline{\partial_2 \partial_t}, \overline{\partial_1 \partial_2}}{\overline{(\partial_1)^2}, \overline{(\partial_2)^2} - \overline{(\partial_1 \partial_2)^2}} \\ U_2 = \frac{-\overline{(\partial_1)^2}, \overline{\partial_2 \partial_t} + \overline{\partial_1 \partial_t}, \overline{\partial_1 \partial_2}}{\overline{(\partial_1)^2}, \overline{(\partial_2)^2} - \overline{(\partial_1 \partial_2)^2}} \end{cases} \quad (4)$$

If mixed derivative correlations are negligible ($\overline{\partial_1 \partial_2} \approx 0$), Eq. 4 simplifies to (Del Álamo & Jiménez, 2009):

$$\begin{cases} U_1 = -\overline{\partial_1 \partial_t} / (\overline{\partial_1})^2 \\ U_2 = -\overline{\partial_2 \partial_t} / (\overline{\partial_2})^2 \end{cases} \quad (5)$$

If direction 1 is homogeneous and streamwise, the result reduces further to:

$$\begin{cases} U_1 = -\overline{\partial_1 \partial_t} / (\overline{\partial_1})^2 \\ U_2 = 0 \end{cases} \quad (6)$$

2.0.3. From the celerity of temperature to that of velocity fluctuations

The linear relationship between U_u and U_τ (Eq. 1) motivates using Eqs. 4–6 to compute the celerity U_T of temperature fluctuations, and then relate U_T back to U_τ through U_u . Because momentum and thermal diffusivities approximately coincide only for $Pr = \nu/\alpha = 1$, this conversion requires accounting for their different diffusivities. Hetsroni et al. (2004) investigated a turbulent flume and provided a criterion linking temperature and velocity celerities through a Prandtl-number-dependent coefficient:

$$U_T^+ = c_T \cdot U_u^+ = c_T \cdot c_U \cdot U_\tau \quad (7)$$

$$c_T = Pr^{-1/3} \quad \text{for isothermal B.C.} \quad (8)$$

$$c_T = Pr^{-1/2} \quad \text{for constant heat flux B.C.} \quad (9)$$

2.0.4. From the celerity of velocity fluctuations to friction velocity

The coefficients c_u and c_T have been investigated mainly in canonical flows such as channel flows or boundary layers with zero or adverse pressure gradients. Dròzdź et al. (2023) recently examined the effect of adverse pressure gradients on U_u in a flat-plate boundary layer. Their analysis, based on two-point cross-correlation and several Clauser-type methods for U_τ , showed that for $\beta < 19$ the convective velocity U_u varies little below $Y^+ = 100$, whereas for stronger adverse pressure gradients ($\beta > 19$) it shows significant changes. To date, no general relationship between U_τ and U_u has been established for complex flows with both favorable and adverse pressure gradients and without assumptions such as those required in Clauser-plot approaches.

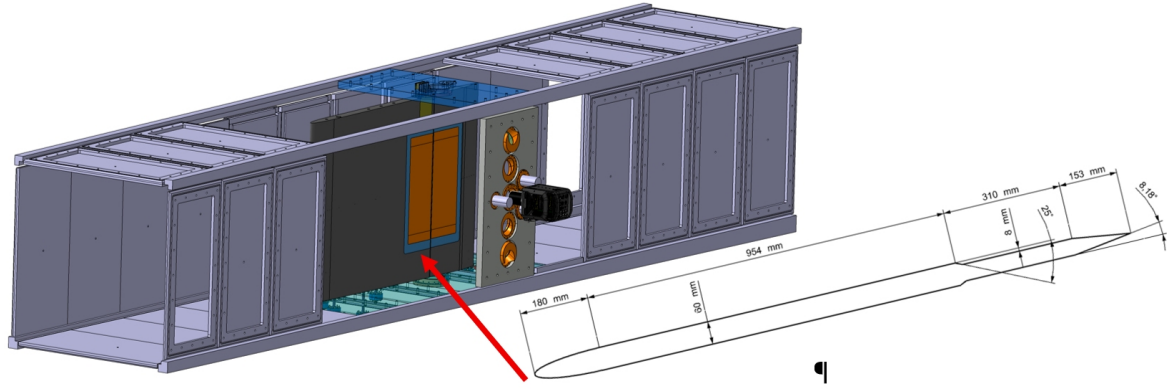


Figure 1. Snapshot of the fluctuating temperature. A detailed description of the surface flow is provided in a range of $\pm 0.2^\circ C$.

3. Application

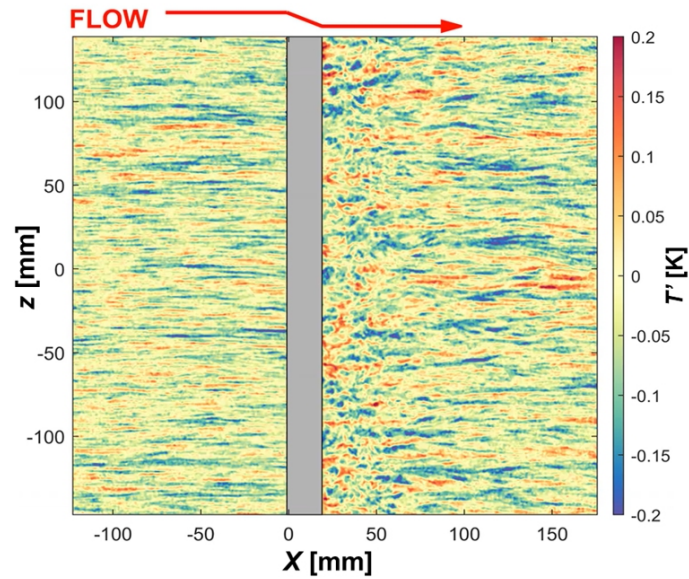


Figure 2. Snapshot of the fluctuating temperature. A detailed description of the surface flow is provided in a range of $\pm 0.2^\circ C$.

Applications of this celerity-based approach (referred to as the TH approach) in comparison with the well-established Shake-the-Box (STB) technique (Schanz et al., 2016) are reported in Miozzi et al. (2024) for studying the flow around a cube immersed in a turbulent boundary layer, and more recently in Costantini et al. (submitted) for a turbulent boundary layer over a flat plate undergoing separation and reattachment induced by a 25° backward-facing ramp. This latter configuration, here examined in greater detail, is illustrated in Figure 1. The experiment was conducted at TU Braunschweig in the GWB water tunnel within the DLR project ADaMant, at Reynolds numbers based on the ramp location x_h (measured from the model leading edge) in the range $Re(x_h) =$

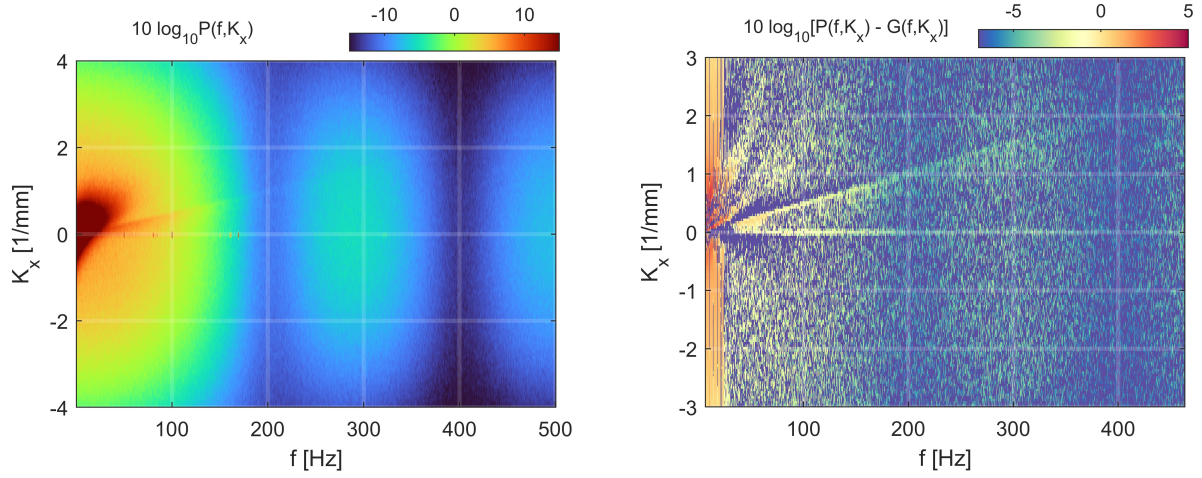


Figure 3. $\kappa - f$ spectrum of the temperature fluctuations in the region before the ramp. Left) Full spectrum. Right) Energy spectrum de-trended by a linear combination of five Gaussian bells. Apart from the noise-derived signal at $K_x = 0$, the evolution of the energy ridge peak is clearly visible.

1.2×10^6 to 2.7×10^6 . Europium-based TSP from Ondrus et al. (2015) was employed for surface temperature measurements.

Figure 2 shows a snapshot of surface temperature fluctuations—defined as the difference between instantaneous temperature maps and the time-averaged temperature map—measured via TSP at $Re(x_h) = 1.2 \times 10^6$. Results exposed in the following will report on this Reynolds number.

A first example of the capability of the fluctuating temperature field to capture the dynamics of the surface flow is provided by illustrating the scale dependency of the celerity of propagation of U_T . Figures 3 (left, right) report the $\kappa - f$ spectrum of T' in the region before the ramp.

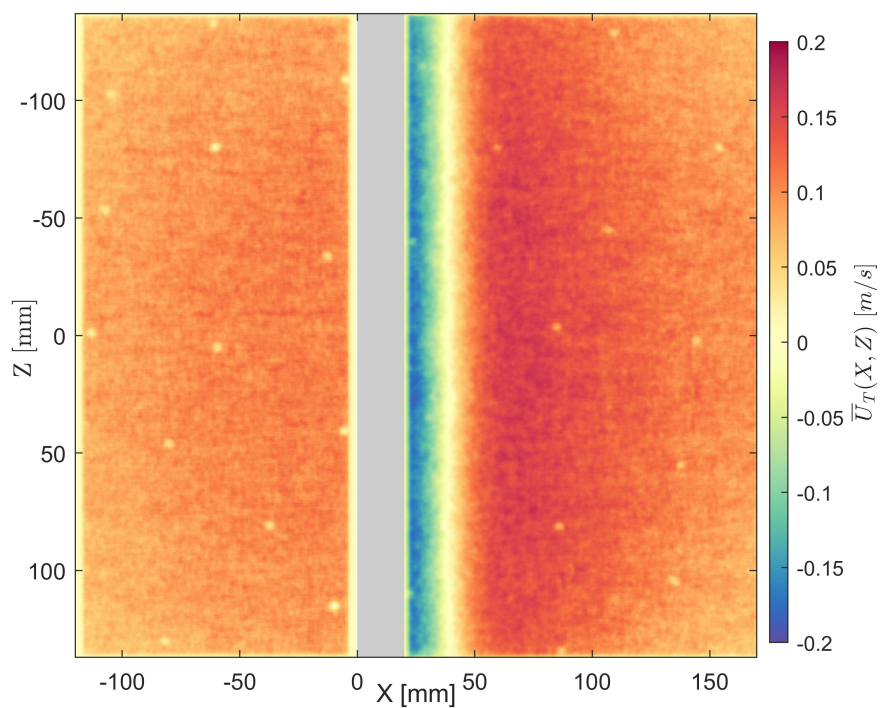


Figure 4. Streamwise, time averaged celerity of propagation $\bar{U}_T$.

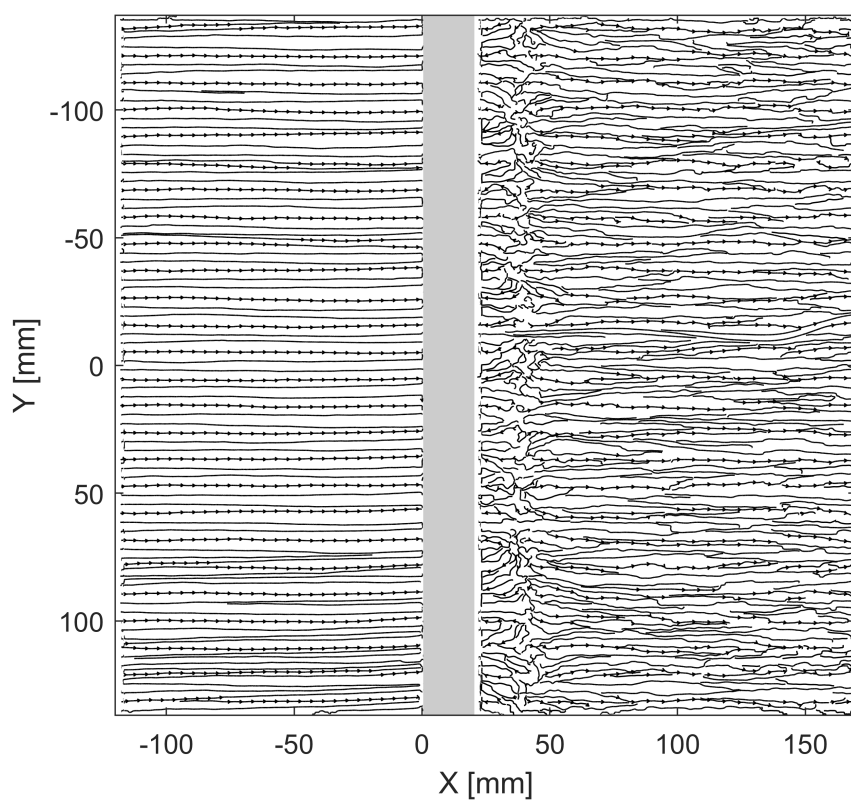


Figure 5. Topology of the time-averaged celerity of propagation vector $\bar{U}_T$

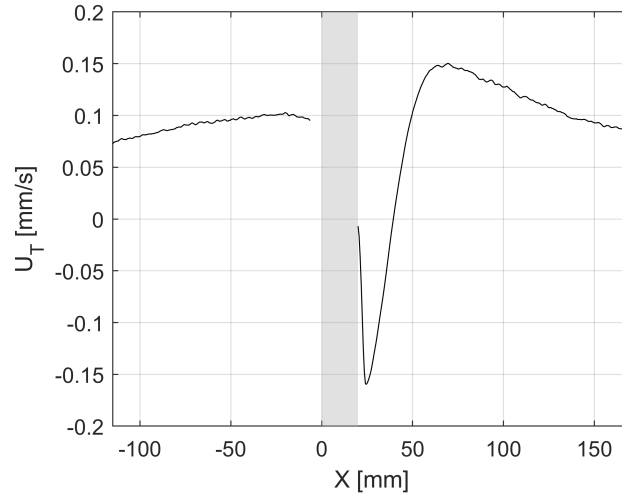


Figure 6. Evolution of the $\langle \bar{U}_T \rangle_Z$ versus X, spanwise and time averaged. The zero crossing of U_T corresponds to the reattachment location, and is in good agreement with the findings of the STB measurements.

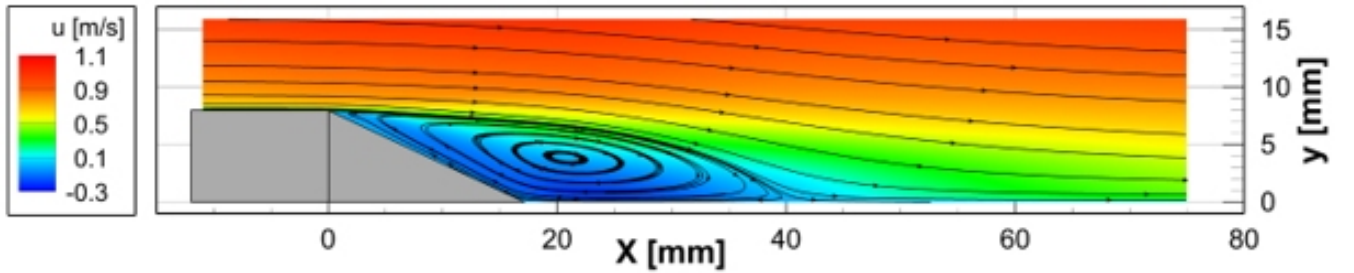


Figure 7. Time- and spanwise-averaged distributions of the streamwise velocity component from STB measurements. Streamlines based on the mean velocity vector field are also shown.

The corresponding time series was processed using the TH algorithm to extract the propagation celerity vector $\mathbf{U}_T$. The streamwise, time-averaged component $\bar{U}_X$ is reported in Figure 4, while Figure 5 shows the topology of $\bar{U}_T$ at the wall. The time and spanwise-averaged $\langle \bar{U}_T \rangle_Z$ profile along X is reported in Figure 6, showing a behavior in linear relationship with the friction coefficient C_f . The zero-crossing at $X \approx 40 \text{ mm}$ marks the change of sign of the skin friction at the wall. On these bases, the TH algorithm clearly identifies a recirculating bubble extending up to $X \approx 40 \text{ mm}$, in remarkable agreement with the recirculating bubble reported in Figure 7, obtained via Lagrangian particle tracking using STB during a companion experimental campaign (Schanz et al., 2025; Costantini et al., submitted).

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